Mathematical Modelling in the Primary School

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Changes in society and the workplace necessitate a rethinking of the nature of the mathematical problem-solving experiences we provide our students across the grades. We need to design experiences that develop a broad range of future-oriented mathematical abilities and processes. Mathematical modelling, which has traditionally been reserved for the secondary school, serves as a powerful vehicle for addressing this need. This paper reports on the second year of a three-year longitudinal study where a class of children and their teachers participated in mathematical modelling activities from the 5th grade through to the 7th grade. The paper explores the processes used by small groups of children as they independently constructed their own mathematical models at the end of their 6th grade.

Our ever-changing global market is making increased demands for workers who possess more flexible, creative, and future-oriented mathematical and technological skills (Clayton, 1999). Of importance here is the ability to make sense of complex systems (or models), examples of which appear regularly in the media (e.g., sophisticated buying, leasing, and loan plans). Being able to interpret and work with such systems involves important mathematical processes that are under-represented in the mathematics curriculum, such as constructing, describing, explaining, predicting, and representing, together with quantifying, coordinating, and organising data. Dealing with systems also requires the ability to work collaboratively on multi-component projects in which planning, monitoring, and communicating results are essential to success (Lesh & Doerr, 2003).

Given these societal and workplace requirements, it is imperative that we rethink the nature of the mathematical problem-solving experiences we provide our students— in terms of content covered, approaches to learning, ways of assessing learning, and ways of increasing our children's access to quality learning. One approach to addressing these issues is through mathematical modelling (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). Mathematical modelling has traditionally been reserved for the secondary school years (e.g., Galbraith, Blum, Booker, & Huntley, 1998), but recent research (e.g., Doerr & English, 2003) has indicated that primary school children can participate successfully in meaningful modelling activities.

This paper draws on data from the second year of a three-year longitudinal study where a class of children and their teachers participated in mathematical modelling activities from the 5th grade through to the 7th grade. The paper explores the processes used by small groups of children as they independently constructed their own mathematical models at the end of their 6th grade. The activity in question was the final in a series of problem situations that required students to create usable rating systems in a range of contexts.

Mathematical Modelling for Children

In the past couple of decades, children's problem solving has engaged them in situations where the "givens," the "goals," and the "legal" solution steps have been specified clearly; that is, the *interpretation processes* for the child have been minimized or eliminated. The difficulty for the solver is simply working out how to get from the given state to the goal state. The solutions to these problems are usually brief answers obtained from applying a previously taught solution strategy, such as "guess and check," or "draw a diagram." Furthermore, although these problems may refer to real-life situations, the mathematics involved in solving them is often not real world and rarely do the problems provide explicit opportunities for learners to generalize and re-apply their learning (English & Lesh, 2003). While not denying the importance of these problem experiences, they do not address adequately the knowledge, processes, and social developments that students require in dealing with the increasingly sophisticated systems of our society. Mathematical modelling activities, in the form of meaningful case studies for children, provide one way in which we can overcome this inadequacy.

As used here, models are systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behaviour of some other experienced system (Doerr & English, 2003). Model-eliciting activities, which provide the basis for subsequent model-exploration and model-application activities, engage children in situations where *key mathematical constructs are embedded within the problem context* and are elicited by the children as they work on the problem. Model-eliciting activities present situations where children (a) are confronted with the *need* to develop a model, (b) clearly recognise the need to *revise* or *refine* their *current ways of thinking* about the given problem situation, (c) are challenged to *express their understandings* in ways that they can test themselves and revise as often as necessary, and (d) develop models that can be *shared* with others and that can be *applied* in other problem situations (Lesh & Yoon, 2004). Engaging students in modelling experiences of this nature is not seen as simply finding a solution to a given isolated problem. Rather, such engagement involves children in multiple activities where significant mathematical constructs are developed, explored, extended, and applied; the end product is a system or model that is reusable in a range of contexts (Doerr & English, 2003).

Unlike traditional non-routine problems, modelling activities are inherently social experiences, where students work in small teams to develop a product that is explicitly sharable. Numerous questions, issues, conflicts, revisions, and resolutions arise as students develop, assess, and prepare to communicate their products. Because the products are to be shared with and used by others, they must hold up under the scrutiny of the team members (Zawojewski, Lesh, & English, 2003).

Design and Methodology

The present study took the form of a three-year, longitudinal teaching experiment involving multilevel collaboration (English, 2003; Lesh & Kelly, 2003). At the first level of collaboration children work on activities involving constructing, refining, and applying mathematical models. At the second and third levels, participating preservice teachers (university undergraduates) and the classroom teachers work collaboratively with the researchers in designing and implementing the children's activities. These activities also serve as challenging and thought-provoking experiences for the teachers as they explore the nature of the mathematical ideas being developed, consider appropriate implementation strategies, and promote learning communities within their classrooms. The focus of the present paper, however, is level 1, that is, the children's model construction.

During the second year of the study (2002; from which data have been drawn for this paper), a series of modelling activities was implemented in the 6^{th} grade classroom from May through to the end of October. One session per week was conducted, with the sessions ranging from 50 minutes to an hour. Meetings with the class teacher were held prior to the introduction of each new modelling activity. The class teacher introduced each modelling activity (described next) in a whole class format, which was followed by small group work.

The teachers and researchers observed the children as they worked the activities. Where appropriate, the observers might ask the children to explain or justify a response but mostly, the observers remained in the background. No explicit teaching was given to the groups. At the end of the activity, each group of children shared with the class their approaches to working the activities, explained and justified the model they had developed, and then invited feedback from their peers. This group reporting was followed by a whole class discussion that compared the features of the mathematical models produced by the various groups.

Modelling Activities

The series of model development activities that was implemented across the year comprised an initial model-eliciting activity (*Sneakers Problem*, described in Doerr & English, 2003), a model exploration activity (*Weather Problem*, also described in Doerr & English, 2003), and two model-application/model-adaptation activities (*Consumer Guide,* and *Car Problem*, the latter of which appears as the appendix). The last two activities provide opportunities for children to transfer and refine the mathematical constructs they have developed in the previous tasks, as well as to refine any representational systems.

The activities involve the core mathematical ideas of ranking, weighting ranks, and selecting and aggregating ranked quantities. These ranking processes entail analysing and transforming entire data sets or meaningful portions thereof, rather than single data points. The sequence is designed such that children can engage in meaningful ways with the problem situation and can create, use and modify quantities (e.g., ranks) in ways that are meaningful to them and in ways that can be shared, generalized, and re-used in new situations. The children in the present study had no specific formal exposure to or instruction on these core mathematical ideas prior to commencing the activities.

Data Collection and Analysis

Data sources included videotapes of whole class sessions and audio- and videotapes of small group responses to each of the modelling activities. Field notes, children's work sheets, and final reports detailing their models and how they developed them were also important data sources. The tapes were transcribed and, for the present paper, analysed for evidence of the processes that the children used in constructing their mathematical models.

Results

Consideration is given first to the modelling processes displayed by one representative group of children (referred to as “Jasmine’s group”) as they worked the Car Problem. Next, a brief description of the models that were produced by other groups is presented.

Model Development by One Group

An analysis of the interactions of Jasmine’s group as they worked the Car Problem indicated a number of diverse processes through which the children cycled as they progressed towards their final model. More specifically, the children displayed the following processes: *interpreting* and *re-interpreting the problem statement, clarifying* and *revisiting the goal, making decisions subjectively versus objectively, applying* and *refining mathematical procedures, posing hypotheses* and *problems, making assumptions* (false or otherwise), *posing arguments* and *counterarguments, considering all options (*versus considering limited options*), asking for justification, thinking metacognitively,* and *applying previous models*.

In the following excerpt, Jasmine's group is commencing the Car Problem with Jasmine restating the goal. In doing so, she makes an incorrect assumption regarding Carl's mother's monetary input; this was not detected by the other group members. Charlotte and Jasmine then suggest beginning with a process of elimination, which instigates a comment from Rachel regarding the need to consider all factors:

> Jasmine: Our job is to find a car for Carl and his mother's paying half of it and she doesn't want it to cost much and we have to make a list and decide which one is best to buy.
>
> Charlotte: I think we should do a process of elimination. (A brief discussion ensued regarding the mother wanting the car to be reliable.)
>
> Jasmine: …Maybe first, maybe we should do a process of elimination so work our way down the list or work our way up.
>
> Rachel: We have to consider all the factors though.
>
> Jasmine: Yeah, but we're going to.
>
> Rachel: Before you start the elimination process you should number the things first then do it. Then do the eliminations.

The group subsequently moved off task for a short period, discussing what they considered to be the best car listed in the table and which car the mother would like. Charlotte reminded the group that the car is "just for Carl"," with Jasmine responding, "Yeah, but he gets to choose. She's not going to make him." Douglas brought the group back on task, indicating that they needed to be objective just as they needed to be in the last modelling activity (The Consumer Guide problem involving the snack chips):

> She (the mother) is helping him. How about we just judge off what the thing says, not by what we think. It goes with the chips too you know. Let's all read through this again and underline all the details that help us.

The group spent a number of minutes revisiting the goal and re-interpreting the problem information. The issue of what Carl needs versus what he wants occupied their discussion ("You can't just think about what he wants, you have to think about what he needs as well."). This discussion led the group to reconsider their elimination process and what they should eliminate first. Considerable argumentation took place as to whether or not the most expensive car (the Honda Legend) should be eliminated. While Charlotte and Jasmine claimed "They are not going to buy the most expensive car so we could cross out the most expensive car but cross out no more than one," Rachel felt they shouldn't delete it because, "The Honda Legend might just have one bad thing and the rest is good." Douglas and Charlotte tried to resolve this dilemma by revisiting the problem information and again emphasising the need to be objective rather than subjective: "Yeah, but we are judging off what this thing (problem description) says. They don't want it to cost that much and it's like the most expensive car." The argumentation continued however, with Rachel claiming that "It (the Honda Legend) could have the best things in it." Rather than entertain Rachel's comment, Douglas repeated his statement regarding objectivity: "We don't know that. We're just judging on facts."

Rachel persisted with her claim that there are other factors to consider before eliminating the Honda Legend and persuaded the group to consider its fuel consumption and mileage, followed by safety factors. Argumentation again followed on which were the important safety factors for the Honda Legend. Douglas, however, was still concerned about the problem criteria: "They also said they don't want something expensive and that's the most expensive car…. weigh the choices up." The remaining group members in the meantime were hypothesising that Carl's mother might think the car "is worth the money because it's got all those safety things and it's going to help." During the argumentation, Jasmine had been examining the table of data herself and declared that the Ford Turbo and Honda Legend were possibilities. Rachel demanded that Jasmine justify her claims:

> Jasmine, tell us why you think these things. We need to know why you think them.

While accepting Jasmine's explanation, Rachel felt the group was not making sufficient progress. Hence, another group member, Lisa, suggested that the members vote to resolve their disagreements. The others, however, did not agree: "We can't just vote. That's not logical." Jasmine then continued to explain the ranking system that she had developed: "The year….the later the year, probably the newer the model so it's more in and so I just did a ranking system of 1,2,3,4,5,6,7, 8—9—and yes, the best one of that was the Hyundai Excel GX X3." Rachel, however, reminded Jasmine that "We have to write a list. It says your job is to create a list for Carl and for his mother." The group then reverted to argumentation over cost factors versus leisure features of the cars, with Lisa suggesting that they consider their own perspective to help resolve their disagreement:

> Maybe we should decide if we were buying a car what would we like….I know we're not buying the car but sometimes that actually helps to think about what we would buy….

Once again, Douglas emphasised:

> We're not deciding on what we like, we're deciding on the facts….we have to look at these factors.

In the next session, however, Jasmine's group decided to resolve this dilemma by voting on what they considered to be the most important features to address. They then constructed Tables 1 and 2 (reproduced from the group's computer file). The group's explanation of their model development appears below Table 1. Notice how the group referred back to the system they used in the previous modelling activity ("the chips," i.e., the Consumer Guide problem).

> Our group used a rating system like the chips. We voted the most important features and then we put them into a table. We put them into categories as safety features, leisurely features, and extras. Then we ticked the cars that had those features as shown. (Hold up picture of table.) Next we rated the mileage. The best was the highest and the worst was the lowest rating. After we came up with the results we did a draft list for Carl and his Mother and also a separate one. We did this by adding the features and the mileage together and the highest was the best. After we got the scores from the table we numbered them 1st, 2nd and so on then we put them in a list for carl and his mum. After doing that we wrote the total so that there was a combined list for carl and his mum. The best for both of them and each of them was the Daewoo Lanos (see Table 2).

Table 1
Car Features Table

CAR	Antilock Brakes	Air bags	Air conditioner	Alloy wheels	Power Windows	Electric Sunroof
Nissan Silva	N/A	N/A	N/A	YES	YES	N/A
Ford Capri Turbo	N/A	N/A	N/A	YES	YES	N/A
Audi 90 Sport	N/A	N/A	YES	YES	N/A	YES
Ford falcon EAS	N/A	N/A	YES	N/A	YES	N/A
Nissan Pulsar LX	N/A	N/A	YES	N/A	N/A	N/A
Hyundai Excel GX	N/A	N/A	YES	N/A	N/A	N/A
Daewoo Lanos SE	N/A	N/A	YES	N/A	N/A	N/A
Honda Legend	YES	YES	N/A	N/A	YES	YES
BMW 318i E36	YES	N/A	YES	YES	N/A	YES

Table 2
Car Preferences for Carl and His Mum

Name of Car	Carl	Mother	Total
Nissan silver	4^{th}	4^{th}	4^{th}
Ford Capri turbo	4^{th}	4^{th}	4^{th}
Audi 90	8^{th}	4^{th}	6^{th}
Ford falcon	3^{rd}	3^{rd}	3^{rd}
Nissan pulsar	5^{th}	5^{th}	5^{th}
Hyundai excel	2^{nd}	2^{nd}	2^{nd}
Daewoo lanos	1^{st}	1^{st}	1^{st}
Honda legend	7^{th}	6^{th}	7^{th}
BMW	6^{th}	7^{th}	7^{th}

Model Development by Other Groups

The remaining four groups varied in their interpretation of the problem information, in particular, in their interpretation of the given criteria (safe, reliable, fun etc) and in identifying factors to define those criteria. While all groups considered the criterion of "safe" to be significant, only two groups also specifically addressed "reliable." One group defined reliable by the year of make and mileage, and listed the cars in descending order with respect to each of these factors. This same group assigned points to the ranked cars as follows: 1^{st} = 5 points, 2^{nd} = 4 points, 3^{rd} = 3 points, 4^{th} = 2 points, 5^{th} = 1 point. To determine the car suitable for Carl, the group totalled the points assigned to the cars for each of the factors, "fun," "cheap," and "good fuel mileage." They did the same for Carl's mother (i.e., totalling the points for "safe" and for "reliable").

The other group began with two categories, namely, "Carl (Fun)" and "Mother (Safe)" and, as they explained, "rated the cars according to how many features they had." They then created a third category, "Cars for Both," which comprised four cars. The group subsequently eliminated three of the four cars by "mainly looking at the mileage and age."

Concluding Points

Modelling problems of the present type provide rich opportunities for children to engage in a range of mathematical processes, along with the development of important mathematical constructs that are embedded within the problem. Because modelling activities are designed for small group work, they are ideal vehicles for developing collaborative problem-solving skills. This paper has provided examples of the mathematical and social processes that children display when working meaningful modelling problems. These processes include interpreting and re-interpreting the problem information (including the goal statement), making appropriate decisions, justifying one's reasoning, posing hypotheses and problems, presenting arguments and counterarguments, applying previous learning, and acting metacognitively. The Car Problem was the final activity in a series of problems that focused on the mathematical ideas of ranking, weighting ranks, and selecting and aggregating ranked quantities. The children applied these ideas in different ways to generate models independently of instruction.

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Appendix

What Car to Buy?

Carl and his mother have been out shopping for cars. Carl wants a car that will be fun to drive around in, gets good gas mileage, but doesn't cost too much. But Carl's mother, who is going to help pay for the car, wants him to have a car that is reliable and safe. Your job is to create a list for Carl and a list for his mother showing which cars are the best. Then they will have to decide which one to buy!

Car Information

Car	Year	Cost	Color	Mileage	litres/ 100 km City	Features	Body Style
Nissan Silva	1992	10,000	Navy Blue	96,000	10	Rear Spoiler, Power Windows, Power Steering, CD Player, Alloy Wheels, Alarm	Coup
Ford Capri Turbo	1989	8,200	Red	105,000	9	Rear Spoiler, Power Windows, Power Steering, CD Player, Alloy Wheels, Alarm	Convertible
Audi 90 Sport	1991	9,500	Silver	97,500	10.5	Rear Spoiler, Power Windows, Power Steering, CD Player, Alloy Wheels, Electric Sunroof, AC	Sedan
Ford Falcon EA S	1988	5,200	Pale Blue	113,500	11.5	Power Steering, Radio Cassette, Tow Bar, AC	Sedan
Nissan Pulsar LX	1993	7,950	Gold	125,000	7.5	Power Steering, Radio Cassette, Tinted Windows, Bull Bar, AC	Sedan
Hyundai Excel GX X3	1999	9,500	Dark Blue	49,000	7.6	Power Steering, CD Player, Rear Spoiler, AC, Tinted Windows	Hatchback
Daewoo Lanos SE	1997	7,250	Azurite Blue	74,118	8.8	6 Stacker CD player, 6 speakers, Amplifier, Tinted Windows, AC	Hatchback
Honda Legend	1993	17,200	Dark Green	154,000	12.5	Dual Airbags, Antilock Brakes, Alarm, Cruise Control, Electric Sunroof, Power Windows, Power Steering,	Sedan
BMW 318i E36	1991	15,000	Blue	164,000	9.5	Radio Cassette, Power Steering, Electric Sunroof, AC, Alloy Wheels, Antilock Brakes,	Sedan