

# “I don’t really understand probability at all”: Final Year Pre-service Teachers’ Understanding of Probability

Nicole Maher

*University of Tasmania*  
Nicole.Maher@utas.edu.au

Tracey Muir

*University of Tasmania*  
Tracey.Muir@utas.edu.au

This paper reports on one aspect of a wider study that investigated a selection of final year pre-service primary teachers’ responses to four probability tasks. The tasks focused on foundational ideas of probability including sample space, independence, variation and expectation. Responses suggested that strongly held intuitions appeared to interfere with understanding probability, which impacted on the pre-service teachers’ ability to identify students’ errors and to confidently provide appropriate teaching suggestions and approaches.

Since Shulman’s (1987) identification of the different knowledge types involved in teaching, other researchers have continued to build upon his ideas and construct frameworks that have proved useful in understanding the relationship between these different knowledge types (e.g., Ball, Thames & Phelps, 2008; Chick, Baker, Pham, & Cheng, 2006; Hill, Ball, & Schilling, 2008; Rowland, Turner, Thwaites, & Huckstep, 2010). Shulman used the term ‘pedagogical content knowledge’ (PCK) to describe “an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction” (Shulman, p. 8). Extensive reference is also made in the literature to mathematical content knowledge (MCK), (e.g., Ball et al., 2008), with concerns raised in relation to pre-service teachers’ understandings of many of the mathematical skills and concepts that they are required to teach.

Within the domain of probability, several studies have explored the common intuitive biases held by some pre-service and practicing teachers that are contradictory to correct probabilistic thinking (Baturu, Cooper, Doyle, & Grant, 2007; Dollard, 2011; Stohl, 2005). While some scholars have suggested ways to address this issue (Dollard, 2011; Stohl, 2005), particularly in undergraduate programs, there is limited discussion in the literature about the effect these misunderstandings have on PCK (e.g., Chick & Baker, 2005). This paper seeks to address this gap through discussing the findings of a study that investigated a selection of final year pre-service teachers’ ability to identify students’ probability misconceptions and then suggest appropriate teaching approaches.

## Theoretical Framework

### *Teacher Knowledge*

A number of frameworks based on the work of Shulman (1987) for identifying and describing pre-service teachers’ and teachers’ MCK and PCK have been developed by mathematics education researchers (e.g., Chick et al., 2006; Hill et al., 2008; Rowland et al., 2010). The framework for mathematical knowledge for teaching (MKT) developed by Hill and colleagues encompasses PCK and subject matter knowledge (SMK) (see Figure 1)

2014. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). *Curriculum in focus: Research guided practice (Proceedings of the 37<sup>th</sup> annual conference of the Mathematics Education Research Group of Australasia)* pp. 437–444. Sydney: MERGA.

and is a refinement of Shulman’s original categorization of subject matter knowledge and PCK. Hill et al.’s domain of SMK is further delineated into common content knowledge (CCK), specialised content knowledge (SCK), and knowledge at the mathematical horizon. CCK is defined as the knowledge used in teaching any discipline that involves the use of mathematics whereas SCK involves being able to represent mathematical ideas accurately and interpret unusual solutions to mathematical problems (Hill et al., 2008). Knowledge at the mathematical horizon is defined as an awareness of how mathematical topics are connected throughout the mathematics curriculum (Ball, et al., 2008). Similarly, PCK is further divided into knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of the curriculum. KCS and KCT are separate from each other as subsets of PCK (see Figure 1). Although KCS focuses on teachers’ understanding of how students learn particular content, KCT is concerned with how best to build on students’ mathematical thinking and how to address student errors (Hill et al., 2008).

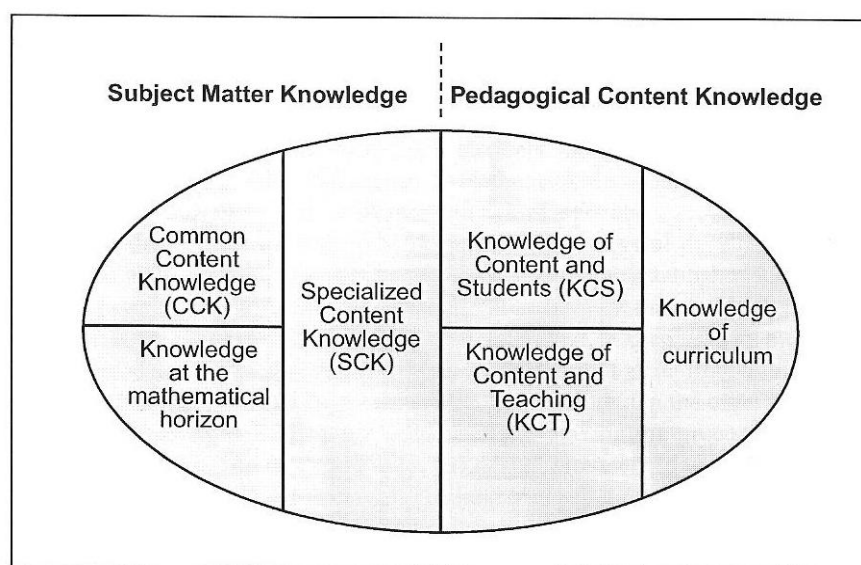


Figure 1. Domain map for mathematical knowledge for teaching.

An alternative framework for analysing PCK developed by Chick and her colleagues is divided into three sections: Clearly PCK, Content Knowledge in a Pedagogical Context and Pedagogical Knowledge in a Content Context (Chick et al., 2006). Each section is further divided into categories relating to mathematics knowledge for teaching. For example the Clearly PCK section includes the category of student thinking, evident when the teacher “addresses student ways of thinking about a concept or typical levels of understanding” (Chick et al., 2006, p. 61). The framework has been used to explore teachers’ PCK and was found to be a useful tool in investigating the similarities and differences in the PCK of two teachers. After due consideration, aspects of Hill et al.’s (2008) and Chick et al.’s (2006) framework informed the construction of our own framework which involved three main categories: knowledge of content, knowledge of learning, and knowledge of teaching.

### *Mathematical understanding*

A strong knowledge of mathematics is necessary for effective mathematics teaching and teachers who lack such knowledge may be limited in their ability to help students develop conceptual and relational understanding of mathematics (Skemp, 1978). In his

seminal work, Skemp distinguished between two kinds of understanding in mathematics: instrumental and relational. Relational understanding is concerned with the underlying principles of a particular mathematical idea whereas instrumental understanding involves following rote learnt rules and procedures, that is “rules without reason” (Skemp, p. 9). Unfortunately, many pre-service teachers’ own mathematics education has been found to be dominated by the rote learning of rules and procedures, (e.g., Maher & Muir, 2013), and although it may enable them to obtain the correct answers to certain mathematical items, it is unlikely to be helpful in terms of providing a sound foundation for teaching others.

Studies have also shown that together with possessing a limited conceptual understanding of fundamental mathematics, pre-service teachers often express a lack of confidence in learning mathematics and display similar misconceptions as held by the students they will one day teach (e.g., Ryan & Williams, 2007). A misconception may be defined as the misapplication of a mathematical rule or an over- or under-generalisation of a mathematical idea. Some of the common misconceptions associated with probabilistic thinking are discussed in the next section.

### *Foundational ideas of probabilistic thinking*

Probability is used to predict the chance of something happening and is applied in situations when the outcome cannot be completely determined in advance (Reys, Lindquist, Lambdin & Smith, 2012). Arguably the best-known misconception in probability is known as the ‘gambler’s fallacy’, where likelihood is based on the pattern of recent events (Ryan & Williams, 2007). Also termed the “negative recency effect” (Fischbein & Scharch, 1997, p. 100), this may be evident when after tossing a coin and obtaining a head three times, a person believes that the fourth toss is more likely to result in a tail, to balance out the number of heads and reflect the expected 50:50 ratio of heads to tails over the long run (in reality every coin toss is an independent event not influenced by previous outcomes). Another misconception identified in the literature has been termed “equiprobability intuition” (Ryan & Williams, 2007, p. 131) and refers to events that are believed to be equally probable when there is strong evidence to the contrary. Students, for example, will often believe that a spinner with ‘unequal’ halves, will give equal probabilities, or that the sum totals from a pair of dice are all equally likely.

In his study into pre-service teachers’ probabilistic thinking, Dollard (2011) found that many participants demonstrated a range of probabilistic misconceptions particularly in relation to subjective (common notion of the ‘likelihood’ of an event), classical (can be described in terms of equally likely outcomes) and frequentist (involving the law of large numbers) interpretations. Dollard alluded to some of the difficulties in teaching probability including the tension between using computer simulations to generate a large number of trials in a very short time and the value of using tangible objects “in order to get a sense of probability as a real phenomenon” (p. 44).

Wang (2001) suggested that one of the obstacles in learning about probability involved personal intuitions held by individuals; such intuitions can get in the way of achieving a “genuine” understanding of key probability concepts (p. 76). In primary school students, for example, this is manifested in such beliefs as ‘the chance of a six coming up on a die is rarer than other numbers’ or that it comes up more often for other people (Ryan & Williams, 2007).

## Methodology

As part of a wider study, a small sample of seven final year pre-service teachers participated in a one-to-one interview that was structured around four key questions or instructions relating to student work samples, two of them involved probability. The interview items are shown in Figures 2 and 3.

A fair coin is tossed 7 times and each time it lands tails up. What is the probability of the coin showing up another tails on the next toss?

**A student in your class responded:**

*It will land on heads because it has already landed on tails 7 times.*

Figure 2. Coin toss interview item

For each of the spinners below what is the probability of getting black



**A student in your class gave the following response:**

*The probability of getting black is  $\frac{1}{2}$  for all three spinners.*

Figure 3. Spinner interview item

The interviews took approximately one hour and were audio-taped and transcribed. Each interview was conducted by the first author and began with the participant being asked to identify whether or not the student's response was correct. If an error was identified, they were then asked to provide the correct response and to identify possible reasons for the student's error. Depending upon the participant's response, further clarifying questions were asked, such as 'What does this work sample tell you about the student's understanding of probability?' Participants were then asked how they would assist the student and to suggest appropriate teaching approaches. Responses were coded and grouped using categories derived from existing frameworks in the literature: knowledge of content, knowledge of learning, and knowledge of teaching. The first is essentially MCK, while knowledge of learning is closely related to Ball et al.'s (2001) SCK in that participants were required to examine and understand student solution methods to problems. Knowledge of teaching refers to PCK, and is particularly aligned with Chick et al.'s (2006) description of 'student thinking' and Ball et al.'s KCS and KCT.

## Results and Discussion

### *Knowledge of content (MCK)*

When asked to respond to the interview item shown in Figure 2, five out of the seven participants incorrectly predicted that a tail would be the more likely outcome after a run of heads. Mia for example gave the following response to the coin toss item on the test instrument: "The coin will probably land tails up because of the number of times tossed already has exceeded the natural probability of 1 in 2 chance". The response suggests that Mia had difficulty reconciling the theoretical expected probability with the variation in the

random process associated with tossing the coin. While Sarah initially said that there was a 1 in 2 (or 50:50) chance of either heads or tails coming up in any toss, she then contradicted this by stating that the next toss would likely be a tail:

In chance every time it is thrown it is to be taken as a singular experiment so in this way 50% chance of landing either way. However, with there already having 4 heads tossed there would be potentially a higher probability of it being tails.

Sarah's response shows a conflicting combination of recognising the notion of independence but then overestimating the predictability of the behaviour of four tosses of the coin. Sarah's reasoning shows an inadequate understanding of the frequentist interpretation of probability (Dollard, 2011) in that she overgeneralised the long-run relative frequency of the proportion of heads to tails.

All seven participants gave correct responses to the spinner item in the interview by attending to the relative proportion of black and white in each of the spinners. The following response is illustrative of the typical answers received when asked to explain why the student's answer was incorrect:

Because the circle showing half and half this one is right (points to the first spinner of the group of three presented in the work sample) but the chance of getting black in this one is only umm 25% so 25% chance and then the opposite chance of getting black in the third one is 75%. [Mia]

### *Knowledge of learning (SCK)*

Although most of the participants were able to identify the errors in the work samples, they were unable to provide clear and appropriate explanations for these errors in either one or both work samples. While Janet's response, for example, implied an understanding that each coin toss is an independent event, she focused on variation without linking to long run probability:

Ok well the child's ... answered that because it's landed each time on tails and it believes that after a certain time it must therefore land on heads but because there is only the two options it may well land again on tails because history really means nothing for the next time we toss the coin; these are all individual umm items and I think this child is looking for a pattern perhaps to explain the probability whereas we all know that there's not it's just random and lands as it does.

Five participants were able to identify the equi-probability misconception evident in the students' spinner work sample, although none of the participants used this term. The following response from Sarah typifies the comments received by most participants:

It's [the number of] colours rather than actually looking at the fraction, it's fifty percent of being black and fifty percent of being right; there are no other colours, just black and white, so they've said fifty/fifty regardless of how much of it is black and how much white.

In contrast Mia and Janet demonstrated limited confidence in explaining the misconception with the spinner work sample:

Because the circle showing half and half is right so the chance of getting black in this one is only umm 25% so 25% chance and then the opposite chance of getting black in the third one is 75% so umm the understanding of it being  $\frac{1}{2}$ ; I don't really know where that has come from with the student; to be honest probability is my weakest part. [Mia]

Well now I'm thinking the child is correct because there's only the two in there (black and white) so I'm not sure if I'm getting myself confused now in relation to the quantities shaded or not I'm now very confused (laughs at herself). Probability is a biggy it really is ummm I go a bit muddled when I think really, really hard. [Janet]

While the concept of the classical interpretation (Dollard, 2011) might seem straight forward, the above responses show that this is a source of confusion with these pre-service teachers.

### *Knowledge of teaching (PCK/KCT)*

It was in this category that the confusion between the concepts of long run probability (expectation) and variation were particularly evident. Two of the seven participants, however, made an attempt to integrate these ideas in their suggested teaching strategies. For example, the following response from Mia suggests some appreciation of the perceived conflict between the concepts of expectation and variation in probability:

No matter how many times you toss it, it might show 9 heads and 1 tails one time and then 5 and 5 another time; just because there's an equal chance of it happening doesn't mean that it will, which I guess is a difficult um concept to understand because it's so abstract. I know you're not going to sit and toss a coin 1000 times to see if you get 500 tails and 500 heads because you can explain that the more times you do it the more even it's going to become because there is the even chance of it happening, so the more times you do it, I guess, the closer the difference between them (number of heads and tails) will get.

Mia's response was the closest to explaining how random processes, in this case the act of tossing the coin, can show short-run variation and long-run stability. Interestingly, she also expressed an appreciation for why students may find these concepts difficult to understand. In contrast, the other five participants had difficulty reconciling short-run variation with the predictability of the long-run relative frequency of the coin turning up heads (approaching one half). This idea was clearly unresolved for Larissa who had reservations about the usefulness of conducting an experiment to address the student's misconception in the coin toss work sample:

I don't know because you can't keep tossing the coin because you could just reinforce their misconception that I mean it has to land on heads eventually but who is to say it has to land on heads that next throw. So I'm not sure how you would help them.

Similarly Larissa grappled with the notion of uncertainty in relation to the spinner work sample.

Larissa: I'd probably get them to do it (spin the spinner) and count how many times out of 10 they got white and how many times they got black and then the same with the other ones (spinners) and see if there were any differences [ pauses and ponders for a minute or so]. That's more relying on chance than probability.

Researcher: What do you mean by chance and probability?

Larissa: Well you could still get the arrow to still land on the white bit with chance.

While an appropriate teaching suggestion would be to conduct a large number of trials to more accurately predict the probability of the middle spinner, for example, landing on black, Larissa limited her suggestion to 10 trials and still seemed doubtful that this would be of assistance. Larissa seemed perplexed that the proportion of black and white would be hard to predict over a small number of trials (10) but was unable to offer an adequate approach, or consider a larger number of trials, to address the equiprobability misconception in the spinner work sample.

## Conclusions and Implications

The results showed that the participants lacked confidence in their own understanding of some of the foundational ideas of probability and responses consistently indicated that this area of mathematics is a source of confusion for many pre-service teachers. Responses suggested that many participants had particular difficulty with reconciling the concepts of variation and expectation. Although many participants recognised the need to carry out a number of trials, there was a strong tendency to overestimate the predictability of outcomes from a small number of repeated trials. The teaching strategies identified by the participants were often tenuous, with some responses being an admission of a lack of understanding. While two participants made reference to long run probability, neither articulated a clear teaching strategy, or, if they did, suggested a small number of trials. This has particular implications for teachers and teacher educators as it is reasonable to assume that this limited understanding is likely to impact upon teaching these concepts to school students.

Although the study was limited in terms of its sample size and situation in one university, one of the benefits from the study was the insight that was provided to the teacher educator who conducted the interviews with the participants. It is often difficult to engage in such discussions in tutorial classes, whereas the interview situation provided the opportunity for the teacher educator to gain an insight into the probabilistic thinking of these future teachers, which could then provide the basis for addressing their current intuitions and misconceptions. According to Dollard (2011), teacher educators cannot assume that their students understand that probability is a measure of the likelihood of events, and it is only through opportunities to engage in activities such as carrying out experiments with repeated trials and reflecting upon the results that some of their intuitive beliefs about probability can be revealed and addressed. The study highlighted that this selection of pre-service teachers at least would benefit from such activities and that it is reasonable to conclude, as Chick and Baker (2005) found, that content knowledge is an important aspect of PCK.

## References

- Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special. *Journal of Teacher Education*, 59(5), 389-407.
- Baturo, A., Cooper, T., Doyle, K., & Grant, E. (2007). Using three levels in design of effective teacher-education tasks: The case of promoting conflicts with intuitive understandings in probability. *Journal of Mathematics Teacher Education*, 10 (4), 251-259.
- Chick, H., & Baker, M. (2005). Teaching elementary probability: Not leaving it to chance. In P. C. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Theory, research and practice* (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia, pp. 233-240). Sydney: MERGA.
- Chick, H., Baker, M., Pham, T., & Cheng, & H. (2006). Aspects of teachers' pedagogical knowledge for decimals. In J. Novotna, H. Moranova, M. Kraka & N. Stehlikova (Eds.), *Proceedings of the 30th annual conference of the International Group for the Psychology of Mathematics Education*. Prague: PME.
- Dollard, C. (2011). Preservice elementary teachers and the fundamentals of probability. *Statistics Education Research Journal*, 10(2), 27-47.
- Fischbein, E. & Schnarch, D. (1997). The evolution with age of probabilistic intuitively based misconceptions. *Journal for Research in Mathematics Education*, 28(1), 96-105.
- Hill, H., Ball, D.L., & Schilling, S. (2008). Unpacking pedagogical content knowledge: Conceptualising and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 33(4), 372-400.

- Maher, N., & Muir, T. (2013). "I know you have to put down a zero but I don't know why": Exploring the link between teachers' content knowledge and pedagogical content knowledge. *Mathematics Teacher Education and Development*, 15(1), 72-87.
- Reys, R., Lindquist, M. M., Lambdin, D. V., & Smith, N. L. (2012). *Helping children learn mathematics* (10th ed.). Hoboken, NJ: John Wiley & Sons, Inc.
- Rowland, T., Turner, F., Thwaites, A., & Huckstep, P. (2010). *Developing primary mathematics teaching*. London, UK: SAGE.
- Ryan, J. & Williams, J. (2007). *Children's mathematics 4-15: Learning from errors and misconceptions*. New York, NY: Open University Press.
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1-22.
- Skemp, R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher*, 26(3), 9-15.
- Stohl, H. (2005). Probability in Teacher Education and Development. In G. Jones (Ed.), *Exploring Probability in School: Challenges for teaching and learning* (pp. 345-366). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Wang, A. L. (2001) How much can be taught about stochastic processes and to whom? In C. Batanero (Ed.), *Training researchers in the use of statistics*, (pp. 73-85). Granada: International Association for Statistical Education and International Statistical Institute.