Students' Relationships with Mathematics: Affect and Identity

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In this paper, an examination of students' relationships with mathematics is informed by affective research into internal mathematical structures and identity research into students' narratives. By analysing the perceptions of a class of 31 adolescents, five interacting elements emerged: students' views, feelings, mathematical knowledge, identities, and habits of engagement. These elements contributed to the context within which students engaged in mathematics and resulted in their unique learning experiences. This framework has potential for researching aspects of students' mathematical journeys and can be used by teachers to get to know individual students' unique connection to the subject of mathematics.

Introduction

A secondary school mathematics classroom is a physical space shared by a teacher and a group of students who have a set of shared norms. They generally work on the same mathematical tasks. Despite these similarities, students engage in mathematics in different ways. Some relish the experience, investigating and discussing further possibilities. Some, bored and restless, follow the necessary steps to get the task over with as quickly as they can. Some steel themselves to have a go, checking the answer frequently and feel lucky if they get it correct. Others avoid the situation by chatting socially or sharpening their pencil.

Students engage in mathematics in different ways because they have unique relationships with the subject. A *student's relationship with mathematics* is defined in this paper as the dynamic connections between the student and the subject of mathematics. This concept has strong links to notions of *mathematical self* or *self-identity* found in affective and identity research. This literature informed the examination of a group of students' relationships with mathematics. This paper reports specifically on these relationships as one aspect of a larger, longitudinal study (Ingram, 2011). The elements of these relationships are specified in this paper and the potential for using this framework in research and practice is explored.

Affect

Learning mathematics is an emotional practice that generates a range of affective responses. *Affect* describes the experience of feelings and emotions (McLeod, 1992). Research into affect in mathematics education explores these as well as other elements in the affective domain such as motivation, anxiety, engagement, attitudes, identity, and beliefs. These elements interact in complex ways and holistically researching across elements is valuable (Grootenboer, 2003).

One aspect of affective research in mathematics education is the conceptualisation of individuals having stable internal structures that relate to mathematics. These have been variously described as a *global affective structure* (DeBellis & Goldin, 2006), *self-system,* (Malmivuori, 2006), *mathematical disposition* (Op 't Eynde, De Corte, & Verschaffel, 2002), or *identity* (Op 't Eynde, De Corte, & Verschaffel, 2006). These structures generally contain the following elements:

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- *Beliefs about mathematics* which incorporate students' personal, internal and shared subjective conceptions about mathematics, mathematics teaching and learning, about themselves in relation to mathematics, and about the context (Malmivuori, 2006; Op 't Eynde et al., 2006);
- Related *goals and needs* related to autonomy, competency, and social belonging (Hannula, 2006);
- Other global affects such as *values and attitudes* (DeBellis & Goldin, 2006);
- *Mathematical content knowledge* such as the facts, symbols, concepts, and rules that constitute mathematics (Malmivuori, 2006). Strategies for accessing and using knowledge to solve problems (Op 't Eynde et al., 2006);
- *Meta-knowledge*, which involves knowledge about meta-cognitive functioning and knowledge about affect and its use (Malmivuori, 2006);
- *Habitual affective pathways* and behaviours in mathematics, including affective skills (DeBellis & Goldin, 2006).

These structures develop from students' previous experiences with mathematics in social environments (Malmivuori, 2006). They form part of the context within which students learn mathematics. When learning, students interpret the mathematical situation according to their internal structure. As a result, they experience a wide range of unique affective responses, which can be unstable, hot emotions, with accompanying physiological arousal such as anxiety or joy, or they can be less hot responses such as boredom or interest. These provide information for the individual about their progress towards their needs and related goals and may disrupt or distract the learning process and affect the level of capability while performing mathematics. This information activates self-appraisals, which thus determine how a student approaches the mathematical task, depending on their current level of awareness, control, and regulation capacities. These processes result in unique performances and new learning experiences. Students' interpretations of these experiences reinforce or, if sufficiently powerful or repeated often enough, alter these structures.

This research generally views students' learning as a product of individual cognitive processes and students are usually researched outside of a classroom context in problem solving situations, rather than within the social context of the mathematical classroom. Furthermore, there are few examples in the affective literature of students' perspectives of how their affect and learning are associated.

There has been some recognition of learning as a social process and connections made between affect and identity. Op 't Eynde et al. (2006) see learning as taking place through engagement in the language, rules, and practices that govern activities in the community of the mathematics classroom. They connect affect and identity:

> [Students'] understanding of and behaviour in the mathematics classroom is a function of the interplay between who they are (their identity), and the specific classroom context. Who they are, what they value, what matters to them in what way in this situation is revealed to them through their emotions" (p. 194).

The elements of a student's internal structure related to mathematics need to be viewed as both collectively and individually constituted through participation in the shared practices of the mathematics classroom. To understand better how students' learn mathematics, there seems to be potential in better understanding connections between the notions of a student having stable internal structures relating to mathematics and ideas of mathematical identity. It is these connections that are now explored.

Identity

Identity is variously seen in mathematics education research as how an individual names themselves and how they are looked on by others (Grootenboer, Smith, & Lowrie, 2006), self-concept (McFeetors & Mason, 2005), a performance (Darragh, 2014), or a narrative about a person (Kaasila, Hannula, Laine, & Pehkonen, 2005). Many researchers in mathematics education (e.g., Boaler, 2000; Op 't Eynde & Hannula, 2006) are informed by Wenger (1998) who defined identity as a constant becoming of who one is in a particular social context.

Sfard and Prusak (2005a, 2005b) take a dynamic view of identity powered by their investigation into the differences in mathematical learning processes between immigrant students from the Soviet Union and native Israelis. They dispute any process of defining identity as *who one is*, just as they reject notions of *God-given* personality, ethnicity, and nature; essentialist visions of identity, which "seem to be saying that there is a thing beyond one's actions that stays the same when the actions occur" (Sfard & Prusak, 2005b, p. 15). They developed a narrative approach to identity and see identity formation to be a form of communicational practice. In their view, identities are the stories that surround a person. "No, no mistake here: We did not say that identities were finding their expression in stories – we said they were stories" (Sfard & Prusak, 2005b, p. 14). Specifically, Sfard and Prusak (2005a, 2005b) and later Sfard (2008), equated identities to be those stories surrounding a person which are:

- Reifying – the transformation of an action into a state which suggests repetitious behaviour through the use of the verbs be, have, can, and the adverbs always, never, usually.
- Endorsable – the identified person (the person the story is about) endorses that the story reflects the actual or expected state of affairs.
- Significant – if any change in it is likely to affect the storyteller's feelings about the identified person particularly with regard to membership of a community.

A person has a number of stories told about them by multiple narrators, including themselves. Stories consist of a person's self-dialogue (thinking), spoken-out-loud stories about themselves or other people, stories told about them by other people, interactions with other people, and reactions to events. There are also those stories told about that person by other narrators. Identities, according to Sfard and Prusak (2005a, 2005b) also included extra-discursive (or mind-independent) stories, such as examination results, certificates, and report grades, referred to as institutional narratives.

Sfard and Prusak (2005a, 2005b) divide a person's multiple identities into two sets of identities. Actual identities are attempts to overcome the fluidity of change by freezing the picture (Sfard & Prusak, 2005a, 2005b). These stories are factual assertions about a person, and can be identified by the use of *I am* or *he is* sentences told in the present tense, such as *I am bad at maths* or *He is a good mathematician*. Designated identities – *I should be* stories – have the potential to become part of one's actual identity, and influence one's actions to a great extent. Sfard and Prusak (2005a, 2005b) usefully link affect, learning, and identity because they suggest there is likely to be a sense of unhappiness in a person when there is a perceived and persistent gap between a student's actual and designated identities.

In the affective research, students are conceptualised as having internal structures that connect themselves and mathematics. Viewing identity as a narrative does not discount this view. Students' designated identities are similar to the affective notions of self-directive constructions (Malmivuori, 2006) and needs (Hannula, 2006). Hannula (2006) described a students' needs as relatively stable and there was stability evident in the students' sets of designated identities in Sfard and Prusak's research (2005a, 2005b) because of their cultural basis. This view of identity as a narrative adds the social to the elements in the internal structure and adds to understanding about how students' internal structures change. Using the phrase *internal structur*e from the affective literature now seems too static to describe this very dynamic process. *Students' relationships with mathematics*' seems a better fit.

Learning is seen here as engagement in practices of the mathematics classroom and in other communities of practice. The students negotiate the meanings constructed from their interpretations of their learning experiences and these meanings either reinforce or alter the elements of their relationship with mathematics. A student's relationship with mathematics is therefore understood in this paper to have both individual and shared elements that are constantly changing. It is these elements that this research seeks to identify. Specifically, this research seeks to investigate the nature of students' relationships with mathematics and how these relationships are associated with mathematical learning.

Methodology

The 31 participants attended a co-educational school in New Zealand. They were from the same class so the social norms and views of the class as a whole could be examined as well as the affect and identities of the individual students. Students in Year 10 (aged 14-15 years) were researched because understanding adolescents' relationships with mathematics is vital because they are on the "brink of deciding whether or not to pursue mathematical studies" (Nardi & Steward, 2003, p. 346).

The methodology of this research was informed by the affective research into students' internal structures and Sfard and Prusak's (2005a, 2005b) narrative view of identity. Sfard and Prusak (2005a, 2005b) operationalised the notion of identity by gathering evidence of students' spoken identities. Their research is based around what students say, rather than on the researcher or teacher's perceptions of what is going on in the classroom.

A qualitative framework was employed in this research. The data collected included observations of mathematics and English classes, interviews, metaphors for mathematics, drawings of mathematicians, personal journey graphs, questionnaires, exercise books, assessment results, reports, prizes, and attendance. The teachers were interviewed. Informed by Evans (2000), affective indicators were sought such as verbal expressions of feelings, the use of metaphors, negative or positive self-talk, body language, avoidance, and resistance. Other data collected were students' reflections on their experiences, their views of mathematics, and the language they used to describe mathematics. The students' identity stories were collected mainly through the interviews. Decision-making permeated the process of data collection and analysis.

The data was analysed using a grounded theory approach of constant comparison to seek, refine, and understand the interrelationship of the emerging elements of a students' relationship with mathematics. A data analysis software package NVivo (QSR International, 2006), helped to manage the large data set and aid the analysis.

Results

The students described relationships with mathematics that had five elements:

1. *Views of mathematics:* Subjective conceptions students hold to be true about mathematics. The students had views about the nature, uniqueness, importance, and difficulty of mathematics and perceptions of how boring the subject was.
2. *Macro-feelings:* Coined by the students, *macro-feelings* are a student's overall feelings about the subject of mathematics. These feelings contributed to the context within which they engaged in a specific mathematical activity. When a student had negative macro-feelings for the subject of mathematics, they were more likely to have negative *micro-feelings;* the feelings they experience during each mathematical situation.
3. *Identities:* The students each had a unique set of identities related to their view of their mathematical ability. They had designated identities – overall expectations about mathematics, which included commonly held expectations of class placement, individual expectations related to class positioning and how they expected the subject to contribute to their future life. They also had actual identities – perceptions of how good they were at mathematics, which developed through their interactions with others and through their experiences of success and failure when they engaged in the mathematics.
4. *Mathematical Knowledge:* The students had different levels of mathematical knowledge, which students talked about in relation to their knowledge of facts and mathematical rules that they knew *off by heart*.
5. *Habits of engagement:* The students engaged in mathematics in habitual ways that developed over time. Among were the students' pathways of engagement – the ways they usually engaged in the mathematical tasks.

The elements of students' relationships with mathematics were both shared by the classroom community and unique to the individual. For example, the class shared common views about their expectations of their teachers, yet individual students had unique macro-feelings about mathematics and unique perceptions of their own mathematical ability. The elements also interacted in complex ways. The students' macro-feelings about the subject of mathematics were associated with their views of mathematics and were situated in the gap between their actual and designated identities. The students' mathematical knowledge was closely linked to their views of the nature of mathematics. The ways the students habitually engaged in mathematics were associated with their macro-feelings, their views of mathematics, and their identities.

Figure 1 summarises the process of change in students' relationships with mathematics. Their relationship with mathematics contributed to the context within which they engaged in the task. Students' views of mathematics led them to judge the task's importance and difficulty. Their identities led them to have expectations of success. The ways they habitually engaged in mathematics, interacting with the other elements, affected their engagement in the task. Macro-feelings contributed to the micro-feelings they experienced during the task. Furthermore, when the students engaged in a mathematical task, they were each situated in a unique *context of the moment.* Even when they were experiencing the same classroom conditions – the same teacher, at the same time of day – the students each interpreted the context in a unique way. Students' engagement in the mathematical task was therefore determined by the complex negotiation between elements of their relationship with mathematics and the context of the moment.

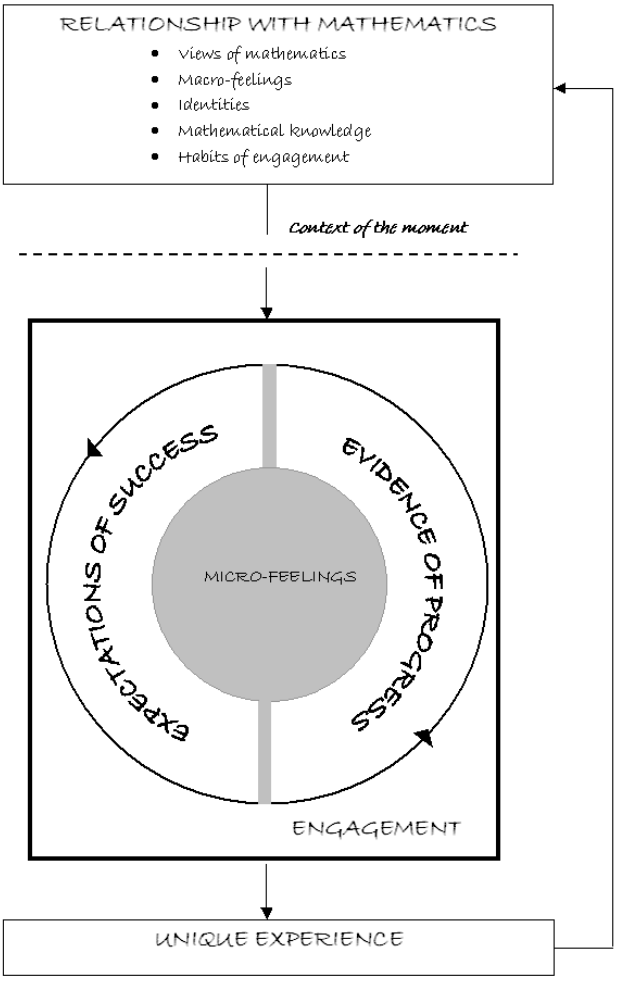

Figure 1. Students' relationships with mathematics

During students' engagement in the task, they collected evidence of their progress. They experienced micro-feelings as they interpreted whether or not their progress met their expectations of success. In Figure 1, the students' expectations and evidence of progress are represented within a circle to show that they surround a student's micro-feelings, and the arrows around this circle show that students' progress can alter expectations of success or vice versa. The way students engaged in the task contributed to their individual experiences and performances, with the way they interpreted these experiences, in turn, reinforcing or altering components of their relationship. These elements, described above, emerged from examining students' perspective of their mathematical learning, yet there are some similarities between these and the components of a student's internal structure, described in the affective research. Both include elements relating to knowledge, beliefs, affect, expectations, and habits. Both include aspects of change and stability. The students' views about school mathematics are similar to the beliefs about mathematics that other researchers found, but these categories emerged from the students' perspective, rather than in response to prompts in a questionnaire. Similar to Op 't Eynde et al.'s (2006) conception of a belief, students' views of mathematics were socially constructed and situated in the context of the mathematics classroom, and dynamic.

Mathematical knowledge is generally defined as the facts, symbols, concepts, and rules that constitute the contents of mathematics as a subject field, as perceived by the community of mathematicians (Op 't Eynde et al., 2002). When students in the current research talked about mathematical knowledge, they usually meant the rules they had been taught by their teacher. The students' knowledge was co-created by the community of the classroom, and may be different to how mathematicians might conceive of mathematics. As discussed by Schoenfeld (1992), the students' conception of knowledge was related to the way the students were taught mathematics – as a series of rules, given with specific examples, and reinforced by practice of that rule from the textbook.

A student's macro-feelings are similar to DeBellis and Goldin's (2006) conceptualisation of *global affect* and McLeod's (1992) notion of a student's *attitude* to mathematics: stable over time compared to transitory emotions. The students' macro-feelings in this research were relatively stable compared to their micro-feelings. The students also described, in some detail, the pathways they usually took when attempting a mathematical task (i.e., their pathways of engagement), a term adapted from Goldin's (2004) use of the term *affective pathway*s to describe individual's dynamic problem solving processes at a task level. The students used it in a more macro sense to describe their habitual pathways of engagement, as described in Ingram (2013).

Conclusions and Implications

Combining the affective concept of students having internal mathematical structures with identity research into narratives informed this examination of students' relationships with mathematics. Students' relationships with mathematics were found to have five elements: views of mathematics, macro-feelings, identities, mathematical knowledge, and habits of engagement. These elements provided part of the context within which the students' engaged in mathematics and contributed to their unique learning experiences. The students' interpretation of these learning experiences reinforced or changed the elements of their relationship with mathematics.

This paper has captured the relationships with mathematics of students in one class. These relationships are connected with that particular context, although there were similarities with students in other classrooms, both in New Zealand (Averill, 2009) and internationally (Boaler, 2000). Despite this, the potential of defining the elements in a student's relationship with mathematics has begun to be realised. The framework of elements was used to analyse the 31 students over two years of their mathematical journeys as they continued to participate, or not to participate, in mathematics (Ingram, 2011). It was used to provide a context for a closer examination of students' engagement (Ingram, 2013), the influence of the parents and teachers, and to explore the tensions between social and mathematical identities (Ingram, 2008). The elements of students' relationships with mathematics have also been communicated to both in-service and pre-service mathematics teachers in New Zealand to provide understanding of how affect and identity play a part in students' learning and to provide a framework for getting to know their students.

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