Convergence or Divergence? Students, Maple, and Mathematics Learning

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We continue to report on research into outcomes that emerge when symbolic manipulator software forms a central component of undergraduate teaching. Preliminary data led to the identification of 7 categories that encompassed the types of student questions and blockages arising during interactive laboratory sessions. Examples illustrative of the categories are provided and implications discussed. Additionally we report and discuss the results of successive applications of test items designed to assess the impact of complexity (defined in terms of syntax, and function choice and specification), on task demand.

Over a decade ago James Fey surveyed developments in the use of technology in mathematics education to that date. In noting that there was no lack of speculative writing on the promise of the revolution that was expected to follow he drew attention to the paucity of data available to back extravagant claims.

> It is very difficult to determine the real impact of those ideas and development projects in the daily life of mathematics classrooms, and there is very little solid research evidence validating the nearly boundless optimism of technophiles in our field. (Fey, 1989)

This comment seems as relevant today as it was over a decade ago, even if the questions have become more refined. The literature confirms the existence of diverse factors that impact on the development and testing of theoretical frameworks, and on the conduct of practice. Among these sources, papers addressing the use of technology in undergraduate mathematics make for interesting and varied reading. For example:

> The impact upon educational practice of powerful software like Mathematica has been less profound than optimists hoped or pessimists feared … (there is a) tendency to begin by looking for electronic ways of doing the familiar jobs previously done by textbooks and lectures. (Ramsden, 1997).

> Of all the flaws in our mathematics training this seemed to us to be the most dangerous and insidious, for as we removed mathematics from our courses in response to 'student failings', the need for mathematics to do real science was in fact increasing … firstly there was the pious hope that a computer assisted approach would require less staff … problems arose from attempts to use Mathematica in two ways-which were incompatible. Was software an arena for exploration of mathematical ideas, or a channel for their transmission? (Templer et al, 1998)

> There is growing evidence (in the UK and elsewhere) of a general decline in the mathematics preparedness of science and engineering undergraduates … one response has been to simply reduce the mathematics content and to rely on computer-based tools to do much of the mathematical computation … difficult questions (emerge) at the intersection of cognitive and epistemological domains; to what extent must the structure of mathematics be understood in order for it (technology) to be used effectively as a tool? (Kent & Stevenson, 1999)

Such are some of the challenging and problematic issues that have emerged, and continue to challenge, undergraduate mathematics education.

B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas (Eds.) *Mathematics Education in the South Pacific* (Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia, Auckland, pp. 285-292). Sydney: MERGA. ©2002 MERGA Inc.

Technology-aided Mathematics Programs

The form of computer-based instruction indicates a range of beliefs among program designers and instructors - both about mathematics, and the nature of mathematics learning. Olsen (1999: 31) discusses one of the most extensive examples of technology used to provide automated instruction. She describes how politicians visiting Virginia Tech's Mathematics Emporium, a 58 000 square foot (1.5-acre) computer classroom:

> see a model of institutional productivity; a vision of the future in which machines handle many kinds of undergraduate teaching duties-and universities pay fewer professors to lecture ... On weekdays from 9 am to midnight dozens of tutors and helpers stroll along the hexagonal pods on which the computers are located. They are trying to spot the students who are stuck on a problem and need help.

This factory model assumes that mathematics is something primarily to be delivered and consumed. By contrast Shneiderman et al (1998) describe a model, in which courses are scheduled into electronic classrooms following a competitive proposal process, and in which active engagement with an interactive, multimedia environment. is mandated.

In between the extremes exist a variety of models of instruction, concerned in varying degrees with factory production on the one hand, and student understanding and engagement on the other, and it is instructive to read comments describing the characteristics of such programs. Here are some selections:

Templer et al (1998) noted problems accompanying efforts to provide meaningful learning that were perceived to arise as a direct result of a symbolic manipulator (*Mathematica*) environment. They noted that typically having mastered the rudiments;

> the majority of students began to hurtle through the work, hell bent on finishing everything in the shortest possible time.

The following comment, or a close relative, was noted as occurring frequently among the students:

> I just don't understand what I'm learning here. I mean all I have to do is ask the machine to solve the problem and it's done. What have I learned?

Kent and Stevenson (1999) in elaborating on their concerns about student quality (see above), question whether mathematical procedures can be learned effectively without an appreciation of their place in the structure of mathematics. They argue that unless some kind of breakdown in the functionality of some concept or procedure (say integration) is provoked, students do not focus on the essential aspects of that concept or procedure. On the other hand they observe that the formal demands of a programming environment serves both to expose any fragility in understanding, and to support the building and conjecturing required in the re (construction) of concepts by learners. This comment engages a debate about whether computer technology should be employed following prior understanding of mathematical concepts and procedures (Harris, 2000), or as a means integral to the development of such understanding (Roddick, 2001). Clearly this matter is not yet resolved.

As noted previously, increasing uses of technology have been linked with perceived decreases in the prior mathematics preparation of undergraduates, and in some quarters at least, there seems to be an implicit belief in the existence of 'good old days'. In fact learning problems have always existed (see Gray (1975) for an Australian example), and their recent escalation is a dubious justification for the introduction or increase of computer-assisted learning. Ultimately the success or failure of any teaching approach resides in the quality with which students engage the learning mediums provided, and the extent to which mathematical integrity, rather than medium specific properties is the arbiter of understanding and quality. Mathematically related problems specifically associated with the introduction of screen-based learning are increasingly being documented (Templer et.al., 1998; Kent & Stevenson, 1999; Guin & Trouche, 1999; Lagrange, 1999; Doerr & Zangor, 2000). Doerr & Zangor refer to student preference for calculator output over contextual reality, whereby students insisted on working with 6 decimal places on problems whose data involved using crude measuring devices. Guin & Trouche report on the erroneous impact of graphic representation of functions. Left alone to experiment students inferred results that were wrong-for example the number of solutions to the equation tan x = x deduced from a screen display featuring six intersections. This gap between real mathematics and the image of mathematics depicted by a screen suggests the emergence of a new 'tyranny of the screen' as an authoritative source, replacing, and possibly more insidious, than the traditional 'tyranny of the text'. Lagrange identified problems associated with the interpretation of output from symbolic manipulator (Derive) software. Specifically, rather than building mathematical meaning from screen feedback as expected, students' perceptions were invaded by the properties of the software. For example, in noting the output of the 'Expand' command on the square of algebraic sums, rather than focusing on mathematical regularities such as the number of terms in the sum and in the expansion, students instead focused on the order of terms in the expansion, a regularity linked only to the software, and of no mathematical significance.

The Study Context

The first-year undergraduate mathematics course, forming the context of this study provides for a population of several hundred students studying within Science and Engineering degree courses. As presented in 1999 and 2000 the courses comprised a lecture series complemented by weekly workshops, in which approximately 40 students were timetabled into a laboratory containing networked computers equipped with *Maple* software. The lecture room was fitted with computer display facilities so *Maple* processing was an integral and continuing part of the lecture presentation. During laboratory workshops two tutors and frequently the lecturer also, were available to assist the students working on tasks structured through the provision of weekly worksheets, and to answer their questions. Unscheduled additional access to the laboratory was available for approximately 5 hours per week. Preliminary observation of students, taken together with issues raised in the literature, led to the formulation of three research questions.

1. What is the distribution, and characteristic of questions raised by students while engaged in learning mathematics within a symbolic manipulator environment?
2. What types of blockages cause progress to be stalled?
3. What factors contribute to the demand of tasks in which *Maple* and mathematics interact?

Data Sources

On the basis of a review of 1999 data (Galbraith & Pemberton, 2000) a set of categories (see Table 1) was defined to structure the 2000 data collection. For each laboratory session the course tutors recorded the range of questions asked of them by students in terms of these categories, together with examples of each type. Analysis of the collated responses enabled the first two research questions to be addressed. A second source of data was derived from a test given 7 weeks after the program started. This test, a voluntary exercise, comprised a series of questions to be addressed with the assistance of *Maple* in its laboratory context. It provided formative feedback to the students on their performance, and was relevant to preparing for their formal assessment. Questions ranged from simple school level manipulations to new material introduced in the tertiary program. The set of basic test questions (13) remained substantially the same with three of the original questions replaced in 2000. Sample questions are included in the appendix.

Complexity

In addressing question 3 we note the importance firstly of accurate syntactical representation of common elementary operations and terms, and secondly the demand imposed by the correct selection and specification of functions necessary to achieve identified mathematical ends. We seek to relate performance and hence task demand to the influence of these two factors (labelled respectively *Syntax* and *Function*). For use in analyses these descriptors were redefined with greater precision than the more generic interpretations adequate for the descriptive levels appropriate for questions 1 and 2.

Syntax: refers to the general *Maple* definitions necessary for the successful execution of commands. These include the correct use of brackets in general expressions, and common symbols representing a specific syntax different from that normally used in scripting mathematical statements (such as *, ^, Pi, g:=).

Function: refers to the selection and specification of particular functions appropriate to the task at hand. Specific internal syntax required in specifying a function is regarded as part of the *Function* component, including brackets when used for this purpose. Complexity is represented by a simple count of the individual components required in successful operation. We illustrate how these definitions work, by applying them to two of the examples given in the appendix.

Q1. *Syntax*: Incidence of ^ [4] *plus* ( ) [2]; total=6.
Function: General structural form of evalf (argument); evalf [1] *plus* ( ) [1] *plus* argument [1]; total=3.
Q11. *Syntax*: Incidence of*[2] *plus* ( ) [3]; *plus* y [1] *plus* := [1]; total=7.
Function: General structural form of plot(function, domain); sub-total [5] *plus* domain specification [1]; General structural form of int(y, integ interval); sub-total [5] *plus* (subtraction) [1] *plus* integration interval specifications [2]; total=14.

Similar pairs were assigned to each of the questions forming the test samples. Scoring was on a correct/incorrect basis, with the success rate on the questions defined by the fraction of students obtaining the correct answer, given that the question was attempted. This differs from the approach to the preliminary data described in an earlier paper (Galbraith & Pemberton, 2000) where the absence of an attempt was defined as failure. The success rate may be interpreted as a measure of the probability of success, of a student on respective questions. We note that for purposes of analysis the population is the set of mathematical tasks (questions) in which *Maple* is invoked as the means of solution. The student responses are used to assign the probabilities of success for the sample of questions used in the study.

Results

A summary of responses relevant to research questions 1 and 2 is contained in Table 1, showing the distribution of student questions and comments; and illustrated in the structured collection of substantive examples (Figure 1). A degree of judgement is occasionally involved in allocating questions and comments to one category rather than another, but the distribution (generated from approximately 1300 items) is believed to be a robust representation.

Table 1
Student Question Distribution (2000 data)

Question Category	Number
1. Identify problem caused by a typo (TYPO)	109 (8.4%)
2. Resolve syntax error (SYNTAX)	333 (25.7%)
3. Problem with function choice (FCHCE)	60 (4.6%)
4. Problem specifying function (FSPEC)	192 (14.8%)
5. Stuck on mathematics (MATHSTUCK)	197 (15.2%)
6. Stuck on *Maple* (MAPLESTUCK)	254 (19.6%)
7. Interpreting aspects of output (INTERPRET)	152 (11.7%)

It is significant that both mathematical issues and software issues feature prominently (Table 1), a characteristic made explicit through the examples illustrating the respective categories (Figure 1). It is clear from these examples that both mathematics and *Maple*, separately and together, feature prominently in the questions that students ask, and in the blockages that cause progress to stall.

TYPO:
Spelling, o for 0, Missing parts (= from :=; one of a bracket pair)
SYNTAX:
- *Case sensitivity e.g., pi for Pi;*
- *Missing characters e.g., symbols (* in 2*x etc); brackets (ln3 for ln(3) etc);*
- *Confusion with calculator syntax e.g., e^x for exp(x);*

FUNCTION
Function choice:
- *Confusion between alternatives; e.g., 'simplify' vs 'expand' vs 'factor', 'evalf' vs 'convert', 'fsolve' vs 'solve'; 'angle' vs 'dotprod'.*
- *Problems within a selection field; e.g., left or right or general 'limit', using 'subs' with 'function'.*

Function Specification:
- *Form; not honouring function syntax x=0...1 etc, specifying vector as a matrix, specifying sequences, using hybrid notations e.g., y (x): =x-7 then Plot (y, x=0..8) etc.*
- *Problems within functions; non-matching dimensions in matrix, evalf in function*

STUCK ON MATHS
- Basic Maths*: e.g., order of operations, difference between significant figures and decimal places, common denominators, floating point vs exact form, difference between function and expression.*
- *Advanced Maths (definitions)* e.g., meaning of continuity, meaning of lim x->0+, What is a second order derivative? What is a partial derivative? Classifying stationary points, formal definitions (e.g. continuity and limits), What are orthogonal vectors? What is an AP? What is a Taylor series?

STUCK ON MATHS (CONT)

- *Advanced Maths (procedures)* e.g., Finding partial fractions, how to solve simultaneous equations by hand, how to find a function value to ensure continuity at a point, How does a (Taylor series) expansion around a point work? What does "justify your answer mean?" Finding approximations for errors in series, calculating percentage change, setting up dot products, matrix multiplication.

STUCK ON MAPLE

- *Syntax Blockages* e.g., What does '^' mean? What is the difference between pi and Pi? How do I put a square root sign in Maple? How do I get 5 dec places using %? Mixing together functions, expressions, diff and D; Specific woes e.g. Taylor series syntax.
- *Procedural Issues* e.g., How to insert Maple input and delete previous output? How do I delete lines? Get cursor back? Write text? Print? Cut and paste? Must I use Maple if I can do it in my head?
- *Operational Blockages* e.g., Trying to use already assigned variables; How do I (in Maple) solve 2 simultaneous equations? Do definite integrals? Find first and second derivatives? Approximate Pi? Develop sequences? Do absolute value? Set up systems of equations? Find sums of terms of sequences?

INTERPRETATION

- *General issues* e.g., How do I know if the Maple answer is right? What does this result mean? Is this what I'm meant to be getting? Where did this answer come from? What does undefined mean?
- *Specific Maple issues* e.g., What did Digits:=3 do? What does the 5 in 'evalf (%, 5) do? What does 'collect' do"? What does it mean if 'solve' returns nothing? Is that output of solve (F1, x) right? What does 'zip' do? What is the 'order' term? Many questions about graphical output.

Figure 1. Student response summary (2000).

For research question 3, sample data for the year 2000 are contained in Table 2, with regression analyses for both years summarised in Tables 3 and 4. The analysis sought the impact of *Syntax* and *Function* on probability of success (*Success rate*).

Table 2
Illustrating Complexity and Success Rate (2000)

Question	1	2	3	4	5	6	7	8	9	10	11	12	13
Syntax	6	3	9	5	3	4	6	8	5	10	7	5	2
Function	3	3	3	5	6	11	6	10	5	11	14	6	6
Success rate	0.89	0.92	0.67	0.89	0.96	0.63	0.81	0.63	1	0.56	0.59	0.93	1

Table 3
Regression (2000 & 1999.)

Regression Statistics	2000	1999
Multiple R	0.904	0.884
R Square	0.817	0.782
Adjusted R Square	0.781	0.738
Standard Error	0.078	0.112
Observations	13	13
F value	22.374	7.906
Significance of F	p<0.001***	p<0.001***

Table 4
Regression Continued (2000 & 1999-in brackets)

	Coefficients	Standard Error	t Statistic	P-value
Intercept	1.202 (1.206)	0.063 (0.078)	15.424 (19.005)	<1E-7 (<1E-7)
Syntax	-0.040 (-0.050)	0.010 (0.013)	-4.108 (-3.754)	0.002** (0.004)**
Function	-0.025 (-0.028)	0.007 (0.010)	-3.705 (-2.641)	0.004** (0.025)*

We note the significant impact of the variables *Syntax* and *Function* on success rate is confirmed for both years. *Syntax* errors penalise those who lack sufficient care in expressing their work symbolically, while the demands imposed by *Function* are proportional to the principles and sophistication of the associated mathematics. Note also the substantial amount of variance accounted for (above 70%).

Reflections

Reviewing the scope and nature of symbolic manipulator supported learning, we note the overt purpose of reducing procedural load though the use of automatic processes. Thus we identify a reduction in the demand for procedural knowledge that could ostensibly lead to improved performance. However we note that achieving mastery of procedures also enables them to be encoded as conceptual knowledge, adding to the network of information available for recall and action in future situations. So is a potential gain on procedural swings lost on conceptual roundabouts? The setting up of *Maple* solutions requires not only that all relevant mathematics concepts be activated, but that *Maple* functions also be appropriately selected and accurately specified using precisely defined syntax. It might be argued then that for students with incomplete learning schemas, the conceptual load, as distinct from the procedural load, is increased in comparison with traditional written approaches? The questions and blockages identified from student workshop activity indicate that the mathematics and software are closely entwined, with problems emerging from each separately and from both in combination. In relation to the literature the students do seem to interrogate the screen more carefully than in some reports. Their responses certainly challenge beliefs in factory models of delivery that imply that computers essentially transmit knowledge for consumption and absorption by learners. By their own expressed reactions via the medium of comments and questions, the students also challenge thinking that would see technology as a means of compensating for poor or absent background knowledge. This inference is strengthened by the analysis relevant to our third question, through which we note that while achieving more rapid and efficient closure to algorithmic procedures, the use of *Maple* has not reduced the need for the mathematical attributes of understanding and attention to detail. This can be observed in the significant impact of the variables *Syntax* and *Function* on success rate. So when used as a learning tool our experience suggests that symbolic manipulator software, far from simplifying demands throws into relief learning issues that add substantially both to our understanding of student problems, and to the challenge of meeting them. On the other hand for those students who possess sound understanding and due regard for precision, the *Maple* environment has provided a means to progress rapidly and successfully at a greater rate than could otherwise be achieved. These latter use the software as a *power tool* rather than a *learning tool*. That is they are able to use its capacity to extend the boundaries of their capability in much the same way that experts use programs in pursuit of their own high level goals. Perhaps one of the immediate challenges in technology-aided learning is to distinguish between such alternative approaches, capabilities, and learning needs. This is where real individual differences may lie, rather than in the more popular assumptions associated with the freedoms provided by individual access to machines and flexible scheduling.

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Appendix

Sample Maple Test Questions

The ordered pairs following the sample questions provide measures of: (*Syntax*, *Function*).

1. *Express* $\dfrac{12^5+23^4}{7^8+5^9}$ *first as a fraction, and then as a decimal.* (6,3)

3. *Find a simpler form of* $(x^2+1)^3+(x^2-1)^3-2x^2(x^4+1)$ (9,3)

10. *If* $f(x)=(x^2+1)^{1/2}+(x^2+4)^{1/2}-x$, *for* $x \geq 0$, *find where* $f'(x)=0$. (10,11)

11. *Plot* $f(x)=(x-1)(x-2)(x-3)$.
Hence find the physical area under the graph from $x=1$ *to* $x=3$. (7,14)

12. *Evaluate* $\int_0^1 \sqrt{1+4\sin^2 x}\, dx$ (5,6)