Classroom Culture, Challenging Mathematical Tasks and Student Persistence

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While there are many considerations for effective mathematics teachers, one key factor is the development of a classroom culture that supports the desired form of learning. In examining the opportunities and constraints associated with posing challenging tasks, we are exploring ways that teachers might influence classroom culture positively. The data presented below suggest that it is possible to foster a classroom culture in which teachers pose tasks that challenge students and encourage them to persist when working on those tasks. The key elements seem to be the ways the tasks are posed, the interactive support for students when engaged in the tasks, collaborative reviews of class explorations and assessment against criteria.

One of the goals of teaching mathematics is to prepare students for their future mathematics studies. Another, and perhaps more important, goal is for students to learn sufficiently well to be able to use the mathematics in their future lives and employment. In classrooms in which teachers pursue the former of these goals it seems it is common to emphasise and assess fluency with procedures (see Hollingsworth, Lokan, & McCrae, 2003). In classrooms in which the latter goal is prioritised, the mathematics being learned should ideally be robust, adaptable and able to be transferred to different contexts. This useable mathematics is not a collection of isolated facts and routines, but networks of inter-connected ideas. More importantly, learners ideally would be confident that not only can they connect mathematical ideas to each other but also they know that they can adapt their knowledge to unfamiliar situations. In other words, it is not enough that students can fluently recall procedures. Not only must learners be able to apply their mathematical understandings in new contexts but also they must have the confidence and willingness to devise solutions to problems that they have not previously been shown how to solve. To prepare students for this, it is arguable that all students, and certainly not just the high achievers, should have opportunities to engage with mathematical tasks that are complex and for which they devise their own methods of solution. We describe such tasks as *challenging*, and we anticipate that most students need to be willing to *persist* when working on such tasks if they are to solve them.

Some of the characteristics of these challenging tasks are the requirement for students to connect different aspects of mathematics together, to devise solution strategies for themselves and to explore more than one pathway to solutions. We also consider part of the challenge to be an expectation that students record the steps in their solutions, explain their strategies and justify their thinking to the teacher and other students.

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The following draws on one aspect of a larger project[1] that is recommending that teachers present students with challenging tasks. We are examining what happens when teachers pose challenging tasks to students and are exploring ways in which teachers can encourage students to persist on those tasks. The focus of this discussion is the relationship between classroom culture and students' willingness to engage with challenging tasks.

Elements of classroom culture

The overall project is informed by a framework that proposes that teachers' actions are informed by their intentions which had, in turn, been influenced by their knowledge about mathematics and its teaching, and the associated beliefs and constraints they anticipate they might experience when teaching. It is this latter aspect that is most directly connected to classroom culture. For example, in terms of constraints teachers anticipate they might experience, it seems there is a tendency for teachers to reduce the demand of tasks when planning (e.g., Tzur, 2008), and to over explain tasks during lessons (e.g., Stein, Grover, & Henningsen, 1996) fearing negative student reactions to the risks they are required to take. In an important meta-analysis of 49 research studies on classroom culture between 1991 and 2011, Rollard (2012) described three significant and relevant findings.

First, the meta-analysis found that the middle years of schooling (years 5 to 9) are critical for connecting classroom goal structures and the formation of student attitudes, including an orientation to persist, because it is in these years that parents and teachers become more interested in assessment of success, and there is more overt competition between students (sometimes fostered by parents and teachers). Students in these years are more reluctant to take risks and sometimes avoid the perception of trying hard to avoid censure from other students (see Sullivan, Tobias, & McDonough, 2006). It is at these years that it is most important to create the type of classroom culture that can allow teachers to present students with challenges. The project is focusing on these years.

Second, Rollard (2012) concluded that classrooms that promote mastery, meaning those that focus on the learning of the content rather than competitive performance, are more likely to foster positive student attitudes to learning. Dweck (2000) explained that students who seek mastery of content are more willing to take risks and are less dependent on the affirmation of others. Dweck argued that students who focus on performance, meaning they seek social affirmation rather than understanding of the content, are more likely to avoid persisting. Rollard (2012) suggested that teachers can actively promote a mastery orientation in the students. We interpret this to include the teachers posing challenging tasks, as well as explaining the purpose of the challenges, and encouraging students to persist, along with emphasising the connections between persistence and learning.

Third, and most critical for our project, Rollard concluded from the meta analysis that classrooms in which teachers actively support the learning of the students promote high achievement and effort. We interpret this to refer to ways that teachers can support students in engaging with the challenge of the task, and in maintaining this challenge as distinct from minimising it. Some elements of this active support that we are exploring are:

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- the identification of tasks that are appropriately challenging for most students;
- the provision of preliminary experiences that are pre-requisite for students to engage with the tasks but which do not detract from the challenge of the task;
- the structuring of lessons including differentiating the experience through the use of enabling and extending prompts (see Sullivan, Mousley, & Zevenbergen, 2009) for those students who cannot proceed or those who complete the task quickly;
- the potential of consolidating tasks (see Dooley, 2012), which are similar in structure and complexity to the original task, with which all students can engage even if they have not been successful on the original task;
- the effective conduct of class reviews (see Smith & Stein, 2011) which draw on students' solutions to promote discussions of similarities and differences;
- holistic and descriptive forms of assessment against criteria that are to some extent self referential for the student; and
- the balance between individual thinking time and collaborative group work.

It is interesting to consider the similarities and differences in Rollard's conclusions and other models of classroom culture. For example, the choice of tasks connects to what Cobb and McClain (1999) termed "personally experienced mathematical problems … (which) would constitute opportunities for them to learn" (p. 12). For both Rollard and Cobb and McClain, the classroom culture is not just created by establishing rules in advance, but through the structure of lessons, the ways of interacting around tasks, the ongoing interactions between teachers and students during lessons and the relationship with the teacher. The data and discussion below seek to elaborate these issues further.

The research context and source of data

The data reported below are from reports from two groups of teachers of Year 8 (age 13) students from Australian schools. The classes of these teachers include a mix of student socio-economic backgrounds. In working with the teachers, we are adopting a design research approach which "attempts to support arguments constructed around the results of active innovation and intervention in classrooms" (Kelly, 2003, p. 3). Our *intervention* is that we are suggesting particular tasks for the teachers to pose to the students, as well as recommending particular pedagogies that they might use. The *innovation* refers to the types of tasks we suggest which are different from those commonly used by the teachers. The lessons are also structured differently from conventional lessons mainly due to a focus on only one or two tasks over each lesson. Each iteration consists of posing tasks to match content the teachers are about to teach, anticipating pedagogical challenges, the teachers incorporating the tasks into their planning, and a subsequent meeting to review the experience. The meetings were audio recorded, transcribed and coded to identify themes that address the elements of active support listed above. Some responses were provided in written form.

Even though the teachers incorporated a range of tasks into their lesson sequences, only three of the tasks are discussed here. The tasks, chosen to address the topic of probability identified by the teachers, are presented to illustrate the nature of the challenging tasks we are proposing. The teacher reactions below refer to these tasks.

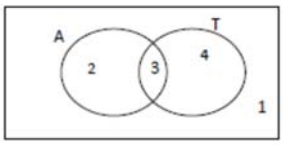
The first task, *Train passengers*, was presented as follows:

> On a train, the probability that a passenger has a backpack is 0.6, and the probability that a passenger has an MP3 player is 0.7. How many passengers might be on the train? What is the maximum and minimum number of possibilities for people who have both a backpack and an MP3 player? Represent each of your solutions in two different ways.

The second task, *Phone and Chocolate*, was:

> At my school there are 100 students in year 8. I know that the probability that a student both owns a mobile phone and likes chocolate is 0.4. The probability that a student does not own a phone and does not like chocolate is 0.35. What is the smallest number of students who might own a mobile phone? What is the largest number of students who might own a mobile phone? Represent your solution in two different ways.

The third task, *Matching representations*, involves sets of cards in the style of Swan (2011). There were five sets of cards, each set consisting of 5 different representations of data related to the names of a group of students. One set of matching cards is shown in Figure 1. There was also a sixth set which did not match that were intended to be used for checking responses:



$\Pr(A') = .5$
$\Pr(T') = .3$

Cathy	Ahmed
Tran	Bret
Travis	Kim
Hassan	Tony
Kitty	Tejinder

Only three students do not have a T in their name.

	A	A'
T	3	4
T'	2	1

Figure 1: Five different representations of students with letters A and T in names.

The explicit intention is that the students are not told how to do any of these tasks but are expected to devise their own methods of solution. These three tasks are substantially more challenging than conventional textbook questions in terms of the requirement for students to connect different ideas, in the variety and sophistication of the representations that the students are expected to connect, in their multistep nature, and the multiple pathways to solutions. The research question that guided this aspect of the project was: "What steps do teachers take to support student learning on challenging tasks, and how do these contribute to building the classroom culture?"

Results

As indicated above, the aspects of classroom culture that seem to support student willingness to engage with challenging tasks are those related to the ways that the lessons are conducted and the expectations set for the students not only in terms of the mathematics but also the ways of learning it. In the post-lesson review of the probability tasks with the teachers, four important aspects related to the classroom culture emerged: the ways tasks are introduced; actions that support students in engaging with the core task; reviews of the student solutions to the task, and non-competitive assessment processes. The following presents some representative responses from teachers that describe the way they incorporated each of these four aspects into their teaching.

Introducing a task.

One of the key aspects of the classroom culture is the way challenging tasks are introduced. Most teachers reported that they considered that their students needed preliminary experiences before they could engage with the tasks. The tension is that teachers need to find ways to introduce the necessary language and forms of representation without detracting from the subsequent student explorations. For example, one of the teachers, in preparation for posing *Train passengers*, explained:

> We discussed games/sports that involved probability. Today's lesson started with a recap of last week's lesson and a starter of how many students voted for the (a football team). I showed the

representation of this on the board as a decimal, fraction and percentage. I then asked who liked chocolate ice cream and showed the representation of this on the board. We had a brief discussion on what questions we could ask from this information.

This helped to introduce students to the different ways of representing data, as well as potentially considering the intersection of those data sets. A different teacher in preparation for posing *Phone and Chocolate*, explained that he had presented a similar task in a previous lesson:

I just gave them a simple example of my own with a similar idea, which was if you roll the die and a one comes up and you win, what are some different ways of representing that story, and so we just brainstormed that you can write it as a fraction, as a decimal, as a percentage, as a bar graph, as a pie chart, as a Venn diagram, all these different ways. So - the representation is there, which I thought helped them all when they came to this phone and chocolate one, was they started using pie charts and diagrams and other diagrams and then that took over a period and then we just presented a couple of the answers to the class at the start of the second period.

In both cases, these preliminary experiences were thoughtfully chosen with an emphasis on the connections between representations. The experiences provided an introduction to the task that did not detract from students' opportunities to determine their own method of solution of the subsequent task.

Supporting students when engaging with the core tasks

Another key aspect of the classroom culture is the ways teachers support students in engaging with the core task. One of the teachers described the method of posing prompts and the reaction of students to *Train passengers*:

I read through the question and asked students to represent their thoughts (method 1). I had a few comments that - this is too hard? I don't understand what to do? I can't start? I let the students have 5 minutes and gave enabling prompt (suggesting that they try and write out the numbers first) I found it very difficult not to give a few of the students BIG HINTS as what to do. I did notice that one girl solved the problem in less than 2 minutes - I asked her to represent her findings in another method. This was not a student whom I would have thought would have got the answer so quick. I gave out 6 enabling prompts (2 students still did not write down anything). Most of the class had one method and 60% had 2 methods. I noticed 5 students with different methods as I went around the class. At this stage I allowed students to talk to their peers as I found a few students needed prompts. I gave 2 students the (extending prompt) task.

This teacher reported some key actions: allowing the students time to struggle individually followed by group collaboration, overcoming the tendency to give "hints", and using prompts that differentiate the task.

In other words, not only had this teacher planned ongoing support for students when engaging in the task, but also had reflected on the challenges of doing this and that she considered the students willing to try. A different teacher reported on the Matching representations task:

Today I stopped them after about 10 minutes and said, "Who understands what this Venn diagram card means and you know what all the things are?" and then they all went, "Yeah," so I got a different group to explain it to them, and they all – bar two groups – understood all the different card types, which I think, you know, is the difference of half a week to a week of teaching it made, you know, them to feel more confident and actually understand what it was without needing too much explanation.

This is an illustration of providing this support without detracting from the student opportunity to understand. It emphasises that awareness of differences between groups.

Another of the teachers also described the responses of the students to the *Matching representation* task:

> ... we were kind of only just getting into probability, ... and they struggled with it quite a bit. So they knew the concepts and they knew what the different terms and the notations meant but they struggled with putting it together, whereas I did it with my other group today and we're a bit further on in the topic and they worked with it a lot better. So even the weaker students were able to kind of get to it eventually, it just took them a little bit longer, and that's why I was tentative to do the other ones too soon, because I feel like in some of them, especially with things like probability where they're unfamiliar with a lot of the terms it would kind of scare them off a little bit and I think like that the card task was a little bit too overwhelming for the group last week, whereas this week was probably a perfect time to present it to them and actually I think boosted their confidence and they thought, "Hey, we can do this. It's not stuff that we've been doing in the textbook but we can apply it now." So I think timing-wise that worked.

The support for the students was as much a matter of the timing of the task within a sequence as it was the particular actions of the teacher. Another related aspect in the report of this teacher was the time the students were engaged in the task since it indicates the level of persistence:

> It took more than a lesson. This group was probably a bit quicker than the first group. The first group didn't get – I started it. I only had one period, I started in the second lesson of a double, and no one finished the task, whereas this group I started halfway into the first lesson of a double, and they were still all working on it at the end of the second lesson. But then it was good because there was that extension task of getting them to create the sets of mismatched card at the end. So I had probably about half the kids working on that (extension) task, while others were still finishing.

The issue here is providing an extension task for those who have finished, allowing those students still working adequate time to engage with the task to a conclusion.

Overall, in terms of creating a culture of support, these teachers had planned task variations for those who needed them and they allowed sustained time working on the task that created the opportunities for students to make the necessary mathematical connections. The timing of tasks in relation to other content is important. The message seems to be that the students are aware that there will be support for them at each stage.

The sharing of student strategies

The processes for managing sharing of student strategies are a critical component of the establishment of an overall culture of support that create opportunities for student learning. As one of the teachers reported:

> At this stage of the lesson I stopped the class and asked the first girl to come up to the smart board and share her method. This student does not always achieve high results in mathematics. The next student I asked does achieve high results and her method was written out - she could not think of another way to represent her findings. The next student wrote his first method but then drew the students (with a coloured table above each person) for his second method. The last student I got up drew a Venn diagram but he didn't know what it was called. This student is (a student with poor literacy skills), he is not very confident with finding out answers for himself. He could explain what he did very well.

In this case, the process of inviting students to contribute to the sharing of strategies allowed some unexpected students to make important contributions. A number of the teachers commented on the opportunities that reviews create for different students:

> I handed out (the second part of *Matching representations*) and asked students to come up with one method. At this stage I expected most students to draw a Venn diagram but this was not the case. One student who did not write anything on the first task drew a Venn diagram and labelled it correctly. He is a student who has been struggling with maths (through external factors).

> To wrap up the lesson I set the task for homework and asked students to come up with 2 methods (they could have a discussion with their parents/ carers if needed). It was very interesting to see my more academic students become uncomfortable with the task.

Teachers commonly report that the students who do well on such tasks are not always the high achievers on more conventional tasks. The key issue though is that observing students when working and choosing students thoughtfully to contribute to explain their strategies makes an important contribution to classroom culture.

Assessment

Another aspect of the Rollard meta-analysis relates to the ways teachers assess. The argument is that if students feel they are being assessed competitively all the time they will be less willing to take the risks that are associated with learning and challenge. One group of participating teachers had devised a descriptive rubric for assessment against criteria, a key aspect of which was them describing their thinking. For example, the statement "The steps I took are shown, and I used examples to explain and justify my thinking" was used as a self-evaluation of progress against process aspects such as reasoning. Such statements reduce the sense of comparisons between students and emphasise evaluation against criteria. In commenting on this, one of the teachers reported:

> Well, I think they've gotten better through the year. Because we've given them a couple of investigation tasks where they've been marked for writing down their thinking and showing their steps, …. one of the students had taken the initiative to write down what she's stuck on. She's now, off her own bat, writing down the problem she's facing half way through the problem. She's like, "Where did the 25 people go?" So she's not just writing down the work – [another student is] not as advanced. She is writing down what she knows and writing the questions that are stumping her… [this is] like the fourth or fifth time this year I've asked them to show them their thinking they're now – they know how to write down their thinking or try things.

This indicates that the use of a rubric based holistic assessment had the effect of influencing the ways that students engaged with a task and encouraged them to reflect on their learning. One of the teachers also commented on student self assessment:

> I also made them mark it ..., I said tick what you think you'll get and then hand it in.

It seems that the mode of assessment had an impact on the ways students engaged with the task and communicated their thinking, and is part of the classroom culture overall.

Conclusion

The overall project of which these results are a part is exploring whether engaging in tasks that are challenging facilitates the learning of mathematics. The review by Rollard suggested that classroom culture is an important determinant of the ways that students learn and that classroom culture is influenced by the actions teachers take in supporting students. In terms of the active support for students, it appears that these project teachers:

- were willing to pose tasks that are substantially more challenging than conventional mathematics tasks in textbooks;
- considered appropriate preliminary experiences that allow access to the task without detracting from the challenge of the task;
- differentiated the experience by using enabling and extending prompts;
- incorporated student sharing of their strategies into the class reviews; and
- used holistic and non competitive assessment as part of the tasks.

We also noted that the timing of challenging tasks within sequences is important, as is allowing sufficient time for students to engage with the tasks and having a considered approach to the various pedagogical issues that will arise in such lessons. It also seems that students may be willing to engage if the support is available. As one of the teachers reported: "Yeah, my students enjoyed that one. Well, at the end of the first one they said, 'Can we do another one of these?'" Overall, these data suggest that a positive classroom culture is not a matter of rules and procedures but the ongoing and interactive support teachers provide that encourages students to take up the challenge of tasks.

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