Mathematics Software Design in Mathematics Education: Factors that Promote Higher-Level Thinking

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This paper presents the results of a qualitative study involving the internet-based software LOGAL Tangible Math. The study was carried out in a secondary school classroom over a period of ten weeks and focused on identifying factors in the software design involved in promoting higher level thinking. Specifically, the roles of cognitive tools, question types, format of student answers, manipulable objects and navigational issues were examined. The authors conclude that higher-level thinking is encouraged by the provision of a rich set of tools, and the presence of manipulable objects that allow students to generate exemplars of a new concept.

Introduction

Imagine a secondary mathematics classroom. Near the window a group of students gathered around a computer monitor examining charts and tables chatter enthusiastically. A number of other students are working alone, intent on the task at hand, tapping at their keyboards. Meanwhile, a facilitator discusses with some other students their progress and helps them with any problems they have encountered. This classroom is fortunate to have a truly student centered learning environment. The computer software enables students to study mathematics at their own pace from high school to beyond early university level. Peer collaboration is encouraged through the software which is also cleverly designed to enhance the development of problem solving skills. The mathematical contexts are realistic and relevant for students aged from 12 through to 17.

It seems we are still an extremely long way from the idealised environment described in the above paragraph. Many questions are yet to be answered regarding the computer's ability as a 'teacher'. We know that computer software can be useful in drill and practice situations, but the focus of education has changed, and teachers' energies are now directed towards teaching students to think, to solve problems, and to be able to learn independently (Gipps, 1994). What is the computer's role in this type of education? Grabinger and Dunlap (1996) verbalised the question this way:

> ...So, how can we design machines to help people learn and think? Does this mean machines need to replicate human processes or that machines support processes? Can we use machines to help make the learning processes visible and more accessible? (p. 688)

Jonassen and Reeves (1996), discussing learning *with* technology, have pointed out the new focus for learners is interpretation of computer output:

> The learner should be responsible for recognising and judging patterns of information and organising it, while the computer should perform calculations, store information, and retrieve it at the learner's command. (pp. 697-698)

With this background in mind, the current study set out to examine the elements of educational computer software which would encourage the use and development of higher-level thinking by students in mathematics classrooms.

Definition

Prior to commencing the study, an operational definition of higher-level thinking was required. This proved to be a difficult task. A review of the literature revealed major differences in the way that higher-level thinking skills are defined. These differences exist not only between major fields of study, such as psychology and philosophy, but also between disciplines taught in secondary school such as English, social science, and mathematics. As a result, the authors searched for a common thread amongst the various views and also for an understanding of higher-level thinking relevant to the subject area in consideration, that is, mathematics. Lauren Resnick's work has been well published and appears to be generally accepted in the mathematics community. Resnick (1987) believes that 'higher-order thinking is difficult to define but easy to recognise when it occurs' (p. 44). In order to be able to recognise higher level thinking when it occurred Resnick produced a list of indicators. These indicators needed modification for the current study so that they more directly addressed the type of data generated. Table 1 contains Resnick's (1987) indicators and the modified version adopted for use in this study.

Table 1
Indicators Used to Recognise Higher-Level Thinking

Resnick's Indicator	Modified Indicator
Higher order thinking is *nonalgorithmic.*	Students make statements that do not obviously rely upon previously learned factual or algorithmic knowledge.
Higher order thinking tends to be *complex.*	Students make statements which associate two or more variables.
Higher order thinking often yields *multiple solutions*, each with costs and benefits.	Students produce a number of potential answers or a list of properties.
Higher order thinking involves *nuanced judgment* and interpretation.	Students provide a statement that demonstrates a thoughtful choice has been made between possible answers.
Higher order thinking involves the application of *multiple criteria*, which sometimes conflict with one another.	Students indicate they have applied two or more criteria in order to arrive at an answer.
Higher order thinking often involves *uncertainty.*	Students indicate uncertainty by taking time to review their solution and experiment with alternatives.
Higher order thinking involves *self-regulation* of the thinking process.	Students make their own decisions regarding which variables to examine, measure or compare.
Higher order thinking involves *imposing meaning*, finding structure in apparent disorder.	Students' responses indicate a search for significant features. Particularly if there is a choice involving which features to report.
Higher order thinking is *effortful.* There is considerable mental work involved in the elaborations and judgements required	This indicator was not used as it appeared to the authors that it always occurred along with one of the other indicators above.

It should be noted that a given instance of higher-level thinking does not require all of these indicators to be present (Newmann, 1990). On the other hand these indicators should not be viewed as discrete as they may occur in a wide variety of combinations.

Method

An interpretive, naturalistic methodology (Erickson, 1998; Denzin & Lincoln, 1998) was utilised. The first author acted as a participant observer and the second author collaborated as a cooperative colleague, observing sessions and participating in interpretation of the data. In working this way results were strengthened through researcher triangulation. The sample consisted of a class of 22 Year 9 students from a private girls school in Perth Western Australia working in pairs and interacting with the software over a series of 10 one hour lessons. During this period their answer files were logged and field notes taken. The operational definition of higher-level thinking was then applied to the data and instances of higher-level thinking identified. Subsequently, a database was created. This data base contained 112 records, one for each screen of the software encountered by the students. The database contained the following information:

- A screen capture from the software
- A description of the student action required from that screen
- A description of any processes required on that screen
- A record of the researcher observations resulting from that screen
- A list of the characterisations of higher-level thinking observed from that screen, either from direct observation or from student answer files
- A list of design elements utilised on that that screen

The screens on which higher-level thinking occurred were then examined further to investigate possible links between instances of higher-level thinking and the screen designs. The following design elements were included in the analysis:

- Tools: Whether or not the students had access to tools (angle measurement, length measurement etc).
- Answer box: Was a space provided for the students to answer a question and, if so, was the size of the space appropriate?
- Question type: what kind of question was asked of the students? Types considered were proof, conjecture, open, closed, simple and complex.
- Manipulable objects: Were there manipulable objects present on the screen?
- Text: What was the nature of the text on the screen? Did it provide instructions or information?
- Navigation: Did students have the ability for free navigation? Were there any hyperlinks?

The Learning Environment

The study was carried out within a broadly constructivist framework. That is, an environment in which the student is viewed as an active constructor of his or her knowledge as opposed to a receiver of knowledge transmitted by the teacher (Noddings, 1990). The reason for selecting this framework was that constructivist approaches to teaching and learning promote student's involvement in their own learning and enhance the likelihood of the development of higher-level thinking skills (Grabinger & Dunlap, 1996; Maor, 1995).

Specifically, a learning environment (within many constraints) similar to the 'Rich Environment for Active Learning' (REAL) as posited by Grabinger and Dunlap (1996) was created. These authors stated:

> One of the main assertions of REAL developers is that REALs improve problem-solving skills and enhance the likelihood that learning transfers to new situations. (p. 682)

It was felt, therefore, that the utilisation of a REAL would be most effective in encouraging higher level thinking, as well as providing a generally beneficial environment for the students. The attributes of the REAL which were implemented are described in Table 2.

Table 2
Attributes of the Rich Environment for Active Learning (REAL)

REAL Attribute	Explanation
Generative Learning Activities	Students learn through active involvement and use the tools available to them to develop and refine their understandings of the concepts being investigated. These type of activities encourage argumentation and reflection.
Cooperative Support	By working in groups the students can support each others learning. The discussion that goes on in the group can help the students in social negotiation of meaning. In addition, students working in groups may feel more confident to engage in the risks associated with tackling complex problems.
Authentic Learning Tasks	Students are provided with learning experiences which are as realistic as possible.
Student Responsibility and Initiative	The environment is student centred, placing a major emphasis on intentional learning. I wanted to encourage student questioning, self-reflection and self-monitoring.
Authentic Assessment Tasks	Students undertake contextualized, complex intellectual challenges (Wiggins, 1989) to assess their progress

The Software

The software selected for this study was the Geometry Inventor (LOGAL, 1997), from LOGAL Software Inc (http://www.riverdeep.net). This software was selected principally because it was designed with constructivist principles in mind. Following is a quote from correspondence with LOGAL Software Inc.:

> All of LOGAL's software is based on the constructivist philosophy that learning is the active construction of knowledge, and that knowledge comes to life when software helps students ask questions and learn through exploration. (Correspondence, 1997).

The Geometry Inventor is part of a suite of mathematics programs from LOGAL Software Inc. The entire suite is titled 'Tangible Math' and consists of components dealing with algebra, geometry, functions, probability and matrices. The geometry module was selected in consultation with the class teacher because it had a rich set of tools for the students to use and it was felt it would complement the students' existing mathematics course well.

Each lesson within the Geometry Inventor is broken into a number of stages. Lessons begin with a warm up activity that is followed by a number of experiments. The results of these experiments are then synthesised by the student in an analysis section. Finally the lessons conclude with a follow-up activity. Individual screens typically consist of some information to the student, a place for students to write their ideas, tools the students can utilise and some form of graphics which are usually manipulable (i.e., they are dynamic rather than static).

Results

Higher-level thinking was observed, according to the operational definition, on 22 of the 112 screens from the Geometry Inventor, However, on most of these screens multiple indicators were observed. In all there were 94 indicators of higher level thinking observed. The following table summarises the occurrences of each indicator as a percentage of the total observations.

Table 3

Indicators of Higher-Level Thinking

Indicator	% Frequency
Non-algorithmic	19
Complex	7
Multiple solutions	3
Nuanced Judgement	14
Multiple Criteria	4
Uncertainty	18
Self Regulation	15
Imposing Meaning	20

The next stage of the analysis was to attempt to match the instances of higher-level thinking with the design elements of the software available on the screen on which the higher-level thinking was observed. Each design element was considered separately and its role with regard to the observed instance of higher-level thinking was analysed. To help the reader to understand this process an example involving the indicator 'higher-level thinking is non-algorithmic' follows.

The indicator that was used for this category was that higher-level thinking was indicated when a student makes a statement that does not obviously rely upon previously learned factual or algorithmic knowledge.

An example of an answer indicating this type of thinking was given by a pair of students who defined a line TR with a positive gradient as follows:

> A line TR has a positive gradient if T is in the top right or bottom left of a square centred at R. (students' answer 14, PLT 23)

The students concerned were previously unfamiliar with the concept of the gradient of a line. The construction of their definition indicates that this answer was not obtained from the teacher or from a book. In the researcher's opinion, the students have constructed this definition themselves as a similar definition has not been seen before. This may be constrasted with a response such as, "The line slopes down to the left." (Student's answer 21, PLT 23), which is a commonly found way of defining a line with a positive gradient. As a result this was not classified as an indicator of higher-level thinking although these students *may* have indulged in higher-level thinking.

Having ascertained an instance of higher-level thinking the attributes of the particular screen stimulating this response were analysed. What were the students doing on this screen to produce this response? In this case students had to manipulate a line and, by viewing a dynamic table, investigate the conditions relating to positive, negative and zero gradients. When they had finally completed their investigation they were to enter their answer in an appropriate section on the screen. Hence, possible significant elements of design were manipulable objects, provision of tools with which to investigate, and the requirement of a student response in an answer box.

It is likely that all three of these played their role in encouraging the students to think beyond a surface level about the question put to them.

The remainder of this paper, due to limitations on publication space, will concentrate on the important design elements we have referred to as *tools* and *manipulable objects*.

Discussion

The Geometry Inventor Tools

The Geometry Inventor provided students with a rich set of tools with which they could measure properties of and/or construct geometric diagrams. For example, students could create any polygon, measure angles and sides, bisect angles, subdivide sides, measure area, create tables, create graphs of relationships and so on. Figures and diagrams created were dynamic, and hence, could be manipulated. All of the concepts developed were done so through the use of the tools, so it could be said that they were a vital element in producing any higher-level thinking in which the students engaged. These tools gave the students much freedom and the opportunity to test and experiment with their own ideas. However, it was observed that this freedom was not generally utilised by the students as they tended to follow the directions given in the software and perform only the investigations that were required of them. Furthermore, the geometry tools were not always the most significant of the design elements in producing the instances of higher-level thinking. In fact, on some occasions the presence of the tools actually inhibited the use of higher-level thinking. This occurred because students tended to immediately use the tools to measure attributes of the geometrical shapes rather than to look for patterns or generalisations. They thus turned lessons designed to produce higher level thinking into routine measurement tasks.

The above also raises another important issue. The authors believe generalisation and mathematical reasoning are higher-level thinking skills. However, in computerised mathematics lessons students tend to quickly generalise correctly because they *know* there is a pattern to find These student generalisations are usually made from too few experimental cases. Knowing something is one thing, knowing why is another, and being able to prove it still another. Lessons that use powerful measuring tools need to be devised to cover the latter two issues. The Geometry Inventor does indeed have some lessons which do this and these proved challenging and thought provoking for the students, and were certainly numbered among those lessons producing evidence of higher-level thinking. Perhaps the answer, in situations where tools are available to students, is to restrict their use at certain critical stages of the reasoning process to ensure the student thinks and doesn't simply measure.

Manipulable Objects

The Geometry Inventor could be classified as dynamic geometry software. That is, the emphasis of the software is on the use of tools to create geometric objects that can be manipulated. Thus, most screens in the lessons the students covered utilised this feature. 64% of the screens on which indicators of higher-level thinking were observed contained manipulable objects. The mere presence of these objects however, does not mean they contributed to the higher-level thinking. So what was their role, and is there evidence to suggest that they are significant in the encouragement of higher-level thinking? Skemp (1987), discussing the principles of learning mathematics states:

> Concepts of a higher-order than those which people already have cannot be communicated to them by a definition, but only by arranging for them to encounter a suitable collection of examples. (p. 18)

Before proceeding, the authors emphasise that they do not wish to equate higher-level thinking with the higher-order concepts that Skemp refers to here. He is talking about students learning a new concept which they didn't previously possess. For example, the Pythagorean theorem. It is the second part of this statement which is particularly relevant to the Geometry Inventor. In order for the students to make the step to a new concept, to generalise or abstract, Skemp suggests they need to be exposed to a suitable collection of examples. This is where the power of the Geometry Inventor is really seen. In manipulating the geometric objects the students are generating for themselves a group of suitable examples and they can generate as many as they like very quickly. This group of examples becomes the foundation for their thinking. We believe this is how these manipulable objects should be viewed, as a potentially infinite set of exemplars to a new concept.

For example, on a particular screen the students have a manipulable object, namely a triangle. They can use this triangle to generate examples of other triangles with all external angles greater than 90 degrees. Following this generation they are required to make a generalisation. It is at this point that the higher-level thinking becomes apparent. This thinking occurs during the search for relationships or patterns, either in the numerical data or in the visual data generated as a result of altering the shape of the triangle. To what extent higher-level thinking occurs during the data generation phase is difficult to say. The students observed generated the data and then examined the table they had produced in a search for patterns. However, it is not inconceivable that a student may manipulate the diagram and use the visual data to produce a conclusion (e.g. deciding visually that a triangle satisfying certain conditions is equilateral), although this was not observed. In such a case, the student may be utilising higher-level thinking skills while actually manipulating the diagram. Speculation aside, it appears from this study that the manipulation of the objects tends to be used to generate data. It is during the analysis of this data, the search for patterns and relationships, that higher-level thinking was observed.

Conclusion

The above discussion can appear to imply that design elements occur individually, or in isolation. In practice this is not the case. They are united in the software to produce a product, a series of lessons, which has effectively engaged the students in higher-level thinking. This fact is supported by the statistic that approximately 1 in 5 screens produced indicators of higher-level thinking, as characterised by Resnick (1987). The two design elements discussed in this paper that appear to significantly encourage higher-level thinking are:

- The provision of a rich set of tools and;
- The presence of manipulable objects allowing students to generate a set of exemplars of a new concept.

Although this paper has concentrated on the above two design elements, future research could focus, not only on these two elements of educational software, but also on the other design elements identified in the 'Method' section of this paper (answer box, question type, and navigation). This research would further our understanding of the link between these elements and higher-level thinking.

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