Connecting Social and Mathematical Thinking: The Use of "Real Life" Contexts

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This paper reports the findings of research into an educational intervention featuring open-ended mathematical problems situated in "real life" contexts and associated pedagogies. "Money and financial mathematics" is the topic in focus, with tasks termed "financial dilemmas" being trialled by 35 teachers in 16 Victorian primary schools. Drawing on the teachers' reactions to one task, "Catching a taxi," the strengths, challenges, and complexities associated with creating and/or selecting meaningful "real life" contexts for mathematics teaching and learning are discussed.

There is a substantial body of literature emphasising the need for school mathematics to prepare numerate citizens who are well-equipped to participate fully in life beyond school. Several researchers have argued that constructivist and sociocultural perspectives have the potential to transform mathematics education through practical or realistic applications. Cobb (1994) argued that that in order to be effective, mathematics education should situate learning within meaningful social and cultural contexts while taking into consideration the differences in individual thinking that might exist as a result of the variety of communities to which students belong. He suggested that constructivist and sociocultural perspectives can be thought of as useful frameworks within which to develop instructional materials and address pedagogical issues.

Similarly, Jablonka (2003, p.78) argued:

> Any attempt at defining "mathematical literacy" faces the problem that it cannot be conceptualised exclusively in terms of mathematical knowledge, because it is about an individual's capacity to *use* and *apply* this knowledge. Thus it has to be conceived of in functional terms as applicable to the situations in which this knowledge is to be used.

Sullivan (2011) argued that contextualised problems that require students to apply both social and mathematical thinking have the potential to prepare students for the challenges of everyday life, while demonstrating that mathematics is useful.

These arguments align with Realistic Mathematics Education (RME), a Dutch reform based on Freudenthal's view that mathematics must be connected to reality, stay close to children, and be relevant to society, in order to be of human value (Freudenthal, 1977 in van den Heuvel-Panhuizen, 2003). Freudenthal believed that mathematics is a human activity, and so mathematics education should give students guided opportunities to explore and experience mathematics by doing it (van den Heuvel-Panhuizen, 2003).

These arguments suggest the need for mathematics teaching and learning to be conceived in such a way that empowers and activates its use. One way this might be achieved is through the use of open-ended mathematical problems involving "real life" contexts. This paper explores the use of one such problem as part of the EPMC project, a design-based research project that involves teachers in trialling and reporting on challenging mathematics tasks and associated pedagogies (for more details about this project, see Sullivan, Askew, Cheeseman, Clarke, Mornane, Roche, & Walker, under

2014. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). Curriculum in focus: Research guided practice *(Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia)* pp. 557–564. Sydney: MERGA.

review). Five financial dilemmas were used by 35 teachers of Years 5 & 6 in 16 Victorian primary schools. There were two research questions:

1. What are the strengths, challenges, and complexities in using meaningful "real life" contexts for mathematics teaching and learning?
2. What are the implications for teacher practice?

Research into Realistic or Practical Mathematics

A number of researchers have highlighted the potential and challenges in creating and/or selecting meaningful "real life" contexts for mathematics teaching and learning. Gravemeijer (1997), for example, argued that problems in textbooks seldom ask students to undertake more than one operation, so "the name of the game becomes finding the proper operation and executing it" (p.390). He suggested that mathematics education might be transformed through problems, pedagogies, and classroom cultures that demonstrate to students that realistic considerations are valued over "flawless but senseless calculations" (p.393).

While it seems sensible that mathematics teaching and learning be situated in meaningful "real life" contexts, doing so is by no means straightforward. Borasi (1986) emphasised that students are the ultimate judges whether a problem is appealing enough to attempt to solve it, and they make this judgement based on the level of difficulty they perceive in the problem, their interest in it, and the importance they ascribe to it. She argued that in making this judgement, students establish the amount of time and effort they are prepared to dedicate to the problem-solving process. In this respect, the choice of context can make or break a mathematical problem – this means there is a need to challenge students with various kinds of richer problems.

Meyer, Dekker, and Querelle (2001) argued that context plays five interdependent roles in mathematics curricula, including: motivating students to explore new mathematics; offering students a chance to apply mathematics; serving as a source of new mathematics; suggesting a source of solution strategy; and providing an anchor for mathematical understanding. They outlined a number of characteristics of high quality contexts, which included that a context should: support the mathematics and not overwhelm it; be real or at least imaginable; be varied; relate to real problems to solve; be sensitive to cultural, gender and racial norms; and not exclude any group of students; and allow the making of models. However, they agreed with Borasi (1986) that a context that interests and motivates one student might hold no interest for another.

It has been argued that student socioeconomic background is an important consideration in this process given the relationship between socioeconomic characteristics and educational attainment. In their review of the national testing system in the United Kingdom, Cooper and Dunne (1998) argued that students' performance on apparently realistic items on the National Curriculum assessment in mathematics varied by sociocultural background, with working class students being disadvantaged. They argued the disparity in achievement could be explained by the different material conditions of life experienced by students. Lubienski (2000) found that contextualising mathematical problems made the purpose of tasks less clear for students she classified as being from lower socioeconomic backgrounds. Jorgensen and Sullivan (2010) wrote about this phenomenon drawing on their experiences in remote Aboriginal contexts. They highlighted the ways and means by which social heritage converts to academic success, giving specific examples of items about money that were included on the 2008 Australian numeracy

assessment. They argued that while particular contexts may be realistic for some students, they are well outside the everyday experiences of others, and so create opportunities for scholastic mortality among those who are already disadvantaged. They argued that "the greater the synergy between the habitus of the student and school mathematics, the greater the probability of success" (p.25). These researchers highlight the need for care to be taken to create and/or select contexts that are in fact meaningful to students given their family backgrounds, characteristics, and interests – and that there is no such thing as a "one size fits all" mathematical problem. Hence, the extent to which teachers know and understand their students can be pivotal.

Numeracy, mathematics, and financial literacy education are closely related. This relationship is represented in the *Australian Curriculum* in two ways. First, the *Australian Curriculum: Mathematics v5.0* includes "Money and financial mathematics" as a substrand of the "Number and Algebra" content strand (Australian Curriculum Assessment and Reporting Authority [ACARA], 2011). Second, numeracy is one of the seven *General Capabilities* that apply across all discipline content. This means that financial literacy is to be taught as part of the school mathematics curriculum, with practical applications of numeracy also featuring in other disciplines where financial literacy topics are identified (i.e., *Economics and Business* and *Civics and Citizenship*).

An Educational Intervention Involving Financial Dilemmas

This paper explores the strengths, challenges, and complexities in using financial dilemmas involving "real life" contexts. A financial dilemma is essentially an open-ended mathematical problem involving a financial context drawn from a "real life" situation that 10-12 year old children might be familiar with and/or interested in and/or able to imagine. One financial dilemma is sufficiently challenging to form the basis of a single mathematics lesson, with enabling, consolidating, and extending versions (Sullivan, Mousley, & Jorgensen, 2009). Financial dilemmas are unique in that they require students to draw on both social and mathematical understandings simultaneously and in synergy, involve multiple solutions, and invite students to share and explain their reasoning. An important goal of this approach is to strengthen students' disposition to draw on both social and mathematical understandings as part of their financial problem-solving and decision-making – something students tend to find challenging. Five financial dilemmas were included in the EPMC project – these represented a range of "real life" situations including shopping for shoes, interpreting value for money, and paying for movie tickets and candy bar deals. The power of any financial dilemma lies in the following associated pedagogies and practices:

- Providing a *rationale* for the lesson by defining financial literacy, explaining the difference between social and mathematical thinking, and emphasising the importance of both to informed financial problem-solving and decision-making.
- Building a strong *introduction* to the lesson through literacy and other strategies that give students confidence to begin problem-solving. Literacy strategies include: ensuring the problem is read aloud to the class; asking students to underline or highlight the information that they think is important to the problem; and asking students to identify any words they do not know or understand so that these can be defined as a class. Other strategies that seem to help students make sense of financial dilemmas include the use of role play and concrete materials (i.e., using notes and coins).

- Emphasising *problem-solving tools and* strategies that might help students, including creating tables to organise information and/or drawing pictures.
- Providing time for *individual thinking and problem-solving*, followed by *small group collaboration* where students can share and discuss their problem solving approaches and solution/s.
- Facilitating *critical whole-class discussions*, including: ensuring that a range of options (mathematical workings and explanations) are recorded, and asking open, sometimes provocative questions to stimulate different ways of thinking.

The Research Context and Data Sources

The findings reported in this paper relate to data collected from 35 teachers of Years 5 & 6 in 16 Victorian primary schools. Government and Catholic, metropolitan and regional schools were included, creating some confidence that the sample represents the range and diversity of schools across Victoria. Data were collected in a number of ways. The teachers were surveyed before and after their involvement in the educational intervention. Twenty teachers also completed and returned one-page lesson reports. These involved short-answer questions and were intended to be completed immediately after teaching each lesson. Finally, the teachers were invited to provide verbal feedback about the educational intervention at a professional learning event.

Findings

Having taught the five lessons, the teachers were asked to vote for the three best financial literacy tasks with respect to: a. mathematics learning; and b. student engagement. They used 1 for the best financial dilemma, 2 for the second best, and 3 for the third best. Table 1 presents the number of times each financial dilemma was voted first, second, and third with respect to mathematics learning and student engagement. The totals provide a way to compare the effectiveness of the five tasks.

Table 1
Tally of Teachers' Votes for the Three Best Financial Literacy Tasks

	With respect to *mathematics learning*				With respect to *student engagement*			
Vote	1	2	3	Total	1	2	3	Total
Shopping for shoes	8	8	6	22	3	11	5	19
Making choices about spending	1	11	5	17	1	9	3	13
Anna and her friends	9	8	6	23	15	6	6	27
Catching a taxi	11	4	9	24	10	5	12	27
The price of meal deals around the world	2	0	4	6	2	0	4	6

The financial dilemma with the highest number of votes for mathematics learning was "Catching a taxi" (24 votes). The financial dilemmas with the equal highest number of votes for student engagement were "Catching a taxi" and "Anna and her friends" (27

votes). This paper focuses on "Catching a taxi" for the reason that teachers evaluated this financial dilemma highly on both dimensions. This financial dilemma was designed to explore how students decide to divide costs when two people share a taxi ride to two different destinations. "Flagfall" is introduced to students as an example of an upfront cost. The fact that the characters Mike and Matt are travelling different distances in the taxi means that sharing the cost of the trip evenly may not be the fairest solution. The task also represents linear relationships. It was presented as follows:

> *Catching a Taxi*
>
> The taxi fare is $3 flagfall (what you pay when you get into the taxi) and then $1.50 per km after that. It does not matter how many people are in the taxi.
>
> Mike and Matt decide to share a taxi because they are going in the same direction but to different houses. The journey to Mike's house is 20 km, then a further 30 km to Matt's house.
>
> How much should each of them pay for the taxi? Explain why your suggestion is fair for both people.

The post-intervention teacher survey included a number of brief statements that teachers were asked to respond to by indicating the extent to which they agreed on a 5-point likert scale (strongly disagree, disagree, unsure, agree, strongly agree). These items and participants' responses are presented in Table 2. In the section that follows, this quantitative data is analysed together with the teachers' qualitative responses about the educational intervention. The latter include an item on the one-page lesson reports that asked, "What is your reaction to the lesson overall?" and an item on the post-intervention teacher survey that asked, "Is there a particular story you would like to share with us? Please tick the task your story relates to, and briefly tell us about your experience." Interpreted together, these data sources revealed the strengths, challenges, and complexities in using "real life" contexts for mathematics teaching and learning.

Table 2
Teachers' Responses to Statements about the Financial Dilemmas

Statement	SD 1	D 2	U 3	A 4	SA 5	Total	Mean
Item about the five financial dilemmas:							
These tasks helped my students see that both social and mathematical thinking play important roles in financial problem solving.	1	0	0	20	10	31	4.23
Items about "Catching a taxi":							
I explained the mathematical purpose to the students.	0	0	0	11	9	20	4.45
Most students learned the main mathematical ideas.	0	0	2	9	9	20	4.35
The contribution of students to the discussion was good.	0	1	0	10	8	19	4.32

Strengths

The teacher participants' feedback on the five financial dilemmas was very positive. All but one strongly agreed or agreed that the financial dilemmas helped their students to see that both social and mathematical thinking play important roles in financial problem-solving (mean = 4.23). This suggests that the educational intervention achieved the aim of building students' awareness that both types of thinking are important to financial problem-solving and decision-making and should be drawn on simultaneously and in synergy to inform and justify an argument.

With regard to "Catching a taxi," all of the teachers strongly agreed or agreed that they explained the mathematical purpose of the lesson to students (mean = 4.45). The vast majority also strongly agreed or agreed that most students learned the main mathematical ideas associated with the task (mean = 4.35). This suggests that the task provided a realistic representation of the usefulness of mathematics beyond school. For example, one teacher reported that "Catching a taxi" gave him/her the opportunity to teach students about up-front costs and the price of the trip varying according to distance travelled. Others reported particular strategies they had used to facilitate mathematics learning, including prompting students to consider that Mike and Matt did not travel all the way together, and selecting various students to share different problem-solving strategies and reasoning with the class over the course of the lesson.

Most of the teachers also strongly agreed or agreed that the contribution of students to the discussion was good (mean = 4.32). One teacher wrote, "This was the third lesson involving financial literacy and I think the children were becoming used to linking mathematics to life experiences." Another teacher commented on how requiring students to draw on mathematics to develop and support an argument about fairness made for productive, if not lively whole-class discussion:

> ["Catching a taxi"] created the most discussion about fairness. Students worked out half the cost each, or one pays the first part of trip and the other pays the last. This showed kids the social aspect of mathematics. This was one of the first times students passionately debated mathematics.

Drawing on social and mathematical thinking as students considered multiple options was an important feature of small group and whole-class discussion reported by the teacher participants. Highlighting that it can be complex to get students to weave social and mathematical thinking to articulate an argument, one teacher wrote, "Most students understood the mathematical concepts, but the reasoning [behind the solution] was challenging for some students to articulate." Related to this, another teacher reported, "A lot just split the bill and didn't think about what was fair and what wasn't." While another teacher made a similar comment, he/she implied that questioning techniques had played an important role in the success of the lesson saying, "They were quite happy to get a mathematical answer and leave it at that, even though it wasn't fair. I had to really question them carefully to get responses that were more reasonable." These responses highlight that the financial dilemmas and associated pedagogies are truly interdependent.

Challenges and Complexities

The qualitative teacher data revealed challenges and complexities in using "real life" contexts for mathematics teaching and learning. First, the importance of establishing the context, including the effectiveness of role play to do this was identified. Second, the issue whether "Catching a taxi" constituted a meaningful "real life" context was raised.

Most of the teachers reported that students were able to related to "Catching a taxi" based on their own previous experiences catching a taxi and/or that using role play enhanced students' understanding of the context. For example, one teacher commented:

> We used role play to introduce the problem. The students were instantly engaged and involved. The three students that acted out the situation were fantastic - way better than we expected! Two grades that did not use the role play did not show as good results as the two grades that used it.

The importance of the use of role play was also mentioned by this teacher, who wrote about the need to identify and discuss a range of options, as follows:

> Many of my students had experience with catching a taxi. In the [lesson] introduction we shared these stories and developed a role play of the scenario. Interestingly a couple of my very strong boys couldn't see past the idea of Mike paying for 20 km and Matt paying for full 50 km, giving the driver a $30 tip. I had to check in with the class several times to discuss an option, let the students work, then share another option. By the end we had about 5 options to decide between.

The above comments highlight these teachers' diligence in weaving the financial dilemma and associated pedagogies to help students visualise the context, discover mathematics by doing it, and share their problem-solving experiences and insights with each other.

Yet while "Catching a taxi" was appropriately socially constructed and situated for financial literacy teaching and learning in some schools, this was not always the case. Several teachers noted that the task was beyond the realms of their students' life experience, and reported that this impacted their students' motivation and ability to connect with the task. Consider this comment about the educational intervention by a teacher who described herself as working with disadvantaged students:

> We found that students couldn't move past the social [thinking] to do the maths. They'd say "We don't go to the movies. We don't pay for films - we download." "We don't pay for taxis." "Why would I pay to go to the gym?" So they couldn't even be objective about answering the maths questions. They were stuck on the social.

The contrasting reports about "Catching a taxi" highlight the challenges and complexities associated with creating and/or selecting meaningful "real life" contexts that take into consideration students' local context, family backgrounds, characteristics, and interests. It is not as simple as that because taxi fares can be expensive, "Catching a taxi" will not be relevant to students from low socioeconomic backgrounds. To illustrate the complexity of the issue, consider this story. A colleague used "Catching a taxi" with a Year 6 class at a school in a low socioeconomic area where the majority of students were from non-English speaking backgrounds. She reported that the students were highly engaged by both the social and mathematical dimensions of the financial dilemma. The students' parents could not drive, and so generally relied on public transport. However, the families also routinely caught taxis to destinations not well-serviced by public transport, and for safety reasons of an evening. Socioeconomic background aside, because the financial dilemma connected with the students' social experiences, the task made for a successful lesson.

It is important to note that socioeconomic background varies even within a local context. So it is not so much a case of knowing what students can afford as knowing what they have observed or experienced outside school. Then again, there is merit in posing contexts that are at least imaginable and might expand students' experiences – i.e., through

role play. Either way, the choice of context and the strategies used to introduce the task are critical.

Conclusion

The educational intervention was well-received by the teachers involved in the EPMC project. The strengths of "Catching a taxi" and the associated pedagogies included:

- helping students to see that both social and mathematical thinking play important roles in financial problem-solving and decision-making;
- enabling students to learn mathematics through doing it and sharing their problem-solving strategies and reasoning with each other; and
- helping students to see that mathematics is useful beyond school.

Creating and/or selecting "real life" contexts for mathematics teaching and learning is both challenging and complex. While there is no such thing as a "one size fits all" context, it seems that role play can be productively used to expand students' experiences. The key implications for teachers include:

- teachers need to carefully consider students' local context, family backgrounds, characteristics, and interests when creating and/or selecting "real life" contexts.
- role play can help students visualise unfamiliar contexts and connect social and mathematical thinking.

Further research to develop, trial, study, and refine new financial dilemmas is needed.

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