Use of Graphics Calculators in School Tests and Examinations

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This paper reports on the outcomes of one component of a study carried out during 2001 to assess the impact of graphics calculator use on Year 12 school-based assessment in a situation where access to the calculator is assumed in the external tertiary entrance examination. The focus of the paper is on test and examination items that form part of the school-based assessment programs contributing to a student's final grade in the Western Australian Year 12 Tertiary Entrance Examination (TEE) subject *Applicable Mathematics*. Assessment items developed by participating schools during 2001 for use with *Applicable Mathematics* were collected and coded. While it was found that each of the participating schools was incorporating use of graphics calculators into questions to approximately the same extent, there was a wide variety of usage apparent within some curriculum components. This suggests there is scope for wider incorporation of the technology.

Background

Students in Western Australia sit for the external Tertiary Entrance Examinations (TEE) at the end of Year 12, and three mathematics subjects, *Applicable Mathematics*, *Calculus* and *Discrete Mathematics*, are examined. *Applicable Mathematics*, the focus of this paper, lies between *Calculus* (the most demanding) and *Discrete Mathematics* (the least demanding) in terms of difficulty. There are five components to *Applicable Mathematics:* Systems of Linear Equations and Matrices (25 hours of tuition time), Graphs and the Solution of Equations (18 hours), Descriptive Statistics (20 hours), Sets, Counting and Probability (18 hours) and Random Variables and their Distributions (24 hours). Each year, approximately 5000 students sit the TEE in *Applicable Mathematics*.

Marks obtained in the TEE contribute 50% towards the student's assessment in the subject, and the other 50% of the assessment is school-based. In *Applicable Mathematics*, the school-based assessment is to consist of 25 to 50% from extended pieces of work and 50 to 75% from other forms of assessment. Tests and examinations figure prominently in the latter but it may include checklists, homework assignments and oral presentations. In this paper, we focus on tests and examinations.

Since 1998, it has been assumed that students sitting for *Applicable Mathematics* and the other TEE mathematics subjects have access to graphics calculators. With the exception of the Hewlett-Packard HP-38G and HP-39G that have limited symbolic capabilities, calculators approved for use are non-symbolic. Given their inclusion in the TEE, it follows that students would make regular use of graphics calculators during mathematics lessons and for assessment tasks throughout Year 11 and 12, in order that

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they become familiar with their capabilities. The syllabus for *Applicable Mathematics* makes clear the expectation that students should "select and use appropriate technologies" and "appreciate the benefits of using technology in mathematics" (Curriculum Council, 2001, p. 37).

Literature Review

Internationally, the use of graphics calculators is permitted in public university entrance examinations in the USA and United Kingdom, amongst other countries. In Australia, Victoria and Western Australia were the first two states to allow use of the technology in external statewide examinations at the end of secondary schooling and this occurred in 1997 and 1998, respectively. South Australia has recently followed suit and the technology is allowed for the alternative entry paths in mathematics in Queensland. However, few empirical studies examine school-based assessment in the presence of graphics calculators at the secondary level. The study by Senk, Beckham, and Thompson (1997) into assessment in eight high schools in the USA is an exception. They identified that inclusion of the calculators and other computer technologies, even when readily available, was limited to approximately 3% of assessment items.

Amongst the literature, several articles have focused on the incorporation of graphics calculators in *Calculus* examinations, while relatively few deal with other areas of mathematics.

Forster and Mueller carried out a longitudinal study on the impact of the inclusion of graphics calculators in the *Calculus* TEE in Western Australia. This study encompassed tertiary entrance examinations from three years before and three years after the introduction of the graphics calculators. Aspects of the study included misuse and misconceptions and problems in students' use of the technology (Mueller and Forster, 1999, Forster and Mueller, 2000a). Their analysis of characteristics of questions revealed (Forster and Mueller, 2000a, 2000b) an increase of questions set in real-life contexts from 1996-1997 (pre graphics calculators) to 1998-1999 (post graphics calculators), a greater role for diagrams in the solution of problems and a change in the level of difficulty of examination questions in some curriculum components. The findings of Forster and Mueller are mirrored in the paper by Anderson, Bloom, Mueller, and Pedler (1999) who investigated the impact of graphics calculators on the assessment of calculus and modelling at the undergraduate level. They also found that the new skills needed to do mathematics in this environment necessitated a change in the tasks set to assess student achievement and envisaged questions of analysis and interpretation assuming greater importance.

Little work has been done in relation to the subjects covered in *Applicable Mathematics*. Bradley (1999) looked at the efficiency of graphics calculator use in the 1998 *Applicable Mathematics* TEE in Western Australia and Mueller, Pedler, Anderson, and Bloom (1998) evaluated students' uptake of graphics calculators in an undergraduate linear algebra unit with content that overlaps *Applicable Mathematics*.

Apart from those of Anderson et al. (1999) and Mueller et al. (1998), each of the studies reported above used as its focus data from Tertiary Entrance Examinations in Western Australia. The present study differs in that the data consist of items developed in schools as part of the school-based component of the student's assessment for Applicable Mathematics. Its focus is on how teachers are responding to the availability of graphics calculators when developing test and examination questions.

A coding schema developed by Senk et al. (1997) in their investigation of assessment and grading in high schools in three states in the USA was adapted (Table 1) to facilitate analysis of these items. The most significant change to their schema was in the coding characteristic of 'technology'. We narrowed 'technology' to 'graphics calculator' and expanded their categories of active, neutral and inactive to essential, advantageous, neutral, checking, and inactive. The categories 'essential' and 'advantageous' refer to uses over and above those possible on a scientific calculator.

Table 1
Item Characteristics and Categories

Characteristic Category	Description
Skill	
Yes	Solution requires the application of a well-known procedure. The item does not require translation between representations. (Word descriptions that aren't merely procedural, tables, graphs, algebraic.)
No	No algorithm is generally taught for answering such questions, or the item requires translation across representations.
Level	
Low	Using the most efficient method, a typical student in that course would use less than four steps for a solution.
Other	A typical student in that course would use four or more steps for a solution, or the content is new to the course.
Reasoning Required	
Yes	The item requires justification, explanation or proof.
No	No justification, explanation or proof is required. (By itself, "Show your work" is not considered reasoning.)
Role of Diagram	
Interpret	A graph or diagram is given and must be interpreted to answer the question.
Make	From some non-graphical representation (data, equation, verbal description) student must male a graph or diagram.
Assist	A diagram could assist the solution.
None	No graphical representation give is needed.
Graphics Calculator	
Essential	Use of the calculator is necessary to obtain a solution.
Advantageous	Use of the calculator greatly simplifies the work needed to get a solution.
Neutral	It is possible to use the calculator to obtain part or all of the solution; but the question could be reasonably be answered without the tool.
Checking	Calculator can only be used for checking and not for obtaining the solution.
Inactive	Use of the calculator is not possible, or can be done using a scientific calculator.

(Adapted from Senk, et al., 1997)

The purpose of the current study was twofold: to ascertain the extent to which the availability of the graphics calculator was reflected in teachers' assessment practices; and to develop a means of categorising assessment items that teachers might find of use when developing tasks for their students.

Data Collection

The authors sought the advice of the Western Australian Curriculum Council to determine schools where teachers were proactive in the use of graphics calculators. Of the eight schools nominated and approached, all agreed to participate in the study. A mathematics teacher involved in the teaching of *Applicable Mathematics* in each school became the contact person and accepted the responsibility of passing on to the authors each test and examination item used for the school's *Applicable Mathematics* program during 2001. All of the test and examination items collected were classified first in accordance with the role played by the graphics calculator in answering the question. Next, items other than those coded inactive for calculator use were coded further using the modified categories described above.

Each item was coded independently by two of the authors. In the first instance it was determined whether the use of the calculator was essential, advantageous, neutral, for checking only, or inactive. Questions other than those where the calculator was coded as inactive were further coded under the characteristics of 'skill', 'level', 'reasoning required', and 'role of diagram'. Where our coding varied, we negotiated agreement. The results of the coding were collated in order to produce an assessment profile for each of the participating schools. Examples that illustrate use of the schema are included in the results section that follows.

Results

Table 2 shows percentages of marks allocated to test and exam items used by the eight schools in the study during 2001 for which graphics calculator use was coded either essential or advantageous. These are displayed both for the five curriculum components of *Applicable Mathematics* and overall. Complete test and examination results from three schools were unavailable at the time of analysis and, because data from School C were limited, it has been excluded when calculating means.

It can be seen from Table 2 that curriculum components 1 and 5 have the highest average percentages of marks allocated to essential/advantageous graphics calculator use (25.3% and 29.7% respectively). The results indicate these are the curriculum areas in which teachers find it most appropriate and/or convenient to devise test and examination that exploit the capabilities of the instrument. Matrix functions on the graphics calculator lend themselves readily to component 1, with its focus on systems of equations and matrices. The capacity of the calculator to evaluate integrals and determine probabilities associated with the various probability distributions appear to make it a valuable tool in test and examination items for component 5. Ignoring School C under curriculum component 1, the percentage of total marks allocated to items requiring essential/advantageous use ranged from 41.6% for School F to 15.2% for School G. In curriculum component 5, percentages ranged from 45.4% for School H to 12.8% for School A.

For curriculum component 2, graphics calculator items are either reasonably well represented in tests and exams (from Schools B, D E and G with around 25% of marks allocated to items which included essential/advantageous calculator use), or modestly represented (ranging from 8.1 % to 13.4% from the remaining schools). This suggests teachers in the latter schools may not be as aware of how use of the graphics calculator can be incorporated in this component or are choosing not to include the technology.

Ignoring School A under curriculum component 3, it can be seen that School G stands out here with a 22.8% of marks allocated to items including essential/advantageous graphics calculator use. The percentage drops to 14.3% and 12.4% for Schools F and H respectively, with a considerable drop in percentages for the remaining schools. Most of the measures discussed in this curriculum component are available on scientific calculators, including fitting exponential curves to data. Where a graphics calculator offers additional benefit for the student is in enabling a quick visual assessment of the quality of the fit of a particular curve to data and in the analysis of time series. It is therefore not surprising that the percentages of questions coded as advantageous or essential in regard to graphics calculator use is not as high as in components 1 or 5.

Table 2
Percentages of Marks for which Use of the Graphics Calculator was Coded Essential or Advantageous in school-based tests and examinations

School			Curriculum Component			
	(1) Systems of linear equations and matrices	(2) Graphs and the solution of equations	(3) Descriptive statistics	(4) Sets, counting and probability	(5) Random variables and their distributions	Across all curriculum component s
A[1]	31.2	9.2	0	0	12.8	13.5
B	23.7	28.9	7.3	0	38.4	19.9
C[2]	0	8.1	3.7	0	0	5.0
D	37.7	26.4	6.3	0.9	39.4	19.3
E[1]	37.5	24.8	6.2	0	19.0	13.9
F	41.6	8.6	14.3	0	13.6	16.1
G	15.2	22.6	22.8	0	39.5	19.9
H	27.8	13.4	12.4	0	45.4	14.8
Mean[3]	25.3	19.1	9.9	0.1	29.7	16.8

[1] These two schools used a commercially prepared final examination and this was not included in the analysis.
[2] This school only provided limited data.
[3] School C was excluded when calculating means.

Regarding curriculum component 4 (sets, counting and probability), an examination of test and exam items used by schools indicates that the activities students are called upon to complete in these topics do not lend themselves to simplification through use of the graphics calculator. The usage of a graphics calculator in, and hence its impact on, this curriculum area is negligible and this is reflected in the table.

The right hand column of the table gives the school percentages of total marks from test and examination items across all curriculum components in which graphics calculator usage was essential or advantageous. Again ignoring School C, it can be seen that the percentages from all the other schools are remarkably consistent around the average of 17%, ranging from a low of 13.5% to a high of 19.9%.

We now illustrate the use of the schema (Table 1) by coding three examples taken from those provided by the participating schools and indicating the reasons for our decisions on coding. Example 1 is from curriculum component 1, Example 2 from component 2, and Example 3 from component 5.

Example 1

*A company produces three products X, Y and Z. Each of these products undergo three processes on two different floors in the factory. The time required for each process, in hours, is given by matrices **A** (floor 1) and **B** (floor 2).*

$$\textit{Floor 1} \qquad\qquad\qquad\qquad \textit{Floor 2}$$

$$\boldsymbol{A} = \begin{array}{r} \\ \text{Cutting} \\ \text{Sanding} \\ \text{Zinc Coating} \end{array} \begin{array}{c} \begin{array}{ccc} \text{X} & \text{Y} & \text{Z} \end{array} \\ \begin{pmatrix} 3 & 1.5 & 1.1 \\ 6 & 2.8 & 1.2 \\ 1.3 & 1.5 & 1.1 \end{pmatrix} \end{array} \qquad \boldsymbol{B} = \begin{array}{r} \\ \text{Assembly} \\ \text{Polishing} \\ \text{Packing} \end{array} \begin{array}{c} \begin{array}{ccc} \text{X} & \text{Y} & \text{Z} \end{array} \\ \begin{pmatrix} 0.7 & 1.8 & 2.1 \\ 1.2 & 1.3 & 2.3 \\ 0.3 & 0.2 & 1.5 \end{pmatrix} \end{array}$$

*The cost for each process per hour is given by matrices **C** and **D**.*

$$\boldsymbol{C} = \begin{array}{c} \begin{array}{ccc} \text{Cutting} & \text{Sanding} & \text{Zinc Coating} \end{array} \\ (\$10.30 \quad \$5.57 \quad \$12.80) \end{array} \qquad \boldsymbol{D} = \begin{array}{c} \begin{array}{ccc} \text{Assembly} & \text{Polishing} & \text{Packing} \end{array} \\ (\$12.50 \quad \$4.20 \quad \$1.53) \end{array}$$

Determine whether the following expressions would have meaning in the context of the problem. If so, find the answer and explain what the resulting matrix represents.

(i) $\boldsymbol{A} + \boldsymbol{B}$ *(ii)* $\boldsymbol{AB}$ *(iii)* $\boldsymbol{AC}$ *(iv)* $\boldsymbol{AD}$

(v) $\boldsymbol{AC}^T$ *(vi)* $\boldsymbol{B}^T\boldsymbol{D}^T$

Note: $\boldsymbol{M}^T$ *refers to the transpose of matrix **M** (columns and rows interchanged).*

We coded use of the graphics calculator as being 'advantageous' for this question because, although multiplication of 3 x 3 matrices can be done without it, it does take away the tedium of doing so and helps reduce the incidence of error. The question was coded as 'no' for 'skill' since it did not rely solely on the application of algorithms, 'other' for 'level' because of the number of steps involved to answer the question as a whole; 'yes' to 'reasoning required' because explanation was needed; and 'none' for 'role of diagram'. We chose to present this question because of the graphics calculator use it entailed and thought that it was a good question because the student was referred back to the context of the question in order to provide a solution. The question is an example of how access to the graphics calculator can allow greater scope for setting questions in a real-life context.

Example 2

(a) *Solve the following inequality, correct to 5 decimal places.*

$$5-\frac{3}{x}<6-x^2$$

(*b*) *What integral values of k will give three solutions to the equation below?*

$$5-\frac{3}{x}=k-x^2$$

We coded use of the graphics calculator 'essential' for both parts of this question. For the first part it is necessary to find the points of intersection of the two graphs. This can be done either via the equation solver of the graphics calculator or via graphs of the functions. Consideration of the graphs of the functions helps to ascertain that there is only one point of intersection of the two functions and assists in determining the lower bound of the interval and so might be more beneficial than using the equation solver. For part (b) consideration of the graphs in part (a) will alert students to the fact that translation of the parabola will lead to an increased number of points of intersection and so greatly helps in finding the least integer value of k required in part (b). Both parts were coded 'yes' for skill, since a procedure is taught; 'low' for level, as each required less than four steps to solve when using the graphics calculator; 'no' for reasoning required, as no explanation was sought; and 'assist' for role of diagram.

Example 3

Suppose that x is a continuous random variable with a probability density function given by:

$$f(x) = k(1-x^2) \qquad \text{for } -1 \le x \le 1$$

(*a*) *Find k.*

(*b*) *Evaluate* $P(-1 < x < 0.5)$.

(*c*) *Find t (to two decimal places), if* $P(x > t) = 0.35$.

Use of the graphics calculator was coded 'advantageous' for each part of this question since utilisation of the solve and integration facilities could be used to automate the work. Each part was coded 'yes' for skill since a procedure is taught, 'low' for level as no more than three steps were required, 'no' for reasoning required as no explanation was sought, and 'none' for role of diagram.

Discussion

To measure the impact of graphics calculators on test and examination questions, a coding schema was adapted from one developed for a previous study (Senk, et al., 1997). Application of this schema revealed that around 17% of the marks allocated in tests and examinations were to questions for which use of the graphics calculator was either essential or advantageous.

Closer examination of the coding revealed a wide discrepancy between participating schools in their incorporation of the use of the graphics calculator in four of the five curriculum components of *Applicable Mathematics*. In the fifth of these, Sets, Counting and Probability, the content does not readily align to functions that are unique to the graphics calculator, but uses functions also on a scientific calculator. The wide range of incorporation in the other four components, for example 12.8% to 45.4% in Random Variables and their Distributions, suggests some teachers may be better informed about the capabilities of the calculator as they relate to the *Applicable Mathematics* curriculum or are choosing to retain traditional approaches. Analysis of the sample items collected indicates there is considerable scope for the calculator to have a wider impact on assessment practices in *Applicable Mathematics*.

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