Using Percentages to Describe and Calculate Change

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This study reports on the use of formative, diagnostic online assessments for the topic percentages. Two new item formats (drag-drop and slider) are described. About one-third of the school students (Years 7 to 9) could, using a slider, estimate "80% more than" a given length, in contrast with over two-thirds who could estimate "90% of" a given length. While four-fifths of the school students could, using drag-drop cards, choose the 2-step calculation of a reduced price after a 35% discount, only one-third could choose the corresponding 1-step calculation.

The aim of this paper is to present items from two formative, diagnostic online assessments for the topic of percentage and the results of trialling with samples of secondary school students. These formative assessments are referred to as 'smart tests', i.e. *Specific Mathematics Assessments that Reveal Thinking*, which the authors have created and made available online to schools. The purpose of the smart test system is to provide information to teachers about their students' knowledge, conceptions and misconceptions. Detailed diagnostic information is available as soon as students complete the tests, so that the teacher can use it for planning and in delivering lessons. Stacey (2013) and the smart test system website (HREF1) provide further details.

Baratta, Price, Stacey, Steinle, and Gvozdenko (2010) reported the results of two early smart tests on percentages. The items in those tests were short word problems with multiple choice responses. Since then, the range of item formats that can be programmed and readily used on school computer equipment has expanded markedly so that many of the smart tests now include more interactive items using drag-drop and slider formats. As well as being more engaging for students, these formats enable us to test different aspects of students' knowledge in an online environment. While online testing might not enable the determination of the depth of understanding that might be available from paper and pencil tests and interviews (see for example, Koay, 1998) the smart test system is attempting to capitalise on the potential of an online assessment system with automatic marking and diagnosis for teachers. The results of several of these interactive items (drag-drop and slider formats) from two percentage tests are reported in this paper.

Background

Dole (2010) proposed a four-step procedure for solving the three types of percent problems found in school textbooks. The four steps in the procedure are structured in order to highlight the proportional nature of percent and occur as follows: (a) identification of the elements within a percent situation; (b) representation of the percent situation as a proportion visually on a dual-scale number line; (c) translation of the visual information into a statement of proportion; and (d) calculation of the proportion equation.

The order in which the fundamental percentage ideas are introduced to school students varies between countries. For example, Van den Heuvel-Panhuizen (2003) described how,

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within the Dutch approach to mathematics education called Realistic Mathematics Education (RME), models are used to provoke students' growth in understanding of mathematics. She described the use of a bar model within a longitudinal trajectory on percentage that was designed for the Mathematics in Context curriculum. This trajectory included the following: (1) Informally understanding the meaning of a percent as a part of the total given in terms of a whole called 100, (2) Expressing one quantity as a percentage of another with a flexible approach rather than with a fixed strategy, (3) Adding a 1% benchmark strategy for calculating percentages in an approximate way, (4) Formal method for expressing *a* out of *b* as a percent, (5) Percentage change calculated in both an additive 2-step way and a multiplicative 1-step way, (6) Calculation of a repeated change, (7) Calculation of multiple changes.

Both the Van den Heuvel-Panhuizen (2003) teaching sequence and the list of outcomes for each year level listed in the Australian Curriculum for mathematics (HREF2) begin with understanding the concept of a percentage, progress through simple calculations of a percentage, finding a percentage change through first 2-step and then 1-step methods, and reach calculation of multiple changes. The first outcomes, (i.e. a basic understanding out of 100, estimation of given and changed quantities and estimating the percentage of relative size) are the foci of the smart test *Percentage Estimation*. The *Percentage Change* test considers finding a percentage change through 2-step and 1-step methods, and the calculation of multiple changes.

Method

Participants and Procedure

Smart tests have been available online from 2010 to 2013 inclusive, and the data reported in this paper includes responses during this period to certain items within the tests *Percentage Estimation* and *Percentage Change*. As the *Percentage Change* test was revised at the end of 2012, the data for this test only includes responses from the new items used in 2013. The sample consists of school students (Year 7 to 10) as shown in Table 1. The school students were from the classes of more than sixty, mainly Victorian, secondary school teachers, working at a range of schools throughout the state and also from a small number of schools from outside Victoria and internationally. The teachers are those who elected to sign up for smart tests and administer one or more of these tests to their students. Most teachers, replying to an online questionnaire, have indicated that these tests have been used prior to the teaching of the topic with the aim of enabling teaching to be well-targeted to the students in the class. Hence, care needs to be taken generalising from this sample; the sample is opportunistic and the results for these school students are likely to be lower than the results from summative tests, for students at the same year level, in other studies.

Table 1
Composition of Sample for each Test

Smart test	Year 7	Year 8	Year 9	Year 10	Total
Percentage Estimation	356	449	142	0	947
Percentage Change	0	58	35	22	115

New Item Formats

In this section, examples are provided of the new drag-drop and slider formats. Figure 1 shows some tasks within the *Percentage Estimation* test, as if partially completed by a student. The first vertical bar (slider) has already been pulled up while the other two sliders are in the starting position. The student input to these tasks is assessed automatically, with a generous tolerance. This item can assess (i) basic understanding of the expression "*x*% taller (shorter) than" by checking whether the slider is longer (shorter) than the given base height, (ii) students' accuracy of estimation of 20%, 75% and 35% of a length, and (iii) any confusion between "35% of a height" and "35% shorter than the height".

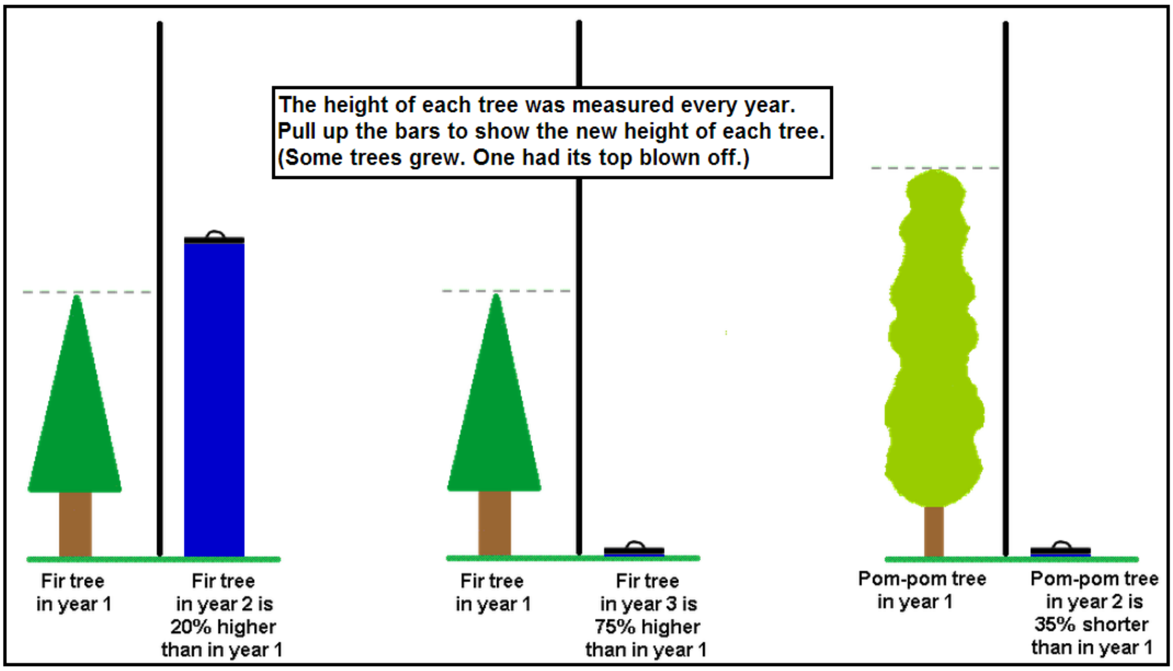

Figure 1. Vertical slider tasks from smart test Percentage Estimation (Quiz A) - modified layout

Figure 2 shows another slider task from the *Percentage Estimation* test; the full item has 12 horizontal slider tasks that assess students' ability to create an image showing a percent and to discriminate between other verbal descriptions of percentage (e.g. 15% as long as, 130% more than the length of, 60% longer than, and 20% of the length).

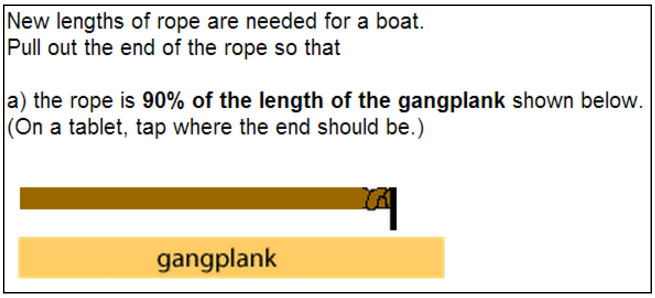

Figure 2. Horizontal slider task from smart test Percentage Estimation (Quiz A).

Figure 3 shows some tasks within an item from the *Percentage Change* test (modified layout). The full item is designed to assess if students are able to use both a 2-step process as well as a 1-step process to calculate the new price of a \$124 tennis racquet after a 35% discount. In the tasks shown in Figure 3, students are expected to drag a sequence of cards into position: one of four cards to show how to calculate the discount, and then one of six cards to show how to find the new price. A similar format (drag-drop cards) is used to determine if students can choose the correct calculation for the new price in 1-step (0.65 × \$124). The choices made by students to the tasks in Figure 3 enable us to provide teachers with several pieces of information (e.g. whether students have a preference for decimal or fraction calculations) as well as misconceptions such as calculating the final price by subtracting the percentage discount from the original price (124 – 35).

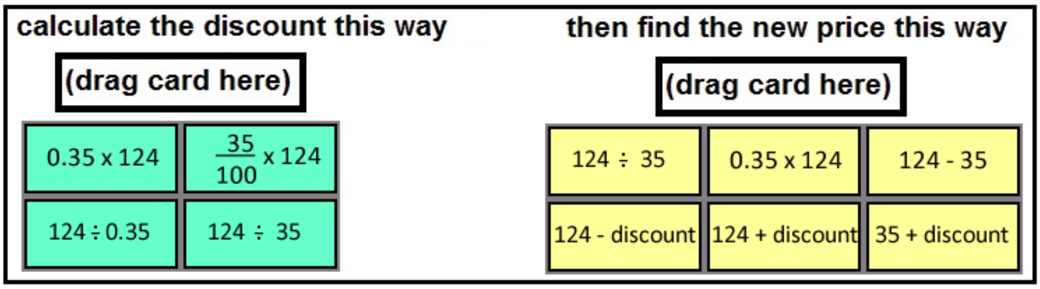

Figure 3. Drag-drop tasks from smart test Percentage Change (Quiz A)- modified layout

Description of the Percentage Estimation Test

Parker and Leinhardt (1995) noted that percent is a multiplicative relationship which causes students particular difficulties. They identified that students experience confusion between multiplicative and additive approaches and showed that the concise, abstract language of percentages often uses misleading additive terminology when the meaning is multiplicative.

This test consists of four items, three of which are reported here. The first purpose of this test was to assess students' skills in estimating a given percent of a whole. This was done in several ways. Most tasks used sliders (vertical sliders as shown in Figure 1 and horizontal sliders as shown in Figure 2). Some of the sliders required a given percent to be shown, (both <100% and >100%) and others required students to show the resultant length after a percentage increase or decrease. Several forms of phrasing were used for the slider tasks to determine how this affected student responses.

The second purpose of this test was to assess students' skills in estimating one quantity as a percentage of another. In one item, a diagram showing the two trees of different heights was shown and students were asked to choose, from a list of options, the height of one tree as a percentage of the other tree.

Description of the Percentage Change test

In contrast with the *Percentage Estimation* test which involved estimation but not calculations, the *Percentage Change* test involved the choice of calculations but did not require any calculating as such. This test consists of four items, two of which are reported here. The first item involves the cost of a tennis racquet at a discounted price as discussed earlier and shown in Figure 3.

The second item involves the growth of a tree with 14% growth rate per year. Students begin by predicting the growth of the tree after 1 year using a 2-step additive method with a drag-drop item similar to that shown in Figure 3. Other items, not discussed further, involve multiple choice tasks, where students select the reasoning that is closest to theirs.

Results and Discussion

Results from the Percentage Estimation test

Table 2 shows results from the three vertical slider tasks shown in Figure 1. These tasks had reasonably similar success rates for students from Year 7 to Year 9, with the percentage decrease task being more difficult than the two percentage increases.

Table 2
Facilities of Estimation Tasks involving Various Percentages using Vertical Sliders

Task: *Show..*	Comment	Yr 7 n=356	Yr 8 n=449	Yr 9 n=142
20% higher than	2 stages, increase <100%	63%	69%	80%
75% higher than	2 stages, increase <100%	69%	66%	67%
35% shorter than	2 stages, decrease <100%	47%	47%	42%

Table 3 shows selected results from the horizontal slider questions; example shown in Figure 2. Here the Year 8 and 9 students had higher facilities on most tasks than the Year 7 students, who were unlikely to have met percentage increases and decreases in their earlier studies of mathematics.

Table 3
Facilities of Estimation Tasks involving Various Percentages using Horizontal Sliders

Task: *Show..*	Comment	Yr 7 n=356	Yr 8 n=449	Yr 9 n=142
90% of	1 stage, <100%	57%	76%	71%
70% as long as	1 stage, <100%	45%	58%	57%
130% as long as	1 stage, >100%	30%	47%	43%
165% of	1 stage, >100%	29%	45%	42%
40% shorter than	2 stages, decrease <100%	28%	33%	32%
80% more than	2 stages, increase <100%	24%	25%	28%
125% longer than	2 stages, increase >100%	15%	14%	17%

The success rates on the slider estimation tasks appear to have been affected by two factors. One was the size of the given percentage, particularly whether it was above or below 100%. The other was whether the question involved a multiplicative operation such as '90% of' (a 1 stage process) or whether it was a change such as '20% higher than' which can be thought of as a 2 stage process.

Percentages close to zero or to 100% were easier to estimate than those further from the 100% reference. When asked to estimate '70% as long', 18% of the school students showed a length greater than the 100% reference, so appeared to have little understanding of the meaning of a percentage.

Percentages greater than 100% were harder to estimate than those less than 100%; only 44% of the school students correctly showed a length that was '130% as long as' the reference length. The remaining students consisted of the 11% who showed a percentage below 100%, 11% who over-estimated but below 200%, 11% who showed 130% as over 200% of the reference length and 24% who omitted this question.

The last three rows of Table 3 involved the percentage increase and decrease questions. Of the school students who did have some facility with estimating a single percentage (as shown by a correct answer for '90% of"), 26% incorrectly showed the change rather than the new amount for '80% more than'. A percentage increase of over 100% (last row, Table 3) was even harder to estimate than a percentage increase or decrease of less than 100%.

Table 4 contains the results of the multiple choice tasks comparing the heights of two trees, students found it easier to express a smaller height as a percentage of a larger height (e.g. 77% of Year 9 were correct) than to express a larger height as a percentage of a smaller height where the percentage was over 100%, (only 37% of the Year 9 students were correct).

Table 4
Facility of Comparing what Percent One Height is of Another

Task: *Choose correct alternative for..*	Yr 7 n=356	Yr 8 n=449	Yr 9 n=142
Shorter tree as a percent of taller (<100%)	65%	72%	77%
Taller tree as a percent of shorter (>100%)	24%	37%	35%

Table 5 contains details of the stages used in the automatically generated report from the *Percentage Estimation* test to teachers about their students, as well as the percentage of students who were allocated to each stage.

Table 5
Stages Reported to Teachers for Percentage Estimation Test

Stage or gap	Yr 7 n=356	Yr 8 n=449	Yr 9 n=142
Stage 0: No understanding	33%	27%	20%
Stage 1: Students can estimate one quantity as a percentage of a larger quantity	44%	43%	46%
Stage 2: and can estimate percentage changes that are less than 100%	19%	19%	21%
Stage 3: and can estimate percentage increases that are more than 100%	4%	8%	12%
Stage 4: and can estimate one quantity as a percentage of a smaller quantity	0%	2%	1%
LL: Language of percent is limited	33%	35%	30%

Note that in order to be diagnosed as Stage 4, for example, students needed to be able to complete a series of items correctly. The last row indicates that about 1 in 3 school students had difficulty with some of the variations in the percentage language used. It is surprising that there is not more of an improvement from Year 7 to Year 9. More testing needs to be conducted with a more representative sample to determine if this is a general result.

Results from the Percentage Change test

The small numbers who have attempted this test to date do not allow for the school year levels to be considered separately. Data for the percentage discount question is shown in Table 6. For the calculation of a percentage change using 2 steps, students first needed to choose a calculation for the change and then a calculation for the new amount.

Table 6
Facilities of Tasks within Percentage Change Item about 35% Discount

Task	Year 8-10 (n=118)
Choose calculation for 35% discount	85%
Choose calculation for new cost	81%
Choose 1-step calculation	33%

It is important that students are able to carry out a percentage change calculation in 1 step (multiply the original amount by a single value), rather than relying only on the 2-step calculation. This helps students to make the later transition to multistep changes. The success rate of the school students on this task was low. This may be due in part to some teachers of Year 7 and Year 8 classes not realising the importance of students progressing to a 1-step method. This is not explicitly mentioned in the Australian Curriculum, but is included the Mathematics Developmental Continuum which is a resource made available by the Victorian Department of Education and Early Childhood Development (HREF3). Results on the growth question for the estimation task and the 2-stage calculation task were similar to those for the 35% discount question and will not be reported in detail here.

Table 7
Stages Reported to Teachers for Percentage Change Test

Stage	Year 8-10, n=118
Stage 0: No understanding of percentage change	16%
Stage 1: Students understand that discount means percentage decrease and that growth means percentage increase	18%
Stage 2: and know how to calculate the change	15%
Stage 3: and how to find the increased or decreased quantity by adding the change to, or subtracting the change from, the original amount	36%
Stage 4: and can calculate the new quantity after a given percentage increase or decrease using a single multiplication	10%
Stage 5: and can calculate a sequence of changes of the same percentage	5%

Table 7 contains details of the stages used in the automatically generated report from the *Percentage Change* test to teachers about their students, as well as the percentage of students who were allocated to each stage. The assignment to stages is as much about alerting a teacher to any gaps in the understanding of the student as about showing the most difficult type of question at which a student can succeed.

Conclusion

An understanding of percentages is crucial for citizens in our information driven society. It is incumbent on us as teachers to enable as many school students as possible to reach a functional level of competence with percentage calculations up to and including the compounding effect of interest rates on unpaid debt. The results provided here confirm that there is a significant group of school students in this sample who have little understanding of percentage change. The bulk of the school students are somewhere on the continuum between no knowledge and mastery. Efforts need to be made to support students to move from 2-stage additive methods to 1-stage multiplicative methods as this approach makes compounding changes more achievable. To help as many students as possible to progress it is important that teaching does not assume that all students already have knowledge appropriate to their year level. Instead there is a need for students to be helped to progress from where they are and this is where diagnostic assessment, such as smart tests, is useful to the classroom teacher.

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