High Performance, Confidence, and Disinclination to Explore: A case study

Gaye Williams
Deakin University
<gaye.williams@deakin.edu.au>

This video-stimulated interview study of problem-solving activity of a high performing Grade 6 girl who displayed confidence in her mathematical ability, provides a microanalysis of tensions she encountered when her findings using concrete aides did not match her rule application. It highlights her disinclination to explore these inconsistencies. This study points to the problematic nature of pedagogical approaches that develop only instrumental understandings and emphasises the need to explicitly value what policies promote; creative and innovative thinking.

The Melbourne Declaration for Young Australians (p. 8) states:

> Successful Learners … are creative, innovative and resourceful, and are able to solve problems in ways that draw upon a range of learning areas and disciplines (Ministerial Council for Education, Early Childhood Development and Youth Affairs, 2008).

The increased valuing of creative and innovative student activity in Australian curriculum and policy documents reflects the growing understanding of how important such activity is for shaping the mathematical literacy of the next generation. Such valuing is not yet reflected in most classroom tests, and National, and International testing though. Many such assessments involve predominantly or only recall of rules and procedures (Shimizu & Williams, 2013) rather than examine students' ability to flexibly use the mathematics they have learnt in unfamiliar situations. Stronger associations have been found between confidence and high mathematical performances on multiple-choice tests items than between confidence and high performance on open-ended tasks (Pajares & Miller, 1997). What might such a finding tell us? This study examines the problem solving activity of a confident student (Aisha) with the ability to perform at a high level in class tests that involved predominantly applying known rules and procedures in familiar situations. It considers associations between her high performances, confidence, and inclination to problem-solve creatively. The research question posed is: What factors are influencing this student's disinclination to explore?

Theoretical Framing

Skemp (1976) distinguished between knowing only rules and procedures and not the reasons why they work (instrumental understanding), and having a connected understanding of mathematics that enabled a student to flexibly recognise when it was appropriate to use this mathematics in unfamiliar situations (relational understanding). Williams (2010) linked such flexibility to Seligman's (2011) 'optimism'. Optimism is a form of resilience associated with a person's orientation to the successes and failures they encounter. For student optimism specific to mathematical problem solving activity (Williams, 2012) 'failure' is 'not yet knowing' and 'success' is 'finding out'. The three dimensions of optimism are Permanent-Temporary, Pervasive-Specific, and Personal-External and their enactment during successful problem solving is as follows. A successful problem solver when encountering something unfamiliar perceives not knowing as *temporary* and able to be overcome through the *personal effort* of looking into the situation

In V. Steinle, L. Ball & C. Bardini (Eds.), Mathematics education: Yesterday, today and tomorrow (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia). Melbourne, VIC: MERGA.

of not knowing to identify what is in their control to change and what is controlled *externally* thus unable to be changed by the student. They make decisions about what *specific* aspect of variables (over which they have control) is likely to be most productive to vary, and vary this. They continue to vary different elements of the problem over time: adjusting their activity as they find out more about what does not work. They perceive the successes they achieve as *permanent* (able to be achieved again) and take these successes on as *pervasive* characteristics of self ("I can do this, I am good at this"). A sub-question becomes, what were the components of optimism this high performing student (Aisha) did not possess, and in what ways did this influence her problem solving activity.

Research Design

This study is part of a broader study of the role of optimism in collaborative problem solving and whether building optimism increases problem-solving capacity. Aisha was a Grade 6 student in a class in which her mathematical competency was frequently affirmed by others because of her high grades on class mathematics tests. Like the types of tests in many Australian schools, these tests predominantly involved recalling rules and procedures not using mathematics in unfamiliar situations. Aisha was part of the broader study for one year only, and the study in this paper focuses on the second problem-solving task she undertook. In this study, Aisha was grouped with Jeff and Belinda. This case was selected because Aisha developed two different and incompatible understandings about the area of the triangle she studied, and then discarded the rule she had learnt in favour of her incorrect finding derived from using concrete materials. Aisha did not evaluate the reasonableness of either approach in making this decision.

The pedagogy employed ('Engaged to Learn Approach)', was developed by the researcher (author), informed by her research (Williams, 2005), and teaching (Williams, 2002). The researcher as teacher (RT) and the classroom teacher (T) team-taught with the RT as primary implementer of the task. Students worked in small groups composed by RT and T (3-4 students) (see Williams, 2008). The RT and T did not affirm pathways taken nor ideas presented but rather asked questions to stimulate further thinking. For more information about the teaching and learning approach, see Williams (2007).

This following section describes the task under focus, the pedagogical approach employed, and the data collection instruments utilized including why they were appropriate for collecting data to answer the research questions.

The Task

The task was accessible through a variety of representations and levels of mathematical sophistication to give groups opportunities to idiosyncratically discover and explore complexities just beyond their present understandings.

The topic 'Areas of Triangles' was not studied in this Grade 6 class prior to this task (Figure 1). This task was designed to be: a) accessible (trial and error in drawing triangles, and trying to cover them with squares as a possible entry point); b) 'conceptual' (supporting the development of understanding of area as the amount of space within a 2D figure); c) developing strategic mathematical thinking (e.g., recognizing that partial squares can be combined to form whole squares); d) developing mathematical language (around properties of figures, areas, triangles, and angles); e) developing language associated with exploratory activity (e.g. varies, constant, and constraints); f) informally drawing attention to perpendicular height; and g) developing the recognition that triangles with the same base length and area have the same perpendicular height.

How Many Triangles

Students are not given directions about how to approach the task.

Make as many different triangles as you can that have a base of 10 cm and use 80 of these little one centimetre-by-one centimetre squares to cover all the space inside them.

While doing that, think about the thinking you are doing to work this out. You are to report to the other groups each 5-10 minutes (with a different reporter each time: at least four reports across two 80-minute sessions). In your report you will talk about the kinds of things you have been thinking of, and what you have found, or are still trying to work out.

Materials for group: pack of 10 coloured squares (1cm x1cm), scissors, ruler, coloured paper, graph paper sheets (1cm, 5mm, 2mm), pencil, overhead transparency sheets, transparency pens, butchers' paper, and texta colours.

Figure 1. Second task class undertook in their Grade 6 year.

Data Collection Techniques

Four video cameras captured the activity of each group in class during their problem solving sessions. The activity of the group under study, their reports to the class, and the responses of other class members to these reports are captured. They provide evidence of types of thinking undertaken, and of Aisha's enactment along dimensions of optimism. After each session, the worksheets and artefacts produced by each group were collected and used as additional stimuli during individual post-lesson video-stimulated student interviews. Aisha was interviewed after Task 3 not this task. This interview provided evidence of her non-optimistic orientation. This matched Aisha's enactment of absence of optimism during Task 2. The interview also provided information about how Aisha learnt mathematics, and how she perceived herself as a learner. Aisha also answered questions including: "How do you think you are going in maths, and how do you decide?" "How do you learn something like that [mathematics associated with the problem solving task]?" and "Does anyone help you with maths at home or outside school? Such questions tended to elicit information about whether Aisha perceived her successes as personal or reliant on external influences, and whether or not she was confident about her mathematical ability. Students displayed indicators of success as personal where they perceived learning as predominantly associated with personal effort in reorganisation and synthesis of previously developed ideas to create new mathematical ideas. Indicators of success as personal were also displayed when students primarily evaluated their mathematical performance internally rather than through external sources like test results, or teacher or parent evaluations. In contrast, where students relied primarily on external judgments of their performance and described the way they learnt as occurring through 'taking in' and repeating of knowledge from external sources, they displayed indicators of success as external. Optimistic or non-optimistic indicators are not always evident in responses to a particular question, and the probes following each question depend upon the student's response. Thus indicators of optimism can be found in various parts of the interview transcript dependent on the responses students give, and the probes the researcher is able to introduce.

Results and Analysis

Excerpts of Aisha's activity and Aisha's interview are presented to illustrate the:

- Types of mathematics she generated in her group
- Way she interacted with her group
- Report Aisha gave to the class for her group and class discussion it stimulated
- Way she learnt, and the mathematical confidence she displayed

Task Activity

When group work commenced, Aisha dominated conversation working rapidly out loud. Excerpts of this talk are now presented. They include some RT talk to the whole class and some conversation in Aisha's group:

1. RT [to class] All you are doing is spending three minutes each of you individually trying to make a triangle with a base of 10 centimetres that is completely filled up with eighty of those little one centimetre by one centimetre squares. … Write one sentence that tells us what happened when you tried that and … what are you going to do next?
2. Aisha [to group] Did you know that to get the area of a right angled triangle you times the bottom by the side and halve that
3. RT [to class] you might even want to write reasons for why you do what you do not only what happens
4. Aisha [Directing group member to make a 16 cm line 'straight up'] that's my (…) because I thought of it
5. RT [to class] you have now got five minutes to work together as a group. Now you can use about two of those minutes to finish what you are doing on your own triangle
6. Aisha (inaudible) [excitedly in a firm voice at fast pace without hesitations tells the group her idea]
7. Aisha I am the most complex [able to give most complex report] so I do the last one [report]

Aisha immediately recalled a rule she knew (half the area of a rectangle gives the area of a triangle) [Line 2]. She was rapidly able to calculate the dimensions of the rectangle she needed [Line 4]. Belinda (in Aisha's group) reported the method Aisha had developed to the class (cut a 10x16 rectangle in half along the diagonal and the two triangles formed are each the required size). See triangles formed in rectangle in Figure 2a. The RT posed several questions after this report for groups to make their own decisions about whether or not they would explore them. Groups then returned to their group work. The RT's questions were:

1. How do we know that will work? This group has done a calculation but they have not told us why they think that calculation should give the amount of space wanted.
2. Is there another way to check whether this triangle is a possible triangle that fits what we were asked to do?

Other groups showed various different shaped triangles they had worked with and ways they had varied the sides when they found they had too few or too many squares. Aisha's group appeared to think they had all possible solutions because they did not mention or try any other triangles when they went back to their group. Nor did they try to work out why their calculations worked even though the other two group members (Jeff and Belinda) made several attempts to start such a conversation.

Just before her report towards the end of the second session, Aisha stated to Jeff or Belinda: “No I am smarter than you though”. When the RT tried to elicit deeper thinking in groups: “Have you gone into why your calculation works?” the other two members of Aisha’s group again tried to get Aisha to think further but she retained the conversation within what she knew. Part of this conversation is listed below:

1. Jeff [exclaims with emphasis] *Aisha-Aisha* we have to actually equal it into eighty …
2. Aisha [firmly states] *this works*! [In a firm voice closing off further conversation] [‘This’ was the calculation].

Towards the end of the second lesson, Aisha gave her report:

> [pointing to Figure 2a] Um well when we actually counted and I don’t think it [their first calculation] would work because up to here [points to bottom of bottom row of squares in the triangle in Figure 2a] it should have been like around forty but it is actually like twenty and another twenty [points to space in triangle below grid] it would be forty so it is actually yeah half of it and I don’t think it [first method] would work

Aisha assumed the space in the triangle covered by grid was the same amount as the larger space at the bottom of the triangle. It is unclear whether she was visualising these spaces as the same size, or did not have a conceptual understanding of what was meant by ‘the amount of space within’. When she finished reporting, she looked around at the class, many of whom looked silently and intently at the board because their own calculations showed them Aisha was not correct. The RT then encouraged comments from the class:

1. RT Now without giving anything away can anybody ask any questions? …
2. Baruch [to Aisha] Are you sure about this?
3. Aisha [Voice firm slow voice] *Yeeah* ‘cause [turns to diagram] that should have been around forty [points to gridded part mentioned] and that should have been around forty [bottom part] and so- but even if that was forty [bottom] and that was another 20 [top] that is still only sixty and still that would be less
4. RT [to Baruch] Nice question … really nice question
5. Aisha ‘Cause even like (pause) everybody like (pause) it seems like they were agreeing to it [group’s first calculation method] and when I actually look- actually I think it’s wrong (pause) so …
6. RT That’s really interesting- you have described your thinking really well Aisha … thankyou very much- next group please … I hope you are all *really* thinking about that …
7. Noella [puts up group’s overhead, see Figure 2b] … we tried Belinda’s strategy because we wanted to see for ourselves whether it has eighty squares- we got 73 squares … but we didn’t count the all non-whole squares [points to the shaded parts along the hypotenuse of the triangle in Figure 2b]

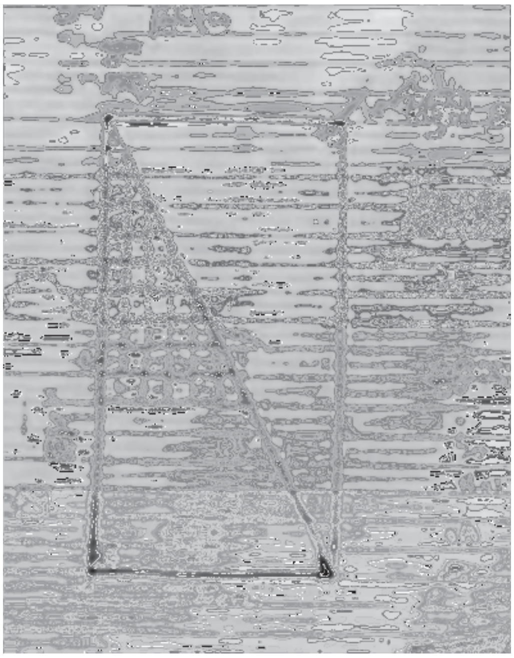

Figure 2a. Diagram used in Aisha's group report.

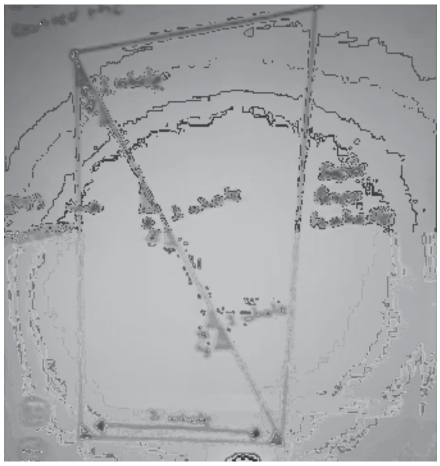

Figure 2b. Diagram used in Noella's report.

The reports of the other groups provided many different opportunities for Aisha to rethink her report but the group retained their current understanding as shown in the next group work session. The group complained to each other that the RT had acknowledged good thinking in reports of others who had 'stolen' ideas from this group. The group showed an absence of understanding that a purpose of the group reports was to adapt and develop ideas arising in the class. The majority of the other groups had found triangles that enclosed a space close to eighty squares and some had found a possible triangle and checked it by another method. One group justified the calculation in Belinda's report using arrays in the rectangle. Noella's group reported after Aisha's. Although not perceived by the class or the teacher as mathematically able relative to Aisha, Noella's report demonstrated a greater depth of understanding of relevant mathematics, as did her response to the questions asked by class members:

1. RT Anyone need to ask anything there?
2. Noella Theresa?
3. Theresa What do you mean by it could be or it couldn't?
4. Noella [Pointed to the small shaded parts of squares] well because we didn't count these little ones it *could actually be eighty* because there was much more than that but we didn't actually draw every single one so it could- *all them* could add up to be eighty but we didn't really count them so we don't really know
5. RT Nice explanation- that is- I am *so glad* that one [Noella's report] came up next- just think about Aisha's report … she doesn't think they are going to come *anywhere near* eighty … and yet this group are telling us that they have counted those squares and they are up to at least … what without the little bits?
6. Noella [looks at page then at RT] 73
7. RT How did that happen? Can both of those reports be on the right track- is something the matter somewhere- you need to think about it- thanks Noella

The RT's questions are intended to stimulate further thinking about the reasons for findings, and evaluation of inconsistencies found. They did not do so for Aisha.

In her interview Aisha confirmed that she learns by listening to others, and that she likes procedures to be explained in slow small steps. Although she liked preparing class presentations, she only liked taking part when she had a clear understanding of the content and not when the presentation was about new ideas:

> I just really *hate* reporting- [smile in voice] because you have to go up … in front of people and *talk* … I don't mind reporting if I know … *very* well what I am going to say -I know that- this is going to happen- this is going to happen- like not last minute changes or being new or something … like *that*

She expressed emphatically in her interview that assessment of her mathematical ability can only be made by an external source not by herself:

> … [if] you are asking me how I think of myself I would say it's not for me to say- it's for … *you* [intensity now develops in voice] … you can't be smart in all ranges of maths- so that is why- I don't really know if I am good at everyth[ing] …

Aisha displayed several indicators of absence of optimism in her interview and in class.

Discussion and Conclusion

Aisha was not optimistic and it was particular elements of this absence of optimism that limited her problem solving capacity. Aisha demonstrated on several occasions that she was extremely confident about her capacity to do mathematics as demonstrated by her articulation that she should be the last reporter—the one who is most able to explain complex mathematical ideas. Aisha perceived success as attained through external sources that explain new work in small steps rather than possessing the optimistic indicator 'success as personal'. When Aisha was confronted with two inconsistent findings, she did not look into the second situation (counting squares) to see if there was anything that needed to change. In addition, her instrumental understanding of the rule she had used to calculate the area of the triangle in her first method was not sufficient to allow her to look into that situation to decide whether or not it was an appropriate rule. Aisha did not perceive 'failure as specific;' she did not demonstrate she could look into a situation that was not working and identify what she could change to increase her chances of success. These are the types of activities a student needs to undertake to be a successful problem-solver. Her non-optimistic characteristics have been conditioned by community responses to high performances on tests that assess instrumental understandings. Aisha had no expert available to find out what to do, so she made a decision that was not based on evidence (the rule must be wrong). If she had not been so definite in her statements about her changed decision, it could be considered that Aisha was succumbing to pressure by agreeing with the method she saw used frequently in the class around her.

This study illuminates how the absence of relational understanding of the formulas and procedure used can limit student ability to evaluate the mathematics they generate, and lead to two incompatible understandings about the mathematics under study. It also provides evidence that students who commence a problem solving activity with little background knowledge of the mathematical concepts that underpin the task can develop relational understanding and be able to flexibly work with new mathematics. Noella's group provide evidence of this.

This study adds confirmatory evidence to the body of knowledge on the limited value of instrumental understanding (Skemp, 1976), and thus adds impetus to Australia's Chief

Scientist's (Chubb, 2012) call for more innovative and creative pedagogical methods to increase the flexibility with which students are able to use the mathematics they learn. The questions raised include: What types of interventions could shift such students to being inclined to explore. This is an area requiring further research.

Acknowledgements

This study was funded by the Australian Research Council DP0986955 and hosted by the International Centre for Classroom Research at the University of Melbourne.

References

Chubb, I. (2012) *Mathematics, Engineering and Science in the National Interest*. Report to the Prime Minister, Australian Government, May. Department of Industry, Innovation, Science, Research and Tertiary Education. Canberra

Ministerial Council for Education, Early Childhood Development and Youth Affairs (2008). *The Melbourne declaration of goals for young Australians*. Retrieved March 21st, 2013, from http://www.mceecdya.edu.au/mceecdya/melbourne_declaration,25979.html

National Curriculum Board. (May, 2009). The shape of the Australian Curriculum: Mathematics.

Pajares, F. & Miller, D. (1997). Mathematics self-efficacy and mathematical problem solving: Implications of using different forms of assessment. *The Journal of Experimental Education*, *65*(3): 213-228.

Schwarz, B. B., Dreyfus, T. and Hershkowitz, R. (Eds.) (2009), *Transformation of Knowledge through Classroom Interaction*. London, UK: Routledge.

Seligman, M. (2011). Flourish: A visionary new understanding of happiness and well-being. William Heinamann; Sydney.

Shimizu, Y. & Williams, G (2013). Studying learners in intercultural contexts. In C. Keitel (Ed.) *Social, political and cultural dimensions in mathematics education*, In K. Clements, A. Bishop, J. Kilpatrick, F. Leung, C. Keitel (Eds.). *Third International Handbook in Mathematics Education (Volume 1,* pp. 145-168*)*, NY: Springer.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, *77*, 20-26.

Williams, G. (2012). Associations between confidence, persistence, and optimism: Illuminating optimistic problem solving activity. *Paper presented in Topic Study Group: Learning and Cognition at the International Congress for Mathematics Education*, Seoul, July 2012. Accessed at http://www.icme12.org/sub/sub02_05.asp on the 28th July 2012. [Study Topic Group 22-> papers]

Williams, G. (2010). Symbiosis between creative mathematical thinking accompanied by high positive affect, and optimism. In Pinto, M., & Kawasaki, T. (Eds.). *Proceedings of 34th conference of the International Group for the Psychology of Mathematics Education*, (Vol. 5, pp. 297-304). Belo Horizonte, Brazil: PME.

Williams, G. (2008). How Group Composition Can Influence Opportunities for Spontaneous Learning. In M. Goos, R. Brown, & K. Maker (Eds.). *Mathematics Education Research Group of Australasia 31 Conference Proceedings*, (Vol. 2, 581-588). Brisbane, Australia: University of Queensland.

Williams, G. (2005). *Improving intellectual and affective quality in mathematics lessons: How autonomy and spontaneity enable creative and insightful thinking*. Unpublished doctoral dissertation, University of Melbourne, Melbourne, Australia. Accessed at: http://repository.unimelb.edu.au/10187/2380 on the 10th November 2005.

Williams, G. (2002). Have faith in students' ability to think mathematically. In C. Vale, J. Roumeliotis & J. Horwood (Eds.), *Valuing mathematics in society* (pp. 114-126). Melbourne, Victoria: Mathematical Association of Victoria.