A Comparative Analysis of the 1996-1999 Calculus TEE Papers

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In this paper we present a comparative analysis of the 1996-1999 Western Australian Calculus Tertiary Entrance Examination papers. A classification scheme from the literature for question characteristics proved to be a useful tool of analysis. We show that the introduction of graphics calculators led to an enhanced role for diagrams and changes in the structure of questions on graphing, with implications for teaching practice.

Introduction

Since 1998 the availability of graphics calculators has been assumed for the West Australian public examinations of Mathematics, Physics and Chemistry at tertiary entrance (Year 12) level. This paper is part of a longitudinal study evaluating the impact of the introduction of the technology on the Calculus examination. Here we consider the characteristics of questions in the 1996-1999 Calculus papers, look in detail at the style of function graphing questions for the four years and consider implications for teaching practice. The analysis is in part a critique of our own practices as examiners for the 1998 and 1999 examinations.

Test and Examination Questions with Graphics Calculators

Examination questions can be graphics calculator *active* where use of the tool is necessary or greatly simplifies a solution, graphics calculator *neutral* where graphics calculator usage and traditional methods are equally viable, or graphics calculator *inactive* where use of the tool is not possible (Harvey, 1992). When the technology was introduced for the Calculus Tertiary Entrance Examination (TEE), the policy was to include questions in all these categories.

The presence of graphics calculators affects the selection of questions for examinations. Graphics calculators can *impact* on traditional questions by enabling alternative methods, *have no impact* because they contribute no more than scientific calculators to a solution, or *trivialise* questions by allowing solutions that require little or no mathematical input from the user (Jones & McCrae, 1996). A solution can also be significantly *reduced in complexity* without the question being trivialised. For an example with complex numbers see Forster and Mueller (1999). Questions can also be specially designed for technology usage, taking a different form to traditional questions and may include functions that students would not be expected to be able to manipulate by hand (Anderson, Bloom, Mueller, & Pedler, 1997).

In an examination, graphics calculators can be used as the first option to generate answers or used to check non-graphics calculator methods (Jones & McCrae, 1996). Checking might involve replication of a hand method, or use of different representations, such as using a graph to verify algebraic working. Where a non-calculator approach is needed for the written answer, students might use the tool to get started (Lauten, Graham, & Ferrini-Mundy, 1994), to help with working and with the final answer. It is expected that procedural work is off-loaded to the technology, for example, for the evaluation of definite integrals (Jones, 1996). Upon the inclusion of technology, the complexity and effectiveness of traditional examination questions must be re-evaluated and changes made to modes of testing students' understanding of some concepts. This paper explores these issues.

Research Method

We restricted our analysis of the Calculus TEE to the 1996-1999 papers on the basis that in 1996 the format of the papers changed to having questions ordered according to their degree of difficulty. Previously, the papers contained two sections. One with routine questions without interdependent parts and the other with longer questions, typically more demanding with interdependent parts. Of the four papers we consider, the 1996 and 1997 papers were set before and 1998 and 1999 papers after the introduction of graphics calculators.

For the analysis we used a coding scheme that we modified from one by Senk, Beckmann and Thompson (1997). First, we independently coded question characteristics according to the original scheme of Senk et al., then modified the scheme to suit the Calculus TEE (see Table 1). Finally, we independently re-coded the questions and, where we varied, we negotiated agreement guided by the official worked solutions.

The role of a diagram depends to a large extent on the ease with which a diagram may be obtained. Graphs can be quickly generated on graphics calculators, whereas without them drawing a graph might be impractical. When coding for 'Role of diagram', we took into account the absence of graphics calculators for 1996 and 1997 and their presence for 1998 and 1999. For interest, we coded 'Active', 'Neutral' and 'Inactive' for 'Graphics Calculator', whether the technology was available to be used (1998 and 1999) or not (1996 and 1997). Some questions belonged to more than one curriculum component and were recorded as belonging to each group.

Results

A summary of our comparative analysis for all questions on the 1996-1999 papers is given in Table 2. We illustrate the coding scheme with the analyses of the questions concerned with graphing of rational functions from 1996 and 1999. Their coding is provided in Table 3 and the questions are provided in below.

1996. Question 10. Let f be the function defined by $f(x) = \dfrac{-4x+5}{(x+2)(5-x)}$.

(a) State the poles of the function.
(b) Evaluate $\lim\limits_{x \to +\infty} f(x)$.
(c) Evaluate $\lim\limits_{x \to -\infty} f(x)$.
(d) Evaluate $\lim\limits_{x \to 4} f(x)$.
(e) State the x and y intercepts.
(f) Show that there are no turning points and sketch the graph of $y = f(x)$, clearly labeling all the important features.

1999. Question 13. If $f(x) = \dfrac{x^2+3x-10}{x^2+x-6}$,

(a) state the domain of f,
(b) evaluate $\lim\limits_{x \to 2} f(x)$,
(c) sketch the graph of f showing the intercepts, asymptotes and any other distinguishing features.

Table 1
Categories for Analysing Examination Questions

Category	Description
Curriculum component	Functions and Limits Theory and Techniques of Calculus Applications of Calculus Vector Calculus Complex Numbers
Skill	
Yes.	Solution requires a well-known algorithm such as solving equations or inequalities or bisecting an angle. Item does not require translation between representations
No.	No algorithm is generally taught for answering such questions, or item requires translation across representations
Level	
Low	A typical student in that course would use no more than three steps to solve.
Other	A typical student in that course would use four or more steps to solve.
Reasoning	
Yes	Item requires justification, explanation or proof or it is necessary to interpret the question before being able to start the answer.
No	No justification, explanation or proof is required. (By itself, 'Show your work is not considered reasoning.)
Realistic context	
Yes	The item is set in a context outside of mathematics (e.g. art, fantasy, science, sports).
No	There is no context outside mathematics.
Role of diagram	
Interpret	A graph or diagram is given and must be interpreted to answer the question.
Make	From some non-graphical representation (data, equation, verbal description) the student must make a graph or diagram.
Assist	The use of a diagram or sketch would simplify a solution, but is not essential for obtaining the answer.
None	No graphical representation is given or needed or a graph or diagram is given but is superfluous to answering the question.
[a]Graphics Calculator	
Active	Use of the tool is necessary to obtain a solution or it greatly simplifies the work needed to get a solution.
Neutral	It is possible to use the tool to obtain part or all the solution, but the question could be answered reasonably without the tool
Inactive	Use of the tool is not possible or is inappropriate.

[a] Over and above scientific calculator capabilities.

Table 2
Percentage of Part-questions in the Calculus Tertiary Entrance Examinations for 1996-1999 per [a] Major Curriculum Component by Characteristic

		Skill-based	Level	Reasoning	Context	Role of Diagram				Technology		
		No	Other	Yes	Yes	Interpret	Make	Assist	None	Active	Neutral	Inactive
Function and limits	1996	21	43	29	0	7	7	0	86	14	57	29
	1997	13	20	27	13	0	7	13	80	7	87	7
	1998	20	40	70	0	0	10	60	30	10	50	40
	1999	15	8	31	0	15	7	46	38	15	54	31
Theory and techniques of calculus	1996	27	45	27	0	0	0	27	73	27	9	64
	1997	0	23	46	8	0	0	8	92	23	15	62
	1998	10	40	20	10	0	0	0	100	10	0	90
	1999	20	40	30	0	0	0	0	100	10	0	90
Applications of calculus	1996	38	77	69	15	8	8	15	69	31	31	38
	1997	25	75	83	33	0	8	33	58	25	17	58
	1998	44	61	67	56	33	11	11	45	17	6	78
	1999	40	40	45	60	14	14	19	52	30	10	60
Complex numbers	1996	43	43	36	0	14	29	0	57	7	36	57
	1997	63	38	38	0	38	0	0	63	13	38	50
	1998	56	56	44	0	0	44	22	33	22	0	78
	1999	80	60	60	0	0	50	20	30	0	30	70
[b]% of total part- questions	1996	30	46	37	13	7	11	9	72	17	33	50
	1997	20	41	45	14	6	4	14	76	18	39	43
	1998	33	53	55	27	12	14	20	53	14	12	73
	1999	35	35	40	28	10	14	21	55	16	23	61

[a] Calculus of trigonometric functions (6hours of coursework) and Vector calculus (10hours) are not included as few part questions fall only into these categories.
[b]number of part-questions were 54 for 1996, 51 for 1997, 49 for 1998, 57 for 1999.

Table 3

Coding for the 1996 and 1999 Calculus TEE Questions on Graphing Rational Functions

		Skill	Level	Reasoning required	[a]Role of diagram	[b]Graphics Calculator
1996						
	a	Yes	Low	No	None	Inactive
	b	Yes	Low	No	None	Neutral
	c	Yes	Low	No	None	Neutral
	d	Yes	Low	No	None	Neutral
	e	Yes	Low	No	None	Neutral
	f	No	Other	Yes	Make	Active
1999						
	a	Yes	Low	No	None	Neutral
	b	Yes	Low	No	Assist	Neutral
	c	No	Other	Yes	Make	Active

Note. [a] No technology available for 1996; [b] answered as though technology was available for 1996.

Changes in the nature of examinations can be attributed to a variety of factors but changes in 1998 and 1999 that seem to be connected with the introduction of graphics calculators relate to skills assessed and the role of diagram. The effects differed with the various curriculum components. These aspects are discussed below.

Skills

For all four examination papers the majority of the questions were skills-based (see Tables 2 and 3). This pattern has not changed to any large extent upon the introduction of graphics calculators, but scrutiny of the questions showed that the skills that are tested have changed. For example, the question, 'Evaluate $(1+i)^7+(1-i)^7$ using de Moivre's rule, from the 1996 examination has been made *redundant* by the presence of graphics calculators The use of de Moivre's rule used to be essential for obtaining the answer to expressions of this type quickly--the other alternative of using the binomial theorem would have been much more time-consuming. Thus the ability to apply de Moivre's rule correctly was an essential skill. With a graphics calculator the simplification requires only a single line entry of the character string. Here, requiring students to use de Moivre's rule is *inappropriate*, just like asking students to use a calculator to find logarithms in order to solve an exponential problem (Jones, 1996).

A further change in the papers is that in 1996 and 1997 testing of integration techniques was based on the evaluation of definite integrals while now indefinite integrals are used exclusively because of the numerical integration capabilities of graphics calculators. Definite integrals have been incorporated to a greater extent in application questions. Similarly, questions that require factorisation of polynomials as the sole task are now absent in view of them being *trivialised*.

Role of Diagram

Diagrams now play a greater role in problem solutions (see Tables 2 and 3). There are more part-questions that *require interpretation* of a diagram or where students are *required to make* a diagram, and more part-questions where a diagram *would assist* the solution. This enhanced role for diagram may be explained in part by the relative ease with which students can obtain graphs of functions, parametric and polar curves and in part by the ease with which markers can generate graphs to 'follow through' students' solutions. In 1999 a question asked students to derive and graph the velocity function from the position $s(t)=(t^2+1)/(t^4+1)$. The derivation is potentially problematic, as is graphing the velocity function with traditional methods. The task of marking the question would have been arduous without having the technology available to check answers that followed-on from the velocity functions students obtained--without the technology available the above position function would not have been used in a question of this type.

Effects on Curriculum Components

The effects described above have impacted on the various curriculum components in differing ways. The summaries in Tables 2 and 3 suggests that diagrams, usually graphics calculator generated graphs, could have assisted in answering questions from the component 'Functions and limits' more in 1999 than previously. This enhanced role of diagrams went hand-in hand with a reduction of the amount of guidance given for graphing, in particular for graphing rational functions. Scrutiny of the 1996 question (see the beginning of the 'Results' section), set prior to the introduction of graphics calculators, shows that it is very highly structured. Part questions, which relied on recognition of properties pertaining to the functions and involved algebraic manipulation, led students item by item through features of the graph and attracted part marks. The final step of drawing the graph required integration of several pieces of information and so was demanding but once the graph was drawn no further interpretation was required. In 1999, two part questions warned students about features of the graph, as did the statement to show 'other distinguishing features'. Choosing to use a graphics calculator meant students needed to interpret the graph on it in light of properties previously established, rather than using the properties to plot the graph. For the identification of all the critical features of the graph students needed to integrate (Boers & Jones, 1994) their reading of the calculator graph with given algebraic information and with mathematical properties established in the previous two part-questions: a highly demanding task that was a major source of error (Forster & Mueller, 2000a).

Another aspect of students being able to graph readily was the use of more complicated functions. For example, in the 1999 examination the function $f(t)=\begin{cases}(1-\cos(2t))/t & \text{for} \quad t\neq 0\\ 0 & \text{for} \quad t=0\end{cases}$ was used to test understanding of limits, continuity and other properties of functions. Without access to a graphics calculator, either the graph would have been supplied or graphing would have dominated the question. Otherwise the requirement for a graph would have been omitted in favour of algebraic methods, thereby making the question too abstract to be a suitable examination question at this level. In its present form,, the question allowed students the opportunity to demonstrate their mathematical insight with an unfamiliar function.

Another change that became evident through the comparative analysis was that questions for 'Theory and techniques of calculus' (integration and differentiation techniques) now largely preclude graphics calculator usage (see Table 2). For 'Applications of calculus', there is no pattern of increased opportunity for calculator usage (see Table 2), but in 1998 and 1999 it is the component for which there were opportunities for flexible problem solving (Gray & Tall, 1991) with both traditional and technology-assisted methods equally viable. For example, graphical or trigonometric methods could have been used in a 1999 question where tidal fluctuations were modeled as simple harmonic motion (Forster & Mueller, 2000b).

The component most affected by the introduction of the calculators (see Table 2) is 'Complex numbers'. Questions have become less skills-based, need a greater number of steps to reach an answer, and call on more reasoning. That is, in general in 1998 and 1999 the questions were harder than those for 1996 and 1997 in all the dimensions that measure difficulty. Diagrams played a greater role, but usually these were not graphics-calculator generated--there was actually reduced opportunity to use graphics calculators in questions on the topic.

Overall, opportunities to use graphics calculators, had they always been available, have not increased on the introduction of the technology for the TEE: see Table 2, which gives the breakdown in usage according to part-questions. On the basis of part-marks, in 1996 and 1997 approximately 53% of all marks could have been obtained through graphics calculator active or neutral part-questions, this percentage was 26% in 1998 and 39% in 1999. The lower mark allocation is largely caused by omission of skills-based questions of the type included in 1996 and 1997 that would be trivial in the presence of graphics calculators. Questions set in real-life contexts doubled from 1996-1997 to 1998-1999 (see Table 2). This is partly attributable to preferences of examiners but also reflects a move away from procedural towards more interpretative questions where graphics calculator usage is an option.

Concluding Discussion

The comparative analysis of the 1996-1999 Calculus TEE papers has made explicit changes in questions that have accompanied the introduction of graphics calculators. The availability of technology has impacted on the way concepts are tested and on what skills are tested. The use of de Moivre's rule to evaluate a complex number raised to a given power has become redundant, and polynomial factorisation questions have become trivialised. Definite integrals are no longer suitable for testing integration techniques. Yet, the concepts associated with de Moivre's rule, polynomials and definite integrals remain important aspects of calculus. There is a need to rethink how to test students' understanding of them, and in the Calculus TEE this involved a greater role for diagrams for questions on complex numbers. An increased role for visual methods is a notable change for the papers overall.

We ask, is it fair that the emphasis in the papers has changed so much in regard to visual methods? Dreyfus (1994) points to difficulties that students have with visualisation. Roth and Bowen (1998) paint dismal pictures of teaching practices associated with graphing and of the resulting confusion experienced by students. However, visualisation is considered helpful in supporting intuition and concept formation in mathematics learning, and the use of technology in particular has the potential of allowing flexible thinking (Dreyfus, 1994).

In 1998 and 1999, the need to integrate algebraic and graphical information when interpreting technology-generated graphs of rational functions called on the flexible thinking to which Gray & Tall (1991), as well as Dreyfus (1994), refer. Based on a sample of 20% of the candidates in the Calculus TEE, errors were widespread in graphing rational functions (Forster & Mueller, 2000a; Mueller & Forster, 1999). The implication for teaching is that interpreting graphs on a calculator should be a subject of instruction, particularly with regard to incorporating algebraic information. More encouraging was the outcome that approximately 45% of students in the sample appeared to adopt a graphical approach for answering the 1999 question on simple harmonic motion, indicating flexible use of the technology.

Current reform documents, for example, the Curriculum Framework for K-12 in Western Australia (Curriculum Council, 1998) promote the use of real-life contexts that are meaningful to students. A move in this direction is evident in the comparative analysis. The possibility of using functions with which students are not familiar when graphics calculators are available opens up opportunities for applications that model real-life situations more realistically than in the past.

In conclusion, implications of the analysis are that some skills have reduced importance with the introduction of graphics calculators, but this does not apply to the concepts to which they relate. Other skills associated with graphical interpretation and visual methods seem to warrant more emphasis when technology is allowed.

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