How Pre-Service Teachers Integrate Knowledge of Students' Difficulties in Understanding the Concept of the Arithmetic Mean into Their Pedagogy

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This research study investigates how pre-service teachers integrate statistical content, students' thinking, and pedagogy as they examine how 11- to 12-year olds develop mathematically. The findings provide insights into: a) how pre-service teachers identify some of the difficulties that students commonly have, and b) what pedagogical approaches pre-service teachers use to address students' difficulties and enhance students' learning. These findings have implications for the design and delivery of professional development that improves teachers' knowledge, understanding, and skills in teaching statistics.

Research literature indicates an array of different domains of knowledge that teachers should know and be able to teach effectively. This includes understanding aspects of pedagogical content knowledge, including expert language and contexts for learning; as well as understanding the thinking of students, including their common difficulties and misconceptions, and how to address those difficulties/misconceptions in order to enhance students' learning. Teachers also need the skills to construct and manage classroom activities efficiently, communicate well, use technology, and reflect on their practice to learn from and improve it continually.

The importance of teaching basic statistics concepts from primary school, coupled with a changing focus in statistics pedagogy (Garfield et al., 2012), has necessitated the importance of preparing pre-service teachers to teach statistics. Elementary teachers generally have little knowledge about the content of statistics and almost never any training in statistics education (Batanero et al., 2011). Estrada and Batanero (2008) argued that teachers have negative attitudes towards statistics that are "linked to perceived difficulty, lack of knowledge and overly formal experience" (p. 5). If teachers see mathematics in a formalistic view, they may encounter problems understanding statistics. Pierce and Chick (2011) argued that, "primary teachers may not consider of themselves as teaching statistics but rather applied number work" (p. 155). Begg and Edwards's (1999) research showed that the majority of their pre-service primary teachers believed that statistics was part of mathematics; their pre-service primary teachers also believed that a good understanding of mathematics was not prerequisite to comprehend fundamental core concepts in Statistics. On the contrary, Chick and Pierce (2008) found that almost half of their pre-service primary teachers believed that one must be good at mathematics in order to understand basic statistical concepts. Primary pre-service teachers appreciate the cross-curricular nature of statistics, but they typically view teaching statistics as part of teaching mathematics (Begg & Edwards, 1999).

Background

Student-centred learning offers a pedagogical approach for mathematics education in the 21st century where the educational paradigm shifts from traditional, teacher and textbook-centred approaches, to situations where the learner is personally challenged and engaged in a social construction of knowledge. There are many ways of organizing the

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knowledge that pre-service teachers need. The core concepts and skills that should be represented as a common curriculum for teachers' education could be organised on three areas of knowledge found in many statements of standards for teaching: 1) knowledge of learners and how they learn and develop within social contexts, including knowledge of language development; 2) understanding of curriculum content and goals, including the subject matter and skills to be taught in light of disciplinary demands, student needs, and the social purposes of education; and 3) understanding of and skills for teaching diverse learners, which are informed by an understanding of assessment and of how to construct and manage a productive classroom (Darling-Hammond, 2006). These three areas of knowledge define a framework for Teaching and Learning, represented visually in Figure 1. Figure 1 depicts effective teaching

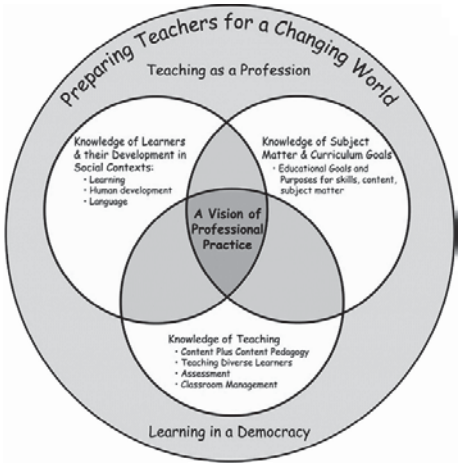

Figure 1. A Framework for Understanding Teaching and Learning (Darling-Hammond & Branford, 2005, p. 11).

as the intersection between learners, content, and teaching. The intersections between these three areas constitute the inherent complexity in good teaching. The implications of this framework for teachers' education are several. Most importantly, it creates the expectation that teachers will be able to understand how students learn and what various students need in order to learn more effectively. Deep understanding of students' learning has not historically formed the foundation of teachers' education; it was usually reserved for curriculum developers who used the knowledge to design texts for curriculum. Teachers only were trained to learn teaching strategies to implement curriculum. Nonetheless, without mastering deep knowledge about how their students learn and how students of different ability learn, teachers lack a foundation that can assist them work out what to do when a particular teaching technique is not effective with all students.

Similarly Lappan and Theule-Lubienski (1992) describe three domains of knowledge that teachers must have in order to teach effectively: 1) mathematics content, 2) pedagogy of mathematics, and 3) students' learning. They argue that teachers' work exists within the union of these three knowledge domains and that these domains should not be taught in isolation from each other, but should be integrated, instead, because any lack of integration between these three domains of knowledge can provide fragmented knowledge of the different aspects of teacher education, leaving pre-service teachers without the appropriate experiences and skills needed to reason and analyse their teaching and students.

Constructing an effective curriculum for students requires incorporating subject matter goals, knowledge of learning, and an understanding of children's development. Connecting the content to be learnt to the learners themselves necessitates curriculum work, even when teachers have access to a range of texts and materials. Furthermore, the work of teaching is viewed as a continuous attempt of teachers to address the problems of practice they encounter and to meet the emerging learning needs of all of their students (Darling-Hammond, & Bransford, 2005). This means that teachers need to develop the disposition to continue try to find answers to difficult problems of teaching and learning and to attain skills to learn not only for practice but to learn from practice as well.

These expectations for teacher knowledge suggest that University programs should not only provide pre-service teachers (and teachers) access to more knowledge, but also to support them learn to incorporate knowledge into their practice and to be able to inquire into their classrooms. Classroom inquiry requires careful observation and reasoned analysis, as well as a sense of commitment and responsibility to students' learning (Shulman, 1996).

Several studies have recognized the complexity of the task for effective teacher education (Cooney, 1994). Darling-Hammond (2006) discusses common features of programs of teacher preparation that confront this "problem of complexity". One critically important feature is the importance of helping teachers-in-training to integrate the theoretical knowledge taught in University with the experience-based knowledge derived from their teaching practice. It is not merely the availability of classroom experience that enables teachers to apply concepts addressed in their academic work; studies of teacher learning suggest that engaging teachers in the materials of practice and working on specific concepts using these materials is very effective for teaching the teachers (Hammerness, et al., 2002). These researchers have argued that analysing samples of student work, teachers' plans and assignments, vignettes of students and teachers in action, and cases of teaching and learning can help teachers connect generalised principles to particular instances of teaching and learning.

Successful education programs help pre-service teachers bridge the gap between theory and practice, in particular when immersing pre-service teachers in purposeful activities such as developing case studies on s tudents, on aspects of schools and teaching by observing and examining students' work, and analysing data when collected. Such cases are powerful tools for professional learning (Shulman, 1996). Other tools−such as "portfolios, teachers' classroom inquiries and research, and analyses of specific classrooms, teachers, or teaching situations when teachers educators provide … guidance, and feedback" (Darling-Hammond, 2006, p. 308) − can be used to connect profession-wide knowledge to particular contexts. The research literature shows that many teachers unconsciously share a variety of difficulties and misconceptions with their students with respect to basic concepts in statistical concepts (Begg & Edwards, 1999). Teachers' difficulties include: having little understanding of the concepts of measures of central tendency, having difficulties in creating or interpreting graphs; using only verbal reasoning to describe variation; and comparing distributions in terms of averages (Batanero et al., 2011). The pedagogical content knowledge required for teaching statistics is often weak (Batanero et al., 2011). For example, in Pierce and Chick's (2011) research, some teachers did not recognize the statistical concepts that could be developed from a specific task and missed opportunities that were inherent in the task. Clearly, teaching statistics does not only need different pedagogical approaches, tasks, and methods, but also requires a profound knowledge of specific student learning difficulties and student cognitive

development in statistics (Batanero et al., 2011). In spite of this need, some researchers suggest that teachers better understand how students learn particular statistical concepts after teaching those concepts (Ponte, 2011).

The intention of this research study is to investigate how pre-service teachers integrate the three domains of knowledge − content in statistics at stage 3, child development, and pedagogy−and to explicate how the three domains of knowledge interact in order to build a model that enables high-quality mathematics teaching.

This paper reports on the findings of a pilot study that analyses pre-service teachers assignments in order to investigate the interconnections that they draw between the key ideas of the statistical content at stage 3, child development, and their own thinking and pedagogy.

The following research questions were used to guide the pilot study and to provide insights about establishing more productive environments that could positively influence our pre-service teachers to learn and teach statistics at stage 3:

- What is the pre-service teachers' understanding of the ways stage 3 students understand statistics?
- What pedagogical approaches do pre-service teachers use in order to address students' difficulties?

The pilot study of analyses of students' assignments is described in the following methodology.

Methodology

The study was undertaken at the end of trimester two, with 176 (158 female, 18 male) third-year primary pre-service teachers, undertaking the Bachelor of Education course at a rural university in NSW. The participants are all continuing students of various ages. The sample was not controlled for age, ability, gender, or cultural background. A general indication of participant ability can be inferred from the expectation set for attending this university course.

The course was designed to cover three topic strands of the Australian mathematics curriculum (ACARA, 2010) including patterns and algebra, number, statistics and probability. Pre-service teachers attended 5 one-hour lectures on statistics and 5 two-hour workshops. The lecture and the workshop were both taught by the author. At the end of trimester two the students were asked to submit an assignment.

The pre-service teachers' work was electronically submitted as part of comprehensive assignment. The assignments included their answers to the questions of assignment 2 (see Figure 2). A total of 176 assignments were submitted. The data included excerpts from the pre-service teachers' responses to the questions, as well as justifications and their reflections on the assignment questions. At the first stage, one of my colleagues who was not involved in the teaching of this University course, informed the pre-service teachers about the research study. Pre-service teachers who wished to participate in the research study emailed their acceptance letter to my colleague who informed me about the pre-service teachers' decisions after the marks of the course were officially released. At that stage I had read the assignments of the students who agreed to participate in the research study, and then wrote extensive field notes during and immediately after reading each assignment.

Assignment 2 – 2000 word Written Task

You will be writing a paper to present at an Australian Teacher Education Annual Association Conference. The paper will be on statistics at stage 3.

1. **Brief overview**
   - What do students at stage 3 need to understand about statistics?
2. **Choose one of the key ideas below to investigate at depth:**
   - Determine the mean (average) for a set of data.
   - Draw a column line and bar graphs using scales of many-to-one correspondence.
   - Read and interpret graphs with scales of many-to-one correspondence.
3. **What difficult(y/ies) may students have with your chosen key idea?**
   - Support your ideas with one or two articles that have been provided to you.
4. **How would you address this difficulty to the kind of thinking that stage 3 students need to do?**
   - Support your ideas from the readings provided to you.

Figure 2. Assignment 2.

I chose twenty focus assignments of students who chose to investigate in depth the idea of determining the mean (average) for a set of data. Those assignments were chosen because those were the most articulate assignments. The ideas expressed by the examples included here were expressed by most of the students, but not as clearly, which makes these examples preferable for the discussion. These pre-service teachers were able to devise their own vocabulary and articulate better their knowledge in a language or other symbolic form in their responses of the assignment questions. By drawing on the notes, the twenty assignments were transformed into plain case accounts for each pre-service student.

The next phase of data analysis used these plain case accounts to develop interpretative analyses. The case analyses became the main focus for subsequent analysis and triggered further phases of progressive focusing (Robson, 1993) to identify key foci for ensuing study. Important similarities and differences between the interpretative case analyses were then identified by constant comparisons of these 20 interpretative case analyses.

Results

This paper concentrates only on analysing assignments of pre-service teachers who chose to investigate the following key idea: "Determine the mean (average) for a set of data." The pre-service teachers elaborated on the content knowledge that students at stage 3 need to learn so they can determine the mean for a set of data. These pre-service teachers also read several relevant pieces of scholarly research on student difficulties with the concept of the mean. The pre-service teachers mentioned the following student misconceptions and difficulties (in the following, "PT" stands for Pre-service Teacher's assignment, and the numbers correspond to individuals):

> PT 5: One misconception common to both students and teachers and circulated by professional learning literature and curriculum documents (Board of Studies NSW, 2002) alike is the language that surrounds mean. In these documents the words mean and average are used interchangeably.…Many other common misconceptions about mean are born of poor conceptual understanding stemming from an algorithm-centric schooling system. When students are deprived of the opportunity to investigate the conceptual knowledge of mean, instead simply memorising an algorithm, the result is shallow understanding of what the mean actually represents.

PT 5 outlined two important difficulties that students commonly have: 1) the language can cause confusion to students, 2) when average is seen as algorithmic procedure, students rely on the algorithm for finding the mean of a set of numbers, as a result students who predominantly use algorithmic procedures are confined only to the detail of procedures, not to the actual meaning of "mean". PT 2 wrote:

PT 2: Students often have difficulty grasping the concept behind averages and the 'mean of data' and can confuse this with the median as they are both a point of balance and fall in the middle area of the data set. These common misconceptions of students have developed historically through early processes of estimation and therefore students are often seeing the average as being the exact middle number (correctly referred to as the median), whilst the average can usually be close to or even the same as this, it is formulated to show the calculated average value from the set of data and not merely the represented middle number shown in the set.

PT 2 referred to students' confusion of the concepts of mean and median. PT 2 mentioned that when students are given a set of data they usually believe that the mean is the midpoint of the data set and the data are arranged in a symmetrical manner so there is an equal number of data above and below the mean. This symmetrical arrangement of data gives an average that is a mean and a median at the same time.

In contrast to PT2, who spoke of the confusion between mean and median, PT 1 and PT 7 both mentioned the confusion between mean and mode, especially the confusion caused by the terminology of "mode" and average:

PT 1: Students are known to confuse concepts of mode and mean; as 'mode' can be seen as a way of representing a majority of information, and occurs most frequently. Mode a concept commonly likened to the term average in many areas of life, students have these misconceptions which cause difficulty for students in understanding and determining the differences between the concepts.

PT 7: Many students give definitions such as normal, most common or the most frequent value when questioned about the meaning of average. It is therefore no surprise that this preconception often permeates their understanding of mean especially as the two words "mode" and "mean" are often used interchangeably and there is little opportunity for students to investigate the underlying concepts.

The pre-service teachers also suggested techniques that they would use to address these difficulties and misconceptions, mentioning the importance of using language clearly and also of developing conceptual understanding:

PT 2: When students begin interpreting data, ensure all students understand: a) the difference between a 'middle number' (median) and an 'average number' (mean) what 'most commonly occurring number' or 'mode' means, b) how to read data in order to identify which central concepts to use, and c) determine data ranges between mode, median and mean.

PT 5: Teachers must be very careful of the language they use when referring to mean in classroom lessons and when engaging students in dialogue about averages....This can go a long way towards helping children separate the two concepts and prevent median and modal interpretations of mean.

PT 1: By providing the time and materials for students to experiment with mean, teachers can give students the opportunity to create solid conceptual understandings. This conceptual knowledge can then be linked to procedural knowledge. It is best to assist students to discover the procedural knowledge for themselves when their conceptual knowledge is sufficiently concrete. This kind of knowledge construction can ensure that students know what the mean represents rather than simply knowing how to calculate it.

The pre-service teachers who participated in this study were engaged in the reading of the existing research literature and reflecting on students' psychological processes. The participants reflected on students' misconceptions and learning difficulties and came up with strategies designed to teach statistics content to stage 3 students. The suggested strategies addressed students' difficulties and seemed to be closely aligned to the general features of student-centered learning, such as collaboration and working on authentic tasks that are relevant to the students. The pre-service teachers generally appeared to acknowledge that the actual difficulties that stage 3 students commonly have hinder students' conceptual understanding and the pre-service teachers attempted to address students' difficulties and help stage 3 students to overcome their conceptual struggles. The

participants engaged in exploring strategies and ideas to provide students with opportunities to work on real-life tasks that would help students' build solid conceptual understandings of the concept of the mean.

Discussion and Conclusion

The results from this pilot study revealed a number of aspects of learning to teach statistical concepts in learner-centred ways that require pre-service teachers to be able to understand how students think and to be informed about possible difficulties students may encounter or common misconception students may hold. This information could be obtained by extensive and intensely supervised clinical work tightly integrated with course work, but in this assignment, it was not feasible to have pre-service teachers conduct a clinical research study and develop case studies on students' understanding of key statistical concepts to investigate students' difficulties in understanding the concept of mean (average). The assignments submitted by the pre-service teachers showed that they had a good knowledge of the subject matter content knowledge required for teaching the concept of mean to stage 3 students and identified some of the difficulties that students commonly have. The analysis of these assignments indicates that most of the participants in this research study were able to provide: (1) a repertoire of teaching strategies for use in the classroom and (2) conceptual and practical tools in light of students' conceptual understandings of determining the mean (average) for a set of data.

There was evidence that the participants were able to integrate the subject content, 11- to 12-year olds' thinking by identifying some of the difficulties that students commonly have and pedagogical approaches used to address students' difficulties in order to enhance students' learning. Pre-service teachers' engagement in reading the recommended research literature about the psychological issues involved with students' understanding of those concepts, helped pre-service teachers to gain insights in primary students' way of thinking and adopt teaching techniques in order to better support students to understand what they need to understand. Notwithstanding, no amount of course work can, by itself, counteract the powerful experiential lessons in teaching Statistics at stage 3. Therefore it is impractical to prepare pre-service teachers to teach statistics without exposing them to the misconceptions that students commonly have in understanding basic statistical concepts.

This kind of pre-service teachers' exposure to "what" hinders students from achieving an understanding of statistics at the primary school should exemplify the shift from disciplines with an all pervading causal interpretation, such as mathematics, to one that is inherently causal, such as statistics. This major fundamental shift in this viewpoint is the major stumbling block to students' understanding of statistics, one that cannot merely be dismissed as an alternative "explanation" when teaching statistics.

In this pilot study, there was evidence that the participants were able to integrate the subject content, 11- to 12-year olds' thinking by identifying some of the difficulties that students commonly have and pedagogical approaches used to address students' difficulties in order to enhance students' learning.

This pilot study was used to provide a starting point for developing new research related to improving the teaching of statistics at the school level and the preparation of teachers to deliver that teaching. A larger main study, including approximately 300 online students as well as a cohort of 55 on-campus students, is scheduled for Trimesters 2 and 3, 2013. This larger sample will provide greater insight into how primary pre-service teachers construct their pedagogical understandings and approaches while integrating content knowledge, and knowledge of their students' specific learning difficulties and cognitive

development. It would be useful to develop models that can be applied in designing procedures or materials directed to educating teachers.

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