# Devising Principles of Design for Numeracy Tasks


Vince Geiger
*Australian Catholic University*
<vincent.geiger@acu.edu.au>

Merrilyn Goos
*The University of Queensland*
<m.goos@uq.edu.au>

Helen Forgasz
*Monash University*
<helen.forgasz@monash.edu>

Anne Bennison
*The University of Queensland*
<a.bennison@uq.edu.au>



Numeracy is a fundamental component of the Australian National Curriculum as a General Capability identified in each F-10 subject. In this paper, we consider the principles of design necessary for the development of numeracy tasks specific to subjects other than mathematics – in this case, the subject of English. We explore the nature of potential design principles by synthesising generic principles of task design from relevant literature, mapping these principles against an episode of classroom practice sourced from a project concerned with enhancing teaching in numeracy, and interrogating this mapping for elements of design that are complementary to aspects identified in the generic principles.


## Introduction

Numeracy is the capacity to make effective use of mathematics in contexts related to the workplace, personal life, and in exercising civil participation and responsibilities. Being numerate involves more than mastering basic mathematics, because numeracy connects the mathematics with life related situations that additionally require problem solving, critical judgment, and making sense of the non-mathematical context (Geiger, Goos, & Dole, 2014). Thus, numeracy also includes the capacity to find, access, and interpret quantitative information in order to make informed judgments and decisions – vital personal skills in an increasingly data drenched world (Steen 2001). The importance of a numerate citizenry to a nation's social cohesion and economic growth and prosperity is receiving increasing recognition internationally. Such attention is evident, for example, in the recently completed Programme for the International Assessment of Adult Competencies (PIACC), which identified numeracy as one of three key information-processing competencies necessary for participating in the labour market, education and training, and social and civic life (OECD, 2013).

Results from national numeracy testing (NAPLAN) and international comparisons of mathematical literacy (a term used for numeracy in some international contexts), such as PISA, however, indicate that too many Australian students fail to meet numeracy benchmarks, and identify a decline in Australia's overall standing as a nation. PISA results, for example, show that 20% of Australian 15 year olds are failing to meet International Proficiency Level 2 for mathematical literacy (indicative of whether students can demonstrate mathematical literacy competencies that will enable them to actively participate in real-life situations) and 43% of students placed below the Australian nationally agreed baseline of Level 3 (Thomson, De Bortoli, & Buckley, 2013). By comparison to the performance of other countries, Australia was ranked 19th for mathematical literacy in 2012, down from 13th in 2009 and 8th in 2006. This is a clear decline in performance over only the past eight years, which indicates potentially debilitating outcomes for individuals and for our nation's competitiveness and growth


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internationally. Numeracy has been a national education priority for more than a decade (e.g., MCEETYA, 1989) and is a *general capability* within each subject of the Australian Curriculum (ACARA, 2014). Yet, at best, there appears to be little progress in students' numeracy performance.

Since the use of mathematical tasks is central to the learning and teaching of mathematics (Artigue & Perrin-Glorian, 1991), attention to how such tasks are designed for, and implemented in, school classrooms holds potential for improving numeracy teaching and learning. As numeracy is a general capability in all school subjects within the Australian Curriculum there is a need to provide support for changing the way teachers design and implement numeracy tasks – especially in subjects outside of mathematics.

The purpose of this paper is to investigate the principles of design for tasks that are specific to the development of numeracy activities within subjects outside of mathematics. The issue will be addressed in the following way. Firstly, general principles of task design are synthesised from relevant literature in mathematics education. Secondly, these principles are mapped against an episode of classroom practice sourced from a project in which teachers were required to design tasks based on a rich model of numeracy (Goos, Geiger, & Dole, 2011). Thirdly, this mapping will be interrogated for elements of design that are complementary to aspects identified within the generic principles of design.

## General Principles for Task Design in Mathematics

As tasks are integral to many dimensions of mathematics learning, including mathematical content, processes, and modes of working, Burkhart and Swan (2013) argue for the importance of task design to improve mathematics instruction. For teachers, task selection, adaptation, and creation are intertwined with choices of pedagogies for realising opportunities that lie within specific tasks (Sullivan & Yang, 2013). Evidence that coherent research and development approaches to task design are effective in improving teaching practice is provided by the long term success of programs such as Connected Mathematics (Lappan & Phillips, 2009). At the same time, Schoenfeld (2009) argues for greater communication between designers and researchers as many designers do not make their design principles explicit, and so it is difficult for others, including teachers, to adopt effective approaches to task creation and adaptation. Thus, partnerships between teachers and researchers, where understandings of principles of task design and the effective integration of tasks with pedagogical approaches are explored, refined and documented, holds potential for improving teaching and learning practices in mathematics.

As most tasks are developed for implementation within specific curriculum and school contexts, the *fit to circumstance* of tasks with local conditions and constraints is a vital consideration for effective implementation (Kiernan, Doorman, & Ohtani, 2013). Such *circumstances* include local curriculum specifications as well as other requirements or restrictions that users can leverage or that must be accommodated, for example, resources available within a particular school.

Challenge is important for students if real learning is to take place (Hiebert & Grouws, 2007). Most guidelines for systemic improvement in learning outcomes stress the need for teachers to extend students' thinking, and to pose extended, realistic, and open-ended problems that challenge students (e.g., City, Elmore, Fiarman, & Teitel, 2009). By posing challenging tasks, and adopting associated pedagogies, teachers provide opportunity for students to take risks, to justify their thinking, to make decisions, and to work with other students (Sullivan, 2011). At the same time, students often resist engaging with challenging tasks and attempt to influence teachers to reduce the demand of an activity

(Sullivan, Clarke, & Clarke, 2013). Thus, for students to engage with the type of tasks that require the use of unfamiliar or developing capabilities, the completion of tasks must appear to be achievable, that is, tasks must be *challenging yet accessible.* In order for students to engage fully with tasks, however, activities must not only be accessible but also *transparent* in relation to their expected outcomes: that is, it is clear what is required of a student to achieve success with a task (Burkhart & Swan, 2013).

As students need to take risks in order to extend their thinking, they must be provided with *opportunity to make decisions*. Such opportunities also provide instances where students can exercise and develop their capacities to use mathematics critically (Geiger, Goos, & Dole, 2014). While closely linked to the notion of challenge, the opportunity to make decisions does not necessarily mean that highly complex or sophisticated mathematics is required to make judgments.

The articulation of carefully constructed principles for the design of a task does not guarantee the effectiveness of an activity as learning is also influenced by the choice of pedagogy. We argue that teachers must also adopt investigative pedagogies to fully realise the numeracy opportunities that such tasks afford (Goos, Geiger, & Dole, 2013). Such pedagogies must provide students with the opportunity to speculate, test ideas, and argue for or defend conjectures (Diezmann, Watters, & English, 2001). In order to be assured of the quality of a task, activities must also be developed, appraised, trialled, evaluated, and retrialed in *iterative cycles of design and improvement* (Maass, Garcia, Mousoulides, & Wake, 2013). Thus effective activities will take time to develop and require a commitment to reflective practice by teachers who aspire to be effective designers of instructional tasks.

## Research Design

The data presented in this paper are drawn from a current project aimed at investigating the potential for enhancing teaching and learning practice in numeracy across the curriculum through teacher professional learning based on a rich model of numeracy (Goos, Geiger, & Dole, 2011). The numeracy model has five dimensions: Context, Mathematical Knowledge, Dispositions, Tools, and Critical Orientation. It has been extensively outlined in the literature (e.g., Geiger, Goos, & Dole 2014; Geiger, Goos, Dole, Forgasz, & Bennison, 2013) and a summary of each dimension is presented in Table 1.

Table 1
*Descriptions of the Elements and Critical Orientation of the Numeracy Model*

| | |
|---|---|
| Mathematical knowledge | Mathematical concepts and skills; problem solving strategies; estimation capacities |
| Contexts | Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings |
| Dispositions | Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge. |
| Tools | Use of material (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners) and digital (computers, software, calculators, internet) tools to mediate and shape thinking |
| Critical orientation | Use of mathematical information to: make decisions and judgements; add support to arguments; challenge an argument or position. |

Twenty-one teachers were recruited from eight schools, four in Queensland and four in Victoria. Schools were selected with the intention of balancing public and private education sectors, socio-economic status, and location (metropolitan versus regional). Consistent with the Loucks-Horsley, Love, Stiles, Mundry, and Hewson (2003) framework for teacher professional development, teachers came together for an initial meeting to develop an understanding of the numeracy model and to participate in activities that exemplified ideas in the model. After this meeting, teachers returned to their schools to trial activities presented at the workshop, or to develop and implement activities based on the numeracy model. After a period that varied from one to two months, members of the research team visited teachers to observe lessons and to conduct interviews with teachers about their initial experiences of implementing numeracy tasks they had designed for their students in their nominated subject area. The project has continued with successive rounds of meetings where teachers were asked to report on the outcomes of their attempts to design effective numeracy tasks, school visits for the purpose of ongoing data collection, and continuing input and support for teachers in developing numeracy teaching practice.

The analysis and discussion presented in this paper are based on data collected during the second round of school visits and are sourced from field notes of two classroom observations, an audio recording of a semi-structured interview with the teacher, and photographic images of artefacts produced during one of the observed lessons.

## Classroom Vignette and Teacher Interview

We observed Kathy teach a lesson to her two Year 8 English classes, each aimed at developing students' understanding of the role of pace when reading poetry. In particular, she was attempting to assist students to improve their oral presentation skills by providing insight into the relationship between the emotions being communicated in a poem and the associated pace at which different sections of a poem should be read. On the second occasion that Kathy taught the lesson (described below), she modified the way she implemented the task based on her experiences from the previous day.

Kathy began the lesson by explaining to students that "pace", when this word was used in relation to poetry, is the speed at which a poem is read and that this speed is affected by both the content and the context of the ideas or events explored in a poem. This stimulated a discussion on how to measure pace, in which students made a number of suggestions including beats per minute, number of words against time, and syllables against time. At this point, Kathy invited one of the students to read a short poem, *So Fast* by Rick Roth, while also asking one group of students to count the number of syllables in the poem and another group to time how long it took for the poem to be read. At the end of the reading, students reported back that there had been 120 syllables read in 24.4 seconds – a rate they concluded was too fast because it did not convey the emotion they felt in the poem.

Kathy asked students what emotions they thought were being communicated within the poem. Students suggested a number of emotions including "regretful", "sad", and "angry". Kathy then asked if different emotions should be read at a matching pace and how emotions and pace might be connected. This led to a discussion about how fast the poem should be read. Students reached a consensus that it should be "slow for sad and faster for angry". At this point, Kathy asked three students to come to the front of the room and for one student to read a section of the poem slowly, another student to read at a pace that related feelings of anger, another for excited, and, on suggestion from the students, for "pumped" (very excited). When they had finished, she asked students to stand across the front of the room in an order related to the pace at which they had read the poem.

The class was then asked to name emotions that should be related to a medium pace, that is, not slow or fast, to which two students replied "happy" and "boredom". These students were also invited to stand at the front of the room at an appropriate spot between the two students representing fast and slow. The remainder of the class was not convinced, however, that they had found a "middle" emotion and so continued the discussion. Eventually they settled on the word "fine". The student who suggested "fine" was asked to stand in the middle of the group of students at the front of the room.

Kathy checked with the class if they were happy with where the students at the front were standing. Members of the class responded by making suggestions about the relative positions of students in relation to each other. After some rearrangement, students were asked to write the emotion they represented on a sticky note and place this is the appropriate position on a line Kathy had drawn on the whiteboard. She then annotated the line by placing scale marks against each sticky note and writing the emotion in larger print so the whole class could inspect the position of each named feeling. After again checking with the class that these emotions were appropriately placed, she asked what emotions were missing. Students provided the additional feelings of *laziness*, *maudlin*, *relaxed*, and *sickened*, which were also recorded on the whiteboard (Figure 1). Kathy explained to the class that they had developed a way of quantifying emotions by placing them on a line and added the title Emotion Scale to the diagram.

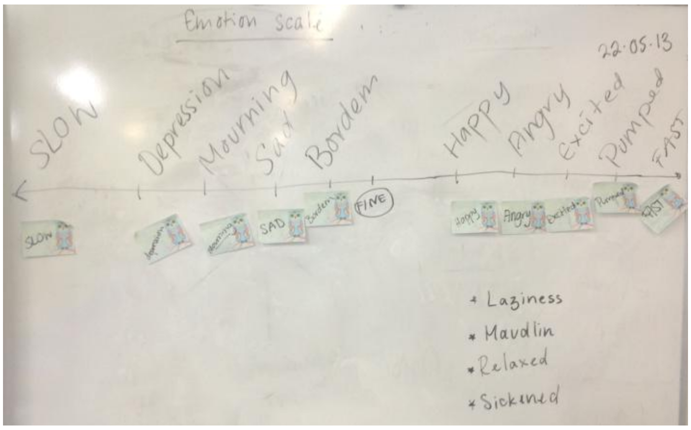


*Figure 1*: Emotion Scale

Following up on the development of the Emotion Scale, Kathy introduced a table for students to use when describing the pace at which a poem should be read. The table was structured around the headings of Poem, Emotion, Evidence, and Pace and students were asked to complete the table for *So Fast*. After students made an attempt to do so as individuals, Kathy orchestrated a discussion with the class that resulted in a completed table displayed on the whiteboard (Table 2).

Table 2
*The Pace of a Poem*

| Poem | Emotion | Evidence (words phrases) | Pace |
|---|---|---|---|
| So fast by Rick Roth | Sad and regretful | Linger ; I wish I haven't gone so fast; I contemplated the past | Moderately slow |

As part of the discussion a new pace, moderately slow, was agreed upon. The lesson concluded with Kathy providing each student with a different poem for analysis against the table she provided in preparation for reading the poem to the class at an appropriate emotional pace in the next lesson.

In the lesson we observed, on the previous day, with Kathy's other Year 8 class, she drew the Emotion Scale on the board with drawings of a snail on the left hand end and a car on the right hand end to represent slow and fast paces, respectively. As students suggested an emotion and the associated pace Kathy added the emotion to the scale. For the lesson described above, Kathy modified the task, on the spur of the moment, by having a student stand across the front of the classroom to represent each of the suggested emotions. This generated discussion amongst students about not only location on the scale but also relative position.

After the lesson we interviewed Kathy about how successful the activity had been and how she went about planning the task. Kathy explained the purpose of the task.

Researcher 1: So, what I'm curious about is how did you go about planning this task from a numeracy perspective?

Kathy: I wanted them to consciously make decisions about how fast they are going to speak, why they're going to speak that fast, and if they could defend their pace they could do so.

This was a task Kathy had not used before. We questioned Kathy about how she had developed the idea. She explained that it had been based on an approach used to illustrate differences of opinions in other English lessons that she had drawn upon and adapted.

Researcher 1: How did you come up with the idea of the scales, almost like a timeline, number line?

Kathy: I remember on prac where … sometimes you ask them to agree or disagree by moving to one end of the scale, and that kind of thing.

Researcher 1: So you'd had an experience before that you somehow drew on, on the spur of the moment – that's pretty impressive actually (laughter)…

We also asked if Kathy had used the numeracy model as part of her planning. She indicated that it was a challenge to include mathematics in an English lesson but she was surprised at how easy she found this to construct.

Researcher 1: Remember the numeracy model that we presented and you tried? Was any of that in your mind at all when you did this?

Kathy: Well a little bit, they actually needed to be able to do the maths; it was quite simple like finding how many syllables per second and we had kids tallying in their books yesterday to come up with how many syllables were in the poems.

Kathy also indicated how naturally her approach had aligned with the teaching of English, that there was a natural fit to using mathematics in developing an important concept in poetic expression.

Researcher 1: It was interesting where you said that um, you, you thought previously the numeracy that you'd been putting in the lesson was a bit superficial and that now you're trying to make it more meaningful… I thought today it just fitted very naturally and sensibly, was that how you felt?

Kathy: It's such an abstract concept, emotions, and I was afraid that we wouldn't be able to agree on where they were going … Um, no I think like I was surprised at how easy it was to do the numeracy in it, and we didn't really say, you know, this IS numeracy – we're DOING numeracy now.

## Discussion and Conclusion

Kathy had attempted to incorporate all of the dimensions of the numeracy model into the task. *Mathematical knowledge* was needed to determine the pace of a poem in the form of a rate: number of syllables per unit of time. This *mathematical knowledge* was necessary to provide a means of matching emotions to an appropriate reading pace – the *context* in which the task was set. Kathy worked with her students to develop a *representational tool* in the form of an emotional scale that aligned the emotions contained within the language of a poem with the pace it should be read. Kathy's inclusion of students in the activity through various teaching devices addressed the need to attend to students' *dispositions* when using mathematics in what would generally be considered a non-mathematical situation. By requiring students to justify their choice of pace through reference to words and phrases in the poem, she provided students with the opportunity to develop a *critical orientation*.

This activity is consistent with the dimensions of the numeracy model and also complies with the generic principles of task design. Kathy found an approach to teaching numeracy in English where the use of mathematics was a natural and complementary vehicle for connecting emotional state and pace. As indicated in the interview which followed the lesson, she considered her method to be consistent with the content and processes associated with the subject of English and so had found a way to *fit* numeracy to the *circumstance* of teaching English in a way that was consistent with the nature of that discipline. Kathy extended students' thinking by *challenging* them to *make decisions* about and justify the pace at which they chose to read sections of poetry, while at the same time providing the support necessary for students to *access* an understanding of the connection between pace and emotion. Through her approach, she also provided students with the opportunity to *investigate* the relationship between a range of emotions and how these were connected with the pace of reading a poem. Kathy had taught the lesson to another class previous to the episode described and had made improvements based on the initial experience; for example, she had incorporated the use of sticky notes in the second version of the lesson to help mark the position of a feeling within the emotion scale. Thus, she had engaged in the first iteration of a *cycle of trial and improvement*.

While Kathy had not been provided with explicit advice on how to design tasks specific to numeracy, it would appear that developing an understanding of numeracy through explicit attention to the numeracy model, complemented by support from researchers on how to implement and improve tasks, resulted in the embedding of a rich numeracy task within a non-mathematical subject – English. Further research is required in order to ascertain if such an approach is as effective in other subjects where mathematics is not considered to be a natural ally.

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