Difficulties Learning Simple Addition Facts: A Persistent Problem

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Research carried out amongst adolescent students with learning difficulties is consequential to the delay/deficit interpretation of performance: Is performance immature and development simply delayed or does a processing deficit underlie performance difficulties so that development is impeded? This study investigated what strategies three students in Year 10 applied to simple addition problems. Performance was analysed on a trial-by-trial basis and strategy variability within problems was examined. After participating in ten years of formal schooling, these students still did not know the basic addition facts (including tie facts, and facts to problems with an addend of two and ten). Their performance based on strategy use was not consistent with a processing deficit interpretation but neither could it be considered simply delayed. Understanding the extent of the difficulty experienced by some students in acquiring basic number facts is imperative to developing appropriate interventions.

If students have difficulty retrieving simple arithmetic facts then they are likely to experience difficulty in other areas of maths learning (Ackerman, Anhalt & Dykman, 1986, Geary, 1994). Inefficiencies in basic fact knowledge have been identified as antecedents of difficulties experienced by students understanding and performing word problems, subtraction and multi-digit addition (Zental, 1990, Cumming & Elkin, 1999).

Students experiencing learning difficulties are not likely to develop a reliance on retrieval to perform simple addition problems in a way that is consonant with their peers (Russell & Ginsburg, 1984; Fleischner & Garnett, 1987; Gray, 1991; Geary, 1990; Goldman, Pellegrino, & Mertz; 1988). These students are referred to in the literature as being: 'learning disabled', 'mathematically disabled', 'not-so-good' and of 'below average ability'.

Developing retrieval strategies to perform simple addition problems involves a complex process of change whereby more efficient procedural strategies supplant less efficient strategies (discovered or constructed by the learner) until eventually answers are retrieved from a network representation of facts (Ashcraft, 1982; Siegler, 1995). Less efficient strategies are referred to as Count All Strategies and include the Long-Sum Strategy and the Sum Strategy (sometimes referred to as the Sum Strategy and Shortcut Sum Strategy respectively). (Using the example 3+5, each strategy is illustrated in Table 1.) More efficient counting strategies are referred to as Count-On Strategies and include the Right Strategy (sometimes referred to as the Max Strategy) and the Min Strategy. Decomposition strategies can also be applied to problems and involve applying a known fact to derive an answer.

Table 1
A Description of Strategies.

Strategy	Description	No. of counts
Longsum	Start at one and count three on one hand (or using any concrete material) then start at one and count five on the other hand, then count each finger up.	16
Sum	Start at one and count to three and then continue to count another five.	8
Right	Start at three and count on five.	5
Min	Start at five and count on three.	3
Decomposition	Eg. double three and count two.	2

Studies based on identifying strategy use by self-report and/or observation have revealed that during certain stages of development, a range of strategies are applied by children to perform simple addition problems: less efficient strategies co-exist with more efficient strategies and continue to compete for selection (Svenson, Hedenborg, & Lingman, 1976; Fuson, 1982; Carpenter & Moser, 1984). For example, Siegler and Jenkins (1989) described one preschool child who applied five different strategies, in mixed order of efficiency, on nine presentations of the problem 5+3. Research into strategy use must take into consideration strategy variability.

Students with learning difficulties do not use retrieval strategies (Geary, Widaman, Little & Cormier, 1987; Geary & Brown, 1991; Russell & Ginsburg, 1984) or decomposition strategies (Gray, 1991) to the same extent as their normally achieving peers. These differences have been interpreted in the literature as being indicative of either a processing delay or a processing deficit (Goldman, Pellegrino, & Mertz, 1988). The delayed view implies that students with learning difficulties do invent for themselves more efficient strategies and that they execute these strategies with increased speed and accuracy over time, but the developmental process is much slower than that documented for 'normal' students. The deficit view implies that students with learning difficulties follow a fundamentally different sequence of skill acquisition for simple addition and require alternative instruction to successfully move-on to develop a reliance on retrieval.

A delay versus deficit interpretation of a learning difficulty remains a topical issue in mathematics (Geary, 1993) as it does in other domains such as reading (Kulak, 1993). An area of enquiry consequential to the delay/deficit debate that has to date been neglected is the simple addition performance of high school students with learning difficulties. This tendency to focus on younger, primary aged children, is common in the learning disabilities research field (Durrant, 1994). Consistent with a delayed view of development, one would expect retrieval to start emerging as the dominant strategy in simple addition for the adolescent student with learning difficulties. Consistent with a deficit view of development, one would expect these students to rely on procedural strategies and other 'unusual' strategies.

This study investigated what strategies were applied to simple addition problems by three adolescent boys (in Year 10) who were experiencing difficulties learning maths. The study was part of a larger study completed in fulfilment of a Ph.D degree (Hopkins, 1999). A single case study methodology was appropriate given the level of detailed, trial by trial analysis required to account for strategy variability (Siegler, 1987; Delaney, Reder, Staszewski, & Ritter, 1998).

Method

The Participants

All three students were in Year 10 at a regular, suburban high school and attended a remedial mathematics class where there was no focus on mastering basic arithmetic facts. Their teacher encouraged them to use a calculator when required and he adhered to a similar but modified curriculum taught in the regular class, which included percentages, indices, compound interest and binomial expansions.

Michael was 16 years old and his performance on the Raven's matrices test (Raven, 1938) was between two and three standard deviations below the mean calculated for his age group (de Lemos, 1989). He was a very enthusiastic student but had difficulty fitting in with his peers at a social level.

Richard was 17 years old and his performance on the Raven's matrices test was between one and two standard deviations below the mean for his age group. He was generally not interested in classroom activities, preferring instead to develop his talent at technical drawing and graffiti art.

Steven was 15 years old and his performance on the Raven's matrices test was also between one and two standard deviations below the mean for his age group. Despite his great difficulty in reading, Steven was a very diligent student who meticulously copied notes from the board and from books.

Procedure

Each student individually came from the classroom to a vacant room within the library to participate in the study. Each school day students performed a practice set of 26 simple addition problems from a problem set of 65 problems until each student had performed the problem set exactly five times (in random order). Due to illness, holidays, and various individual timetable constrains, practice extended over a period of between 16 and 23 school days.

The problem set (displayed in Figure 1) included simple addition problems (problems with addends less than or equal to ten), as well as ten problems with one addend between 10 and 20 to promote the use of decomposition (Siegler & Jenkins, 1989). Only problems with the first addend smaller or equal to the second addend were included to allow for use of the Min Strategy to be distinguished from use of the Right Strategy (Carpenter & Moser, 1984). Problems with an addend of zero were not included.

Problems were displayed on a personal computer in the format m+n. After each trial the student was asked to explain the strategy they had used and they entered their answer into the computer. All performance was videotaped and comments were transcribed. Students were instructed to obtain an answer using whatever strategy they chose and told that speed and accuracy were equally important. The computer was programmed to keep a record of whether the answer was correct or not, and the time taken to perform each problem. (Reaction time results are not reported in this paper). Strategy use was identified by self-report and checked for consistency with observed behavior.

Results

Michael made a total of 34 mistakes, representing an error rate of approximately 10.5%. Richard made 19 errors in total, representing 6.1% of trials (based on a reduced total of 312 trials due to a computer error). Steven made 17 errors representing 5.2% of trials. Only strategy use on correct trials is reported in this paper.

Combining all the correct trials from the five time intervals, the three most common strategies applied were retrieval, decomposition and the Min Strategy. The percentage of trials performed using these three strategies, for each student, is displayed in Table 2.

Table 2
Percentage Strategy Use

Students	Min Strategy	Decomposition Strategy	Retrieval
Michael	71.5	0	22.7
Richard	21.2	8.9	69.3
Steven	64.3	1.0	33.8

There was no evidence of unusual strategy use. On one occasion Michael used a Sum Strategy, a Right Strategy and on two occasions, a multiplication strategy. On one occasion Steven reported using a guessing strategy.

To display constant and variable strategy use for each student, problems were categorised into one of four problem groups depending on what strategies were applied. Constant groups were labelled Allmin, Alldec and Allret problems. If a student correctly retrieved a problem on all five trials then this was classified as an Allret problem. (Classifications were based on performance of correct trials only and therefore may have been based on four trials or less.) Similarly Alldec and Allmin problems refer to problems consistently performed by a student using decomposition or the Min Strategy respectively. The fourth group labelled Variable problems consisted of problems that were performed correctly by a student using more than one strategy. To minimise the possible number of groups for classification purposes, a multiplication and guessing strategy was considered a retrieval strategy and a Sum and Right counting strategy was considered a Min strategy.

Problem groups based on strategy use are displayed in Figures 1, 2 and 3 for Michael, Steven and Richard respectively.

1+1							Allmin		(60.0%)
1+2	2+2						Variable*		(24.6%)
1+3	2+3	3+3					Alldec		0
1+4	2+4	3+4	4+4				Allret		(15.4%)
1+5	2+5	3+5	4+5	5+5					
1+6	2+6	3+6	4+6	5+6	6+6				
1+7	2+7	3+7	4+7	5+7	6+7	7+7			
1+8	2+8	3+8	4+8	5+8	6+8	7+8	8+8		
1+9	2+9	3+9	4+9	5+9	6+9	7+9	8+9	9+9	
1+10	2+10	3+10	4+10	5+10	6+10	7+10	8+10	9+10	10+10
1+13	2+11	3+15	4+14	5+12	6+13	7+11	8+12	9+11	
	2+14								

Figure 1. Problem groups based on strategy use for Michael. *Includes decomposition.

1+1							Allmin		(13.8%)
1+2	2+2						Variable*		(36.9%)
1+3	2+3	3+3					Alldec		0
1+4	2+4	3+4*	4+4				Allret		(49.2%)
1+5	2+5	3+5	4+5*	5+5					
1+6	2+6	3+6	4+6	5+6*	6+6				
1+7	2+7	3+7*	4+7	5+7	6+7*	7+7*			
1+8	2+8	3+8	4+8*	5+8	6+8*	7+8	8+8		
1+9	2+9	3+9	4+9*	5+9*	6+9*	7+9*	8+9*	9+9*	
1+10	2+10	3+10	4+10	5+10	6+10	7+10	8+10	9+10	10+10
1+13	2+11	3+15	4+14	5+12*	6+13*	7+11	8+12	9+11	
	2+14								

Figure 2. Problem groups based on strategy use for Richard. *Includes decomposition.

1+1							Allmin		(55.4%)
1+2	2+2						Variable*		(16.9%)
1+3	[illegible]	[illegible]					Alldec		0
1+4	2+4	[illegible]	4+4				Allret		(27.7%)
1+5	2+5	3+5	4+5	5+5					
1+6	2+6	3+6	4+6	5+6	6+6				
1+7	2+7	3+7	4+7	5+7	6+7	7+7			
1+8	2+8	3+8	4+8	5+8	6+8	7+8	8+8		
1+9	2+9	3+9	4+9	5+9	6+9*	7+9	8+9*	9+9	
1+10	2+10	3+10	4+10	5+10	6+10	7+10	8+10	9+10	10+10
1+13	2+11	[illegible]	[illegible]	[illegible]	[illegible]	7+11	[illegible]	9+11	
	[illegible]								

Figure 3. Problem groups based on strategy use for Steven. *Includes decomposition.

Michael's performance at simple addition was particularly immature. He most dominantly relied on the Min Strategy and on no occasion did he apply a decomposition strategy. Gray (1991) found that children of average-ability at age seven used some decomposition strategies. Michael's lack of decomposition was not surprising given that he only knew (that is, consistently retrieved) two facts: 5+5 and 6+6. He still sometimes relied on the Min Strategy to perform problems that had an addend of ten. His performance was most consistent with that documented by Gray for below-average children, aged between nine and ten.

Richard relied most dominantly on a retrieval strategy and knew most low-tie facts and facts to problems with an addend of one and ten. He still sometimes relied on counting to solve problems with an addend of two. Richard's performance appeared more developmentally mature than that documented for the group of below-average 12 year olds, particularly in the use of decomposition strategies, but still not as mature as that documented for average achieving 12 year olds (Gray, 1991).

Steven relied mostly on the Min Strategy to perform simple addition problems. On problems with a minimum addend of two and problems with an addend of ten, strategy variability between use of a Min and retrieval strategy was still evident. Steven was still either consistently counting problems, or only beginning to retrieve problems that are useful to know and apply as part of a decomposition strategy (tie problems and problems with a sum of 10). Tie facts are usually the first facts to be retrieved (Groen & Parkman, 1972; Ashcraft & Battaglia, 1978). Steven's performance was comparable to that document by Gray (1991) for 10-year-old children of below average ability.

Discussion

Previous studies have established that primary school students with learning difficulties do not move-on to develop a reliance on retrieval for simple addition problems. Findings from this study indicate that for some students, this difficulty persists well on into the secondary years of schooling. The strategies applied by the students in the present study were not different to strategies applied by normal achieving, younger students and so the deficit view of development does not appear applicable. Given the age of these students, to suggest that their performance is consistent with a delayed view of development, without considering the question of whether or not the same asymptotic level of cognitive ability will ever eventually be reached without intervention, is somewhat optimistic.

Geary (1993) argued that a developmental delay in the acquisition of related conceptual counting knowledge mediated procedural strategy differences exhibited by students with learning difficulties, He also argued that a developmentally different (deficit) pattern of processing mediated retrieval differences exhibited by students with learning difficulties. Geary's explanation of the delay/deficit issue is consistent with the present findings. These students had moved on to develop a reliance on an efficient counting strategy. Their lack of decomposition strategies was more a result of not knowing useful facts, than not applying facts. The greatest difficulty faced by these students was developing a reliance on retrieval.

The question of intervention is beyond the scope of this paper. What is noted is the importance of considering what students already know before launching into intervention procedures. For example, the students in the present study did not retrieve all the tie facts or the facts to problems with a sum of ten. Interventions based on direct strategy instruction of decomposition strategies would therefore be inappropriate. The question of what intervention is appropriate remains open for consideration.

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