The Early Numeracy Research Project[1]: Developing a Framework for Describing Early Numeracy Learning

Doug Clarke
Australian Catholic University
<d.clarke@patrick.acu.edu.au>

Peter Sullivan
Australian Catholic University
<p.sullivan@patrick.acu.edu.au>

Jill Cheeseman
Australian Catholic University
<j.cheeseman@patrick.acu.edu.au>

Barbara Clarke
Monash University
<barbara.clarke@education.monash.edu.au>

This paper describes the development of a framework of key aspects of early numeracy learning. The framework was developed as part of a project seeking to identify processes for supporting and enhancing numeracy learning in the early years of school. The framework built on work of others, through considering key research in numeracy learning and others' attempts in developing such frameworks. Data collected from interviews with over 5000 children led to modifications of the framework.

In the past ten years, there has been considerable community and political attention to measuring the outcomes of students' learning (Australian Education Council, 1994; Board of Studies, 2000). Increasingly, state and territory departments of education indicated that the early years of schooling were crucial in providing the kind of positive start to students' literacy and numeracy learning that was needed to develop confident and capable lifelong learners.

In Victoria, the Early Literacy Research Project (Hill & Crevola, 1998) worked with 27 disadvantaged primary schools to bring about substantial improvements in early literacy outcomes. Part of this research involved the development of models and guidelines for teaching, assessment and additional support for young children learning to read. As a result of the research, Hill and Crevola offered a "general design for improving learning outcomes" (p. 122), which they believed had application in literacy, numeracy, and other curriculum areas. The design was greatly influenced by research literature on educational effectiveness, including a finding that the impact of classroom effects on student learning greatly exceeds that of school effects (Creemers & Reezigt, 1996; Hill & Rowe, 1996). The nine elements of the design (Hill & Crevola, 1998, p. 123) included leadership and coordination; standards and targets, monitoring and assessment, classroom teaching programs; professional learning teams; school and class organisation; intervention and special assistance; home, school and community partnerships; and beliefs and understandings.

The Early Numeracy Research Project (ENRP) was established in 1999 by the (then) Victorian Department of Education, with similar aims to those of the Early Literacy Research Project, but with a numeracy focus. The ENRP is now a collaborative venture between Australian Catholic University, Monash University, the Victorian Department of Employment, Education and Training, the Catholic Education Office (Melbourne), and the Association of Independent Schools Victoria. The project is funded to early 2002 in 35 project ("trial") schools and 35 control ("reference") schools (for details see Clarke, 1999). The ENRP has a major professional development component, with teachers meeting with project staff for statewide, regional cluster, and local inservice programs.

Important differences from the literacy project included the need for development of a comprehensive and appropriate learning and assessment framework for early numeracy (such frameworks were well established for reading), and the need to address the personal confidence with and understanding of mathematics of many primary teachers. This paper

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explains the need for the development of the learning and assessment framework, outlines the process for the development of the framework, and presents some results to illustrate the profiles of the students at these levels.

Measuring Numeracy Learning

The impetus for the project was a desire to improve numeracy learning and so it was necessary to quantify such improvement. It would not have been adequate to describe, for example, the effectiveness of the professional development in terms of teachers' professional growth, or the children's engagement, or even to produce some success stories. It was decided to create a framework of key "growth points" in numeracy learning. Students' movement through these growth points in trial schools could then be compared to that of students in the reference schools.

The project team studied available research on key "stages" or "levels" in young children's numeracy learning (e.g., Boulton-Lewis, 1996; Fuson, 1992, McIntosh, Bana, & Farrell, 1995; Mulligan & Mitchelmore, 1995, 1996; Pearn & Merrifield, 1992; Thomas, 1996), as well as some frameworks developed by other authors and groups to describe learning.

A major influence on the project design was the New South Wales Department of Education initiative *Count Me In Too* (Bobis & Gould, 1999; NSW Department of Education and Training, 1998) that developed a learning framework in number (Wright, 1998) that seemed to incorporate most of the desired elements in describing students' learning. It was soundly based on prior research and, in particular, on the Steffe counting stages (see, e.g., Steffe, Cobb, & von Glaserfeld, 1988; Steffe, von Glaserfeld, Richards, & Cobb, 1983), and it formed the basis of an individual interview designed to measure children's learning against the framework.

Discussions with Trish O'Toole and Greg Parker (personal communications, January 10-12, 1999) from the Catholic Education Office (South Australia) were also helpful in considering aspects of the measurement parts of the framework, as was the term "growth point" which they were using in their work.

The draft version of the Victorian *Curriculum and Standards Framework II* (Board of Studies, 1998) was examined but seemed on one hand too specific (in its outcome statements), and on the other hand too general (in its curriculum focus statements). The Numeracy Benchmarks were clear and comprehensive but were limited in scope and described only minimum achievement levels without which a student is unable to progress at school (Curriculum Corporation, 1997).

While the framework documents considered had much to offer, none served the needs of this project without substantial adaption and extension. It was decided to create a framework specifically for this project.

In developing the framework it was intended that the framework would

- reflect the findings of relevant research in mathematics education from Australia and overseas;
- emphasise the "big ideas" of early numeracy in a form and language readily understood and, in time, retained by teachers;
- reflect, where possible, the structure of mathematics;
- allow the description of the mathematical knowledge and understanding of individuals and groups;
- form the basis of planning and teaching;
- provide a basis for task construction for interviews, and the recording and coding process that would follow;
- allow the identification and description of improvement where it exists;

- enable a consideration of those at risk students who may benefit from additional assistance;
- have sufficient "ceiling" to describe the knowledge and understanding of all children in the first three years of school; and
- build on the work of other successful, similar projects such as *Count Me in Too*.

The principles informed the process of developing and refining the framework as is outlined in the next section.

The Development of the Framework

Having assembled relevant research and frameworks for Number and Measurement, the research team conducted a number of "think tank" sessions during which the initial framework was developed. During this time, colleagues with expertise in specific areas were consulted, and their advice incorporated into the development process. For 1999, the decision was taken to focus upon the strands of Number (incorporating the domains of Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies) and Measurement (incorporating the domains of Length, Mass and Time). In 2000, the strand of Space has been added to the framework, but this will not be discussed here.

Within each mathematical domain, growth points were stated with brief descriptors in each case. There were typically five or six growth points in each domain. To illustrate the notion of a growth point, consider the child who is asked to find the total of two collections of objects (say nine objects and another four objects). Many young children will "count all" to find the total ("1, 2, 3, . . . , 11, 12, 13"), even once they are aware that there are nine objects in one set and four in the other. Other children will realise that by starting at 9 and counting on ("10, 11, 12, 13"), they can solve the problem in an easier way. *Counting All* and *Counting On* are therefore two important growth points in children's developing understanding of Addition.

1. Rote counting
   Rote counts the number sequence to at least 20, but is unable to reliably count a collection of that size.
2. Counting collections
   Confidently counts a collection of around 20 objects.
3. Counting by 1s (forward/backward, including variable starting points; before/after)
   Counts forwards and backwards from various starting points between 1 and 100; knows numbers before and after a given number.
4. Counting from 0 by 2s, 5s, and 10s
   Can count from 0 by 2s, 5s, and 10s to a given target.
5. Counting from x ($x > 0$) by 2s, 5s, and 10s
   Given a non-zero starting point, can count by 2s, 5s, and 10s to a given target.
6. Extending and applying counting skills
   Can count from a non-zero starting point by any single digit number, and can apply counting skills in practical tasks.

Figure 1. ENRP growth points for counting.

For clarity this paper presents only some results from the *Counting* domain of the framework. The six growth points for counting are shown in Figure 1. Note that growth point 6 was not added until this year, and will therefore not appear in the discussions of the *Counting* data. These growth points informed the creation of assessment items, and the recording, scoring and subsequent analysis, as is discussed below.

Growth Points, Levels and Stages

The growth points are clearly a key element of this framework. In discussions with teachers, we have come to describe them as key "stepping stones" along paths to mathematical understanding. However, we do not claim that all growth points are passed by every student along the way. For example, one of our growth points in Addition and Subtraction involves "counting back", "counting down to" and "counting up from" in subtraction situations, as appropriate. But there appears to be a number of children who view a subtraction situation (say, 12-9) as "what do I need to add to 9 to give 12?" and do not appear to ever use one of those three strategies.

The interpretation of these growth points reflects the description by Owens and Gould (1999) in the *Count Me In Too* project: "the order is more or less the order in which strategies are likely to emerge and be used by children. . . . intuitive and incidental learning can influence these strategies in unexpected ways" (p. 4). In discussing "higher" level growth points in a given domain, the comments of Clements, Swaminathan, Hannibal, and Sarama (1999) in a geometrical context are helpful: "the adjective *higher* should be understood as a higher level of abstraction and generality, without implying either inherent superiority or the abandonment of lower levels as a consequence of the development of higher levels of thinking" (p. 208).

Also, the growth points should not be regarded as necessarily discrete. As with Wright's (1998) framework, the extent of the overlap is likely to vary widely across young children, and "it is insufficient to think that all children's early arithmetical knowledge develops along a common developmental path" (p. 702).

The Development of the Interview

Once the early drafts of the framework were developed, assessment tasks were created to match the framework. A major feature of the project is a one-to-one interview with every child in trial schools and a random sample of around 40 children in each reference school at the beginning and end of the school year (February/March and November respectively), over a 30- to 40-minute period.

Although the full text of the interview involves around 50 tasks (with several sub-tasks in many cases), no child moves through all of these. The interview is of the form "choose your own ending", in that the interviewer makes one of three decisions after each task. Given success with the task, the interviewer continues with the next task in the given mathematical domain as far as the child can go with success. Given difficulty with the task, the interviewer either abandons that section of the interview and moves on to the next domain or moves into a detour, designed to elaborate more clearly the difficulty a child might be having with a particular content area.

All tasks were piloted with children of ages five to eight in non-project schools, in order to gain a sense of their clarity and their capacity to reveal a wide range of levels of understanding in children. This was followed by a process of refining tasks, further piloting and refinement, and where necessary, adjusting the framework, as shown in Figure 2.

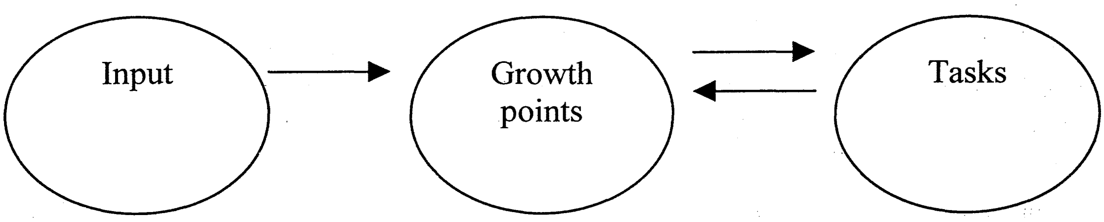

Figure 2. The process of developing the ENRP framework and the interview.

The form and wording of the tasks are influenced by the growth points for which they are intended to provide evidence, while at the same time the consideration of the data provided by a given task can lead to a refining of the wording of a given growth point.

The interview provides information about those growth points achieved by a child in each of the seven domains. Our aim in the interview is to gather information on the most sophisticated strategies that a child accesses in a particular domain. However, depending upon the context and the complexity of the numbers in a given task, a child (or an adult) may use a less sophisticated strategy than they actually possess, as the simpler strategy may do the job quite nicely in that situation.

Wright (1998) warns of the challenge of determining the actual strategy used by a child in solving a problem, as "a child may unwittingly or intentionally describe a strategy different from the one used" (p. 703).

It is important to stress that the growth points are "big mathematical ideas or concepts", with many possible "interim" growth points between them. As a result, a child may have learned several important ideas or skills *necessary* for moving to the next growth point, but perhaps not of themselves *sufficient* to move there. Also, to achieve many of the growth points requires success on several tasks, not just one or some.

Of course, decisions on assigning particular growth points to children are based on a *single* interview on a *single* day, and a teacher's knowledge of a child's learning is informed by a wider range of information, including observations during everyday interactions in classrooms. However, teachers agree that the data from the interviews are revealing of student mathematical understanding and development, in a way that would not be possible without that special opportunity for one-to-one interaction. It appeared that the children also enjoy that special time having the teacher "all to themselves". Teachers report that children appreciated the opportunity to show what they knew and could do.

Data Collection and Results

As well as moving carefully through the 16-page interview schedule, the interviewer completed a four-page Student Record Sheet. The information on this record sheet is then used by a trained team of coders together with a scoring algorithm to assign "achieved growth points" to each child for each content area. The rating of an individual child at a particular growth point is based on his or her responses to a number of different interview tasks. These raters demonstrated extremely high levels (all greater than 90%) of inter-rater reliability, as detailed in Rowley (in preparation).

A key criterion for the framework to be successful is the extent to which it reports on the spread and development of children's learning. Table 1 presents the percentages of children at each growth point in the first interview (March, 1999) for *Counting*, for each grade level.

Table 1
Percentage of Children at Each Counting Growth point, by Grade Level (%)

	Prep *n*=1702	Grade 1 *n*=1658	Grade 2 *n*=1498	Total *n*=4858
0. Not apparent	43.6	5.0	1.1	17.3
1. Rote counting	11.8	7.4	2.1	7.3
2. Counting collections	42.4	65.6	40.4	49.7
3. Forwards/backwards	1.6	12.5	15.6	9.6
4. Skip count from 0	0.5	8.4	32.4	13.1
5. Skip count from x	0.2	1.0	8.3	3.0

There is a clear development from Prep to Grade 1 to Grade 2. This part of the framework seems to allow description of a spread of development within the one grade level, and illustrates development across the grades.

It also seems that these results indicate possible directions or emphases for teachers at this level. For example, at each level, and overall, there is a large group of students who have achieved growth point 2 for *Counting*, but not growth point 3. It is possible that the development to growth point 2 happens naturally; it may be that development to growth point 3 may require some prompting. This possibly has different curriculum implications for each level. Perhaps the imperative at Prep is to assist children at growth point 0 and 1 to progress to growth point 2. In Grade 2, the emphasis may be on providing the experiences to assist the students to move beyond this point. Project teachers will be able to offer insights on this over the course of the project.

The growth points identified are meaningful and seem suitable for describing the learning of the children over time. For example, Table 2 compares the rating of the children at Grade 1 in the trial schools (only) from March to November.

Table 2
Grade 1 Children at Each Counting Growth Point in March and November (%)

	Grade 1 March *n*=1233	Grade 1 November *n*=1223
0. Not apparent	5.4	0.8
1. Rote counting	6.2	1.1
2. Counting collections	64.5	31.2
3. Forwards/backwards	13.2	12.4
4. Skip count from 0	9.8	41.9
5. Skip count from x	0.8	12.5

There is a clear movement in growth points evident in the Grade 1 data. Other grade levels show similar growth. Given the importance of the principle of having an appropriate ceiling for all children, an additional growth point was added (with related interview tasks) for 2000, as shown in Figure 1. Interestingly, 0.1% of Grade 1 children and 0.4% of Grade 2 children achieved this new growth point in the March 2000 interviews. Such data enable a comparison between growth in understanding in trial and reference schools, thereby providing a measure of the effectiveness of the professional development program. These comparisons will not be discussed here.

The framework is designed to allow the quantification of the learning of the children. However, we are more interested in identifying factors that may contribute to such learning. To complement the data on the children's learning a range of other data are being collected, including detailed questionnaires on teachers' beliefs and understandings about numeracy learning, regular journals kept by Early Numeracy Coordinators (the leaders of the professional learning teams in each school), as well as teacher and principal data on the effect of the project on teaching practice and student attitudes to mathematics.

In the third year of the project (2001), major emphasis will be given to studying those teachers and schools who have been shown to be particularly effective in building numeracy understanding.

The Refinement of the Framework

Data collected in March and November 1999 from approximately 5000 children informed the refinement of the framework and interview in preparation for the assessment period in

March 2000. Changes were also made in light of the perceived need to increase the focus on applying understandings in "practical" contexts. The major change to the framework was the incorporation of two domains for the Space strand (Classification of shape and Visualisation and orientation). There were also a number of word and phrase changes, to increase consistency and clarity for teachers and interested others.

Discussion

When the framework was first developed, a major purpose for its creation was to enable a measure of the effectiveness of the professional development part of the project. However, the framework is proving powerful in a variety of other ways.

Teachers are increasingly "owning" the framework, and using it to enhance their own understanding of children's mathematical learning. In the same way as the development of the framework and interview continue alongside each other (as in Figure 1), teachers' understanding of the framework is enhanced by their familiarity with the interview. Similarly, as the framework becomes better known, teachers view student responses during the interview in the light of their understanding of the growth points. Most importantly, the growth points provide a kind of "lens" through which children's mathematical thinking can be viewed, in all individual, small group and whole class interactions.

The framework in conjunction with data from interviews provides a basis for teacher planning and decision making. Teachers, coordinators, principals and the research team are increasingly reporting more focused teaching, in response to information gained in both interview and classroom situations.

The framework provides a way of reporting to parents on what children know and are able to do, in a relatively easily understood way. Parent information evenings at all 35 trial schools have contributed to growing goodwill towards the project.

The framework and interview must still be regarded as "work in progress". However, data to hand indicate the power of this approach in terms of both teacher professional growth and student mathematical learning.

References

Australian Education Council. (1994). Mathematics—A curriculum profile for Australian schools. Carlton, Victoria: Curriculum Corporation.

Board of Studies. (1998, draft). Curriculum and standards framework II: Mathematics. Carlton, Victoria: Author.

Board of Studies. (2000). Curriculum and standards framework II: Mathematics. Carlton, Victoria: Author.

Bobis, J., & Gould, P. (1999). The mathematical achievement of children in the count me in too program. In J. M. Truran & K. M. Truran (Eds.), Making the difference (Proceedings of the 22nd annual conference of the Mathematics Research Group of Australasia, pp. 84-90). Adelaide: MERGA.

Boulton-Lewis, G. (1996). Representations of place value knowledge and implications for teaching addition and subtraction. In J. Mulligan & M. Mitchelmore (Eds.), Children's number learning: A research monograph of MERGA/AAMT (pp. 75-88). Adelaide: Australian Association of Mathematics Teachers.

Clarke, D. M. (1999). Linking assessment and teaching: Building on what children know and can do. In Early Years of Schooling Branch (Eds.), Targeting excellence: Continuing the journey (pp. 8-12). Melbourne: Author.

Clements, D. H., Swaminathan, S., Hannibal, M. A. Z., Sarama, J. (1999). Young children's conceptions of space. Journal for Research in Mathematics Education, 30(2), 192-212.

Creemers, B. P. M., & Reezigt, G. J. (1996). School level conditions affecting the effectiveness of instruction. School Effectiveness and School Improvement, 7(6), 197-228.

Curriculum Corporation. (1997). Background information for benchmarks school trial and consultation. Melbourne: Author.

Fuson, K. (1992). Research on whole number addition and subtraction. In D. A. Grouws (Ed.). Handbook of research on mathematics teaching and learning (pp. 243-275). Macmillan: New York.

Hill, P. W., & Crevola, C. A. (1998). The role of standards in educational reform for the 21st century. In D. D. Marsh (Ed.), Preparing our schools for the 21st century (Association for Supervision and Curriculum Development Yearbook 1999, (pp. 117-142). Alexandria, VA: ASCD.

Hill, P. W., & Rowe, K. J. (1996). Multilevel modeling in school effectiveness research. School Effectiveness and School Improvement, 7(1), 1-34.

McIntosh, A., Bana, J., & Farrell, B. (1995). Mental computation in school mathematics: Preference, attitude and performance of students in years 3, 5, 7, and 9 (MASTEC Monograph Series No. 1). Perth: Mathematics, Science & Technology Education Centre, Edith Cowan University.

Mulligan, J., & Mitchelmore, M. (1995). Children's intuitive models of multiplication and division. In S. Flavel, I. Isaacs, D. Lee, R. Hurley, T. Roberts, A. Richards, R. Laird, & V. M. Ram (Eds.), GALTHA Proceedings of the 18th annual conference of the Mathematics Education Research Group of Australasia (pp. 427-433). Darwin: MERGA.

Mulligan, J., & Mitchelmore, M. (1996). Children's representations of multiplication and division word problems. In J. Mulligan & M. Mitchelmore (Eds.), Children's number learning: A research monograph of MERGA/AAMT (pp. 163-184). Adelaide: AAMT.

New South Wales Department of Education and Training. (1998). Count me in too professional development package. Ryde, NSW: Author.

Owens, K., & Gould, P. (1999). Framework for elementary school space mathematics (discussion paper).

Pearn, C., & Merrifield, M. (1998, October). Mathematics intervention. Paper presented to the Early Numeracy Networks, Department of Education, Melbourne, Victoria.

Steffe, L. P., Cobb, P., & von Glasersfeld, E. (1988). Construction of arithmetical meanings and strategies. New York: Springer-Verlag.

Steffe, L.P., Von Glasersfeld, E., Richards, J., & Cobb, P. (1983). Children's counting types: Philosophy, theory, and application. New York: Praeger.

Thomas, N. (1996). Understanding the number system. In J. Mulligan & M. Mitchelmore (Eds.), Children's number learning: A research monograph of MERGA/AAMT (pp. 89-106). Adelaide: Australian Association of Mathematics Teachers.

Wright, R. (1998). An overview of a research-based framework for assessing and teaching early number learning. In C. Kanes, M. Goos, & E. Warren (Eds.), Teaching mathematics in new times Proceedings of the 21st annual conference of the Mathematics Education Research Group of Australasia (pp. 701-708). Brisbane: MERGA.