Personal Number Sense and New Zealand Pre-Service Teachers

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Results of a personal number sense assessment completed by 1253 students enrolled in the first year of pre-service teacher education between 2010 and 2013 are reported. The assessment consisted of 10 short questions requiring solutions to addition and subtraction problems, selected to promote mental calculations using strategies that implied the use of number sense. Analysis of data revealed the scores gained by the pre-service teachers were not necessarily consistent with the University entrance requirements for numeracy. Findings suggest that personal number sense could make a difference to the outcome of pre-service teachers' final grades for the first year mathematics education course.

The importance of mathematical content knowledge for effective teaching and learning of mathematics is well documented (Ball, 1990; Ma, 1999; Shulman, 1987; Young-Loveridge, Bicknell, & Mills, 2012). The knowledge and ability to confidently apply numbers and number operations to solve mathematical problems, with an understanding of the associated mathematical concepts underpins mathematical content knowledge. The importance of developing this number sense is an essential part of being numerate (Muir, 2012). Number sense is an ability to use the processes of mental computation to work flexibly with numbers in problem situations, in ways that allow judgments to be made about the reasonableness of numerical situations (McIntosh, Reys, & Reys, 1992). The development of number sense is recognised as a lifelong process (Reys, Lindquist, Lambdin, & Smith, 2007).

Internationally, number sense has been highlighted as an important idea in school mathematics (Anghileri, 2000; Australian Education Council, 1991; Cockcroft, 1982; National Research Council, 1989; Ministry of Education, 2009). Children who find mathematics difficult often lack number sense (Shumway, 2011). Teachers play a critical role in supporting children to develop number sense (Anghileri, 2000; Siegler & Booth, 2005). To fulfill this responsibility teachers must have a proficient understanding of number sense themselves (Yang, 2007). This suggests implications for initial teacher education.

A number of studies have reported that pre-service teachers demonstrate low number sense (Biddulph, 1999; Kaminski, 1997; Sengul, 2013; Yang, Reys, & Reys, 2009). Biddulph (1999) reported that 81% of the 242 students entering the first year mathematics education course were able to answer a simple mental subtraction problem. Of those who answered correctly 46% used number sense, while 35% used a traditional rule based method that suggested a lack of number sense. Yang, Reys & Reys (2009) study of 280 pre-service teachers in Taiwan reported that nearly 50% of them had a tendency to use rule-based written methods to solve problems. Sengul's (2013) study of 133 pre-service teachers in Turkey reported similar findings concluding that the participants' number sense was very 'low'. Kaminski's (1997) study involved 43 second year pre-service teachers who participated in a number sense programme developed as a component of a mathematics education unit. These studies highlight the importance of continuing the development of pre-service teachers' number sense in teacher education programmes. Findings suggested

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that pre-service teachers with low number sense are focused on solving, or trying to make sense of the mathematics themselves. An implication from these studies is that pre-service teachers who have yet to develop a strong personal number sense are unlikely to successfully engage in mathematics knowledge for teaching.

Entry to teacher education programmes requires students to provide evidence of achieving a specified level of mathematical knowledge. Initial teacher education providers are required to assess numeracy competency prior to entry to their programmes (New Zealand Teachers Council, 2010). Currently there is no national entry assessment. New Zealand high school students awarded University Entrance (i.e. 14 or more credits in mathematics at NCEA Level 1) are deemed to have the numeracy competency necessary for primary school teaching. Students without University Entrance are required to meet similar numeracy requirements. Despite entry requirements, a growing body of research continues to report on concerning gaps in pre-service teachers' mathematical understanding (Sengul, 2013: Yang, 2007; Young-Loveridge, Bicknell & Mills, 2012).

This paper presents the findings of a study that investigated the personal number sense of 1253 first year pre-service teachers enrolled in a three-year primary teacher education programme in a New Zealand university.

Method

The compulsory first year mathematics education course consists of four hours weekly of face to face lectures and workshops over a nine week period (36 hours). The mathematics focus for this paper is the teaching and learning of addition, subtraction and place value within levels 1-3 of the New Zealand curriculum (Ministry of Education, 2007). The course aims to develop the mathematical knowledge needed for teaching this domain, and provides opportunities for pre-service teachers to continue to develop their personal number sense.

Between 2010 and 2013, 1253 pre-service teachers enrolled in a first year mathematics education paper participated in a personal number sense test. The test contains problems at Level 3 and 4 of the New Zealand Curriculum (Ministry of Education, 2007), common content knowledge for any well educated adult (Ball, Hill, & Bass, 2005). The written number sense test consisted of ten addition and subtraction problems in the form of equations. Participants were instructed to work out the solutions mentally and not to carry out traditional vertically written algorithms. Participants wrote an answer to each problem with a brief explanation of how they arrived at their answer. Words, symbols and diagrams were acceptable forms of recording their thinking. If the problem was solved correctly with working that showed an understanding of the mathematical processes involved in solving the problem one mark was awarded. Traditional vertical written algorithms did not gain marks. This type of response was perceived to be indicative of rule-based instrumental thinking, rather than evidence of relational thinking (Skemp, 1978). The result of this test, a mark out of ten, was recorded as 10% of the participant's final mark for the course. For this analysis the participant's mark for the Personal Number Sense Test was matched with the final grade. A score of 8, 9 or 10 marks was considered to demonstrate a reasonable personal number sense.

Results

The marks participants' attained on the personal number sense assessment (PNS) ranged from 0 to 10. 70% of participants scored in the range 8 to 10 marks, with 29% of participants scoring 10 marks (Table 1). The results show that participants who gained an overall course grade of A or A+ scored 8, 9 or 10 marks on the PNS test (Table 2). Of the participants who received a D+ to D- grade, 66% of them scored 7 or less.

Table 1
The PNS scores of the participants

PNS score	10	9	8	7	6	5	4	3	2	1	0
No. of participants	367	288	219	155	90	63	34	20	11	2	4
% of participants	29	23	17	12	7	5	3	2	1	.1	.3

Table 2
PNS scores and overall grades awarded

Grade	PNS score 8-10	PNS scores 0-7
A+, A, A-	22%	1%
B+, B, B-	31%	10%
C+, C, C-	13%	13%
D+, D, D-	4%	7%

Discussion

Results from this study were of considerable interest to the course tutors, and raised a number of concerns. Undergraduates entering the programme, based on the entry requirements, should have had sufficient mathematical knowledge to successfully complete the personal number sense assessment. Gaps in personal number sense knowledge were not apparent until after the course was completed, as the number sense assessment was presented at the end of the course. The majority of pre-service teachers who demonstrated reasonable number sense successfully achieved the valued outcomes that were taught on the course. Participants with low personal number sense were unlikely to receive an overall grade in the A range. These results indicate that pre-service teachers with reasonable number sense are more able to engage with the mathematics knowledge for teaching, and more likely to meet the outcomes of the course. The 30% of pre-service teachers in this first year paper gaining 0-7 marks on the personal number sense test suggests that the current regulated entry status cannot be used to determine pre-service teachers' level of number sense, a foundation block for mathematical understanding and mathematical knowledge for teaching.

The importance of number sense and the critical role that teachers play in the development of number sense in children has implications for initial teacher education. In the past it has been assumed that the regulated entry requirements for a teacher education programme would ensure those entering have the confidence and ability to use numbers flexibly when solving mathematical problems (personal number sense). Yet this is not the case. Number sense, the foundation block for developing mathematical knowledge for teaching, for a significant number of pre-service teachers is underdeveloped. Herein lies the challenge for pre-service teacher educators. Developing personal number sense takes time. Pre-service teachers often need to be convinced of the importance of developing their number sense and the confidence to let go of their rule-based learning. For these pre-service teachers additional support is needed above and beyond the number of hours dictated in current regulations for compulsory pre-service mathematics education.

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