Foundation Content Knowledge: What Do Pre-Service Teachers Need To Know?

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The mathematics content knowledge of pre-service teachers is a growing area of inquiry. This topic requires further theoretical development due to the limited applicability of current cognitive and practice-oriented frameworks of mathematics content knowledge to beginning pre-service teachers. Foundation content knowledge is an integrated, growth-oriented concept of mathematics content knowledge specifically for beginning pre-service primary teachers. While we acknowledge that our proposal is preliminary and incomplete we also maintain that it addresses a number of important issues faced by pre-service teachers, their initial teacher education providers, and the mathematics education community.

Following international interest in assessing the mathematics content knowledge of qualified teachers (Stephens, 2003), the assessment of mathematics content knowledge of pre-service teachers has emerged as a focal area for inquiry (Norton, 2012; Ryan & McCrae, 2005/6; Senk et al., 2012; Walshaw, 2012). Researchers have established that teachers' mathematics content knowledge is related to student achievement and that this knowledge is multi-dimensional and contextually situated (Bobis, Higgins, Cavanagh, & Roche, 2012; Fennema & Franke, 1992; Hill, Ball, & Schilling, 2008; Rowland & Ruthven, 2011; Senk et al., 2012). Initial teacher education providers in New Zealand are required to ensure that primary pre-service teachers meet the professional standards of a practising teacher (New Zealand Teachers Council, 2010), however, the knowledge base underpinning these professional standards has not been clearly defined. Adding to this a lack of clarity has been the theorising of pre-service teachers mathematics content knowledge from, predominantly, a deficit perspective (Ryan & Williams, 2011). An integrated, growth-oriented concept of mathematics content knowledge is needed for primary pre-service teachers at the start of their initial teacher education programmes rather than a practising teacher-based or misconception-laden model. The concept of foundation content knowledge encompasses a set of at least eight mathematical features that primary pre-service teachers must have at the start of their initial teacher education programmes. Instead of an absence of mathematical knowledge, the concept of foundation content knowledge provides a base from which primary pre-service teachers' prior mathematical knowledge can be built into more expert forms during pedagogical courses, practicum experiences, and beyond. We begin by examining current theories of knowledge for teaching mathematics and consider how features of those theories have shaped the way mathematics content knowledge of pre-service teachers has been conceptualised.

Theoretical Perspectives on Mathematics Content Knowledge

Cognitive Conceptions of Mathematics Content Knowledge

From a cognitive perspective, mathematics content knowledge must be more than a set of memorised laws and well-defined problem solving procedures. Skemp's (1976) early work, which described relational understanding as knowing both what to do and why, and

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instrumental understanding as rules without reasons, has been built-on in later cognitive models (Hiebert & Carpenter, 1992; Ma, 1999; Mason, Stephens, & Watson, 2009). From a cognitive perspective, conceptual knowledge of mathematics is viewed as a web-like representation where prominent pieces of information are interconnected, whereas procedural knowledge is viewed as algorithms for carrying out mathematical tasks and knowledge of our mathematical symbol system (Hiebert & Carpenter, 1992). Procedural knowledge is viewed as requiring minimal cognitive effort because it has limited connections between successive actions, making them efficient to execute. Conceptual knowledge, on the other hand, is required for deciding what procedures to use. Conceptual and procedural knowledge are not alternatives, but intertwined elements of mathematical knowledge which are required for solving problems effectively. Mason, et al. (2009) use the phrase, appreciation of mathematical structure, to indicate how relationships between mathematical concepts can be used for determining the appropriateness of a procedure to solve a particular problem. The importance of an appreciation of mathematical structure is also echoed in Ma's (1999) construct, a profound understanding of fundamental mathematics, which she describes as more than just a sound conceptual understanding. Her concept involves appreciating multiple solution methods, basic principles of mathematics, breadth of understanding through connecting topics to other topics of similar conceptual power, depth of understanding through connecting topics to those of greater conceptual power, and longitudinal coherence of those understandings. Ma found that many U.S. teachers viewed mathematics as a collection of facts, rules, and procedures, whereas Chinese teachers were concerned with knowing why procedures made sense as well as knowing how to carry them out. Ma noted that the Chinese teachers developed this profound understanding of fundamental mathematics during their teaching careers and did not demonstrate it as pre-service teachers, even though they displayed sound conceptual knowledge at that time. Assisting pre-service teachers to be prepared to develop their content knowledge as they become effective and purposeful mathematics teachers requires teacher education providers to consider the practical dimensions of these complex and dynamic forms of knowledge.

Practice-based Conceptions of Mathematics Content Knowledge

A central issue concerning mathematics content knowledge are the links between knowledge and teaching. Shulman's (1986) seminal work suggests that teachers use six facets of knowledge for teaching: content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners, knowledge of educational contexts, and knowledge of educational purposes and values. Rather than considering general pedagogical knowledge and content knowledge separately, he proposes the concept of pedagogical content knowledge, as "the ways of representing and formulating the subject that make it comprehensible to others" and "an understanding of what makes the learning of specific topics easy or difficult" (Shulman, 1986, p. 9).

In mathematics education the boundary between pedagogical content knowledge and content knowledge has been a focus of research. Practice-oriented researchers have noted that teachers use different mathematical representations, analogies, illustrations, and examples in a variety of ways, and that mathematics content knowledge becomes activated as a large, integrated, functioning system that is personal and situated (Fennema & Franke, 1992). The situated nature of knowledge means that the type of mathematical knowledge that a pre-service teacher developed as a school student is likely to be used in a similar form and manner when teaching. The knowledge of mathematics content and pedagogy that pre-service teachers bring with them into their initial teacher education programmes is likely to feature a limited base of school mathematics content and pedagogy, characterised by memorising laws and solving well-defined problems (Fennema & Franke, 1992).

One way that the boundary between pedagogical content knowledge and content knowledge had been clarified is by practice-oriented researchers defining knowledge domains. In Hill et al.'s (2008) model, subject matter knowledge has been subdivided into common content knowledge, specialised content knowledge and knowledge at the mathematical horizon. Common content knowledge is "knowledge that is used in the work of teaching in ways in common with how it is used in many other professions or occupations that also use mathematics" (p. 377). Specialised content knowledge is "the mathematical knowledge that allows teachers to engage in particular teaching tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods" (pp. 377-378). Knowledge at the mathematical horizon involves knowing where mathematical concepts will lead (Ball, 1993). Knowledge at the mathematical horizon, specialised content knowledge, and common content knowledge are defined as constructs independent to knowledge domains related to students or teaching. Even though specialised content knowledge and knowledge at the mathematical horizon are distinct from pedagogical content knowledge, however, they are defined as developing during a teacher's career rather than necessarily being present in pre-service teachers at the start of their university programmes. Common content knowledge, in contrast, is a concept that may be more appropriate to apply to pre-service teachers at the beginning of their initial teacher education programmes. We question, however, whether common content knowledge is appropriate and sufficient to connote the knowledge that primary pre-service teachers need to build upon t o develop into effective and purposeful teachers of mathematics.

Foundation Content Knowledge

Given the multi-faceted and dynamic conception of mathematics content knowledge that has emerged from cognitive approaches and the increasingly defined domains of teacher knowledge from practice-based perspectives, we suggest that pre-service teachers must bring with them a solid mathematical base upon they can build their knowledge and practice as mathematics teachers. We suggest foundation content knowledge may be a more useful concept than common content knowledge when considering the mathematical content knowledge of primary pre-service teachers at the start of their initial teacher education programmes. We propose that foundation content knowledge consists of at least eight features. Each of the features of foundation content knowledge includes intertwined and inseparable elements of conceptual and procedural knowledge. These features are illustrated with knowledge appropriate for primary pre-service teachers.

Features of Foundation Content Knowledge

Modelling. Pre-service teachers should be able to use mathematics to model real world problems. Foundation content knowledge is not merely procedural knowledge, but must be integrated with conceptual knowledge. Integrated knowledge of this kind is needed for solving non-routine problems and will therefore be observed in modelling.

Doing and undoing. Pre-service teachers should be able to make mathematically appropriate choices between evaluating expressions directly and using inverse operations to create alternative statements. They should be able to use multiplicative and proportional reasoning to solve problems on ratios and rates. They should be able to operate additively and multiplicatively on and with fractions. They should also be able to form and solve linear equations.

Reasoning and proving. Pre-service teachers should be able to derive answers to additive and multiplicative problems by using logical arguments about whole numbers. They should also be able to use relationships between geometric figures, and deduce and use angle properties of intersecting and parallel lines.

Using multiple representations. Pre-service teachers should be able to use tables, graphs, and equations to solve problems involving number and spatial patterns. They should be able to choose appropriately between fractions, decimals, and percentages to solve problems and also convert between standard metric units of length, area, volume, capacity, and mass.

Generalising. Pre-service teachers should be able to use the associative, commutative, and distributive properties of number, and write equivalent expressions to solve problems. They should be able to use the place value structure of decimals to multiply and divide by powers of ten. They should also be able to derive and use formulae for perimeters, areas, and volumes of simple shapes.

Working with real numbers. Pre-service teachers should be able to use the idea that there are an infinite number of real numbers between any two given numbers in order to solve problems involving decimals.

Basic facts. Pre-service teachers should have instant recall of the basic facts of addition, subtraction, multiplication, and division in order to use the other features of foundation content knowledge effectively and purposefully.

A growth-oriented disposition. Pre-service teachers should be able to view their mathematical content knowledge as a foundation rather than a pre-requisite. They should be open to transforming and interacting with their current mathematical knowledge so they can act with agency to develop and extend this knowledge.

Foundation content knowledge reflects the conceptual integration of findings from the central elements of learning frameworks, contemporary international studies, and the current formulation of school curricula. The growth-oriented dispositional feature of foundation content knowledge is included to challenge deficit conceptions of learners that underpin traditional modes of mathematics assessment and theoretical frameworks. Each of these features will be discussed in turn.

An Integrated Concept with a Growth Orientation

Contemporary mathematics learning theories. Foundation content knowledge reflects the central elements of learning frameworks and essential understandings present in contemporary mathematics education research. The central elements of learning frameworks, such as the Number Framework (Ministry of Education, 2003) and the van Hiele framework for geometric reasoning (van Hiele, 1986), provide specific details of how mathematical ideas are developed and interconnect. The Number Framework provides specific indicators about the type of number thinking that beginning primary pre-service teachers should hold. The Number Framework is closely aligned to the New Zealand Curriculum and nearly 20% of children attain the highest level of advanced proportional thinking before reaching secondary school (Young-Loveridge, 2010). Advanced proportional thinking allows a person to choose broad range of mental strategies to estimate answers and solve problems involving fractions, decimals, proportions, and ratios.

Mental strategies include partitioning fractions and relating the parts to one, converting decimals to fractions and vice-versa, and using number properties to form equivalent statements. Similarly, the van Hiele framework for geometric reasoning provides specific indicators about the type of geometric reasoning that beginning primary pre-service teachers should be able to demonstrate. The levels, consecutively, are visual recognition, descriptive/analytic, informal deduction, formal deduction, and rigour. It is generally recognised that few children at primary school advance past the descriptive/analytic stage, in which shapes are recognised by checking their properties (Clements & Battista, 1992). Pre-service teachers should demonstrate knowledge at the informal deduction level because the distinction between the descriptive/analytic and informal deduction levels of van Hiele's framework is particularly important. While a person at the descriptive/analytic level would be expected to recognise that a triangle was isosceles by ascertaining that two of the three sides were equal, at the informal deduction level, the expectation would be that the person would deduce that the base angles of an isosceles triangle must be equal because of the symmetry of the sides. The central elements of learning frameworks have been an example of one type of evidence considered as we identified the specific features of foundation content knowledge.

Foundation content knowledge also reflects another type of evidence, a focus on big ideas and essential understandings present in contemporary mathematics education research. Pedagogical and mathematics content summaries about the fundamental ideas in the teaching and learning of mathematics have been generated by a research project that focuses on awareness of big ideas in mathematics classrooms (Kuntze et al., 2011). The big ideas include doing and undoing, proving, using multiple representations, modelling, dealing with infinity, generalising, and dealing with uncertainty and variation. These big ideas were also used as a means to identify the foundation content knowledge in school curricula.

Schooling Systems and their Curricula. Foundation content knowledge reflects the important mathematical ideas present in contemporary school curricula. In New Zealand, the graduating teaching standards require primary pre-service teachers to have mathematics content knowledge to at least Level 4 of the New Zealand Curriculum (Ministry of Education, 2007). The mathematics topics at Level 4 in New Zealand are similar to the Year 7 content descriptions of the Australian Curriculum (Australian Curriculum Assessment and Reporting Authority, 2012). The New Zealand Curriculum prefaces each mathematics achievement objective with the phrase, "In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations that require them to..." (Ministry of Education, 2007, p. 51). The achievement objectives also place emphasis on understanding, generalisation, and using a range of representations. While the school curricula provides details of knowledge in number, algebra, measurement, geometry, statistics, and probability, it does not necessarily differentiate the aspects of content that are important as a foundation for pre-service teachers from those that could be easily learnt prior to teaching children. For example, identifying rectangles by their geometric properties is a foundational aspect of the New Zealand Curriculum, whereas knowing the names of all the types of quadrilaterals is not. In this way, foundation content knowledge not only reflects the appropriate mathematical requirements of school curriculum but it also serves as a guide to determine what mathematical ideas are important for beginning primary pre-service teachers to hold.

One area omitted from the concept of foundation content knowledge has been the topic of statistics. We have specified several key features of foundation content knowledge for mathematics only and not for statistics. Primary teachers in New Zealand are expected to teach mathematics and statistics. Further work will be needed in the area of statistics to expand the definition of foundation content knowledge to include statistical knowledge for beginning pre-service teachers.

Moving Beyond Deficit Models. The growth-oriented dispositional feature of foundation content knowledge is proposed to challenge deficit conceptions of learners that underpin traditional modes of mathematics assessment and theoretical frameworks. The view that mathematics content knowledge is a foundation rather than a pre-requisite has not been a dominant paradigm in the research literature (Ryan & Williams, 2011). The mathematics content knowledge of pre-service primary teachers has been described as a lack of proficiency rather than as a starting point for development (Anthony, Beswick, & Ell, 2012). A number of studies have investigated the mathematics content knowledge of pre-service teachers (Linsell & Anakin, in press; Ryan & McCrae, 2005/6; Senk et al., 2012; Tobias & Itter, 2007). These studies have used a variety of assessment tools so a direct comparison their results is not possible, but their findings are in broad agreement. In each study, less than half of the participating pre-service teachers demonstrated the aspects of foundation content knowledge when assessed in the areas of fractions, place value, and measurement. For over half the participating pre-service teachers, their content knowledge was characterised in terms of gaps, errors, misconceptions, and reliance on pr ocedural rather conceptual knowledge. Unfortunately, there is an absence of conceptual tools with a growth orientation for researchers to investigate and describe the content knowledge of beginning pre-service teachers. A focus on primary pre-service teachers' mathematical knowledge as a starting point, instead of an absence of knowledge, is an important conceptual re-orientation. When knowledge is described in participative terms, then pre-service teachers can act with agency to develop and extend their current knowledge. For example, pre-service teachers can be involved with the interpretation of diagnostic assessment of their own mathematics content knowledge (Ryan & Williams, 2007). The work of Ryan and her colleagues (Ryan & McCrae, 2005/6; Ryan & Williams, 2007, 2011) has broken ground in the use of diagnostic assessment as a way for pre-service teachers to better understand and develop their own mathematics content knowledge. The growth-oriented feature of foundation content knowledge also provides a conceptual basis reconsidering the theoretical frameworks associated with the learning of mathematics and aligning them with socio-constructivist learning environments (Sfard, 1998). By making use of the growth orientation feature of foundation content knowledge, a promising approach is to encourage pre-service teachers to actively participate in identifying their current knowledge, build upon it, and reflect on their learning.

Conclusion

We have questioned the applicability of current models of mathematics content knowledge pre-service for primary teachers. We have argued for a concept that identifies the knowledge base for primary pre-service teachers at the start of their initial teacher education programmes. Foundation content knowledge is an integrated, growth-oriented concept of mathematics content knowledge. This preliminary formulation encompasses a set of eight mathematical features: modelling, doing and undoing, reasoning and proving, using multiple representations, generalising, working with real numbers, basic facts, and a growth-oriented disposition, that are specifically targeted at beginning primary pre-service teachers.

By situating our inquiry within cognitive and practice-oriented frameworks of mathematics teacher knowledge, we found that existing conceptions of content knowledge were inappropriate for describing the nature of knowledge that beginning pre-service teachers need to build into more expert forms of knowledge over the course of their teaching careers. We drew on contemporary mathematics learning theories, the features of the schooling system and their curricula, and research that challenges the deficit models of traditional mathematics assessment and theorising to find possible ways of moving forwards. Mathematics knowledge for teaching has been defined as a multidimensional and contextually situated phenomenon, whereas, the definition of mathematics content knowledge for pre-service teachers has not developed at the same pace.

Research approaches could be formulated from the concept of foundation content knowledge that involve pre-service teachers as agents that develop and extend their own knowledge. The concept of foundation content knowledge will provide greater clarity than the current graduating teacher standards in New Zealand for initial teacher education providers to implement diagnostic assessment and design appropriate mathematics content courses. Foundation content knowledge also re-orients our perspective towards growth rather than deficit theorising. Foundation content knowledge re-positions pre-service teachers as active participants in learning and encourages them to interact with assessment information in order to better understand and develop their own content knowledge.

References

Anthony, G., Beswick, K., & Ell, F. (2012). The professional education and development of practising teachers of mathematics. In B. Perry, T. Lowrie, T. Logan, A. MacDonald & J. Greenlees (Eds.), Research in mathematics education in Australasia 2008-2011 (pp. 291-312). Rotterdam, Netherlands: Sense.

Australian Curriculum Assessment and Reporting Authority. (2012). The Australian Curriculum. Retrieved from http://www.australiancurriculum.edu.au/Mathematics/Rationale

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. The Elementary School Journal, 93(4), 373-397.

Bobis, J., Higgins, J., Cavanagh, M., & Roche, A. (2012). Professional knowledge of practising teachers of mathematics. In B. Perry, T. Lowrie, T. Logan, A. MacDonald & J. Greenlees (Eds.), Research in mathematics education in Australasia 2008-2011 (pp. 313-340). Rotterdam, Netherlands: Sense.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning (pp. 420-464). New York, NY: MacMillan.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning (pp. 147-164). New York, NY: MacMillan.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning (pp. 65-97). New York: MacMillan.

Hill, H., Ball, D., & Schilling, S. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. Journal for Research in Mathematics Education, 39(4), 372-400.
Kuntze, S., Lerman, S., Murphy, B., Kurz-Milcke, E., Siller, H.-S., & Winbourne, P. (2011). Development of pre-service teachers' knowledge related to big ideas in mathematics. In B. Ubuz (Ed.), Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education (Vol. 3, pp. 105-112). Ankara, Turkey: PME.
Linsell, C., & Anakin, M. (in press). Diagnostic assessment of pre-service teachers' mathematical content knowledge. Mathematics Teacher Education and Development, 14(2).
Ma, L. (1999). Knowing and teaching elementary mathematics. Mahwah, NJ: Lawrence Erlbaum Associates.
Mason, J., Stephens, M., & Watson, A. (2009). Appreciating mathematical structure for all. Mathematics Education Research Journal, 21(2), 10-32.
Ministry of Education. (2003). The Number Framework. Wellington, New Zealand: Ministry of Education.
Ministry of Education. (2007). The New Zealand Curriculum. Wellington, New Zealand: Learning Media.
New Zealand Teachers Council. (2010). Approval, Review and Monitoring Processes and Requirements for Initial Teacher Education Programmes. Retrieved from http://www.teacherscouncil.govt.nz/te/itefinal.stm
Norton, S. (2012). Prior study of mathematics as a predictor of pre-service teacher's success on tests of mathematics and pedagogical content knowledge. Mathematics Teacher Education and Development, 14(1), 2-26.
Rowland, T., & Ruthven. (2011). Introduction: Mathematical knowledge in teaching. In T. Rowland & K. Ruthven (Eds.), Mathematical Knowledge in Teaching (pp. 1-5). Dordrecht, Netherlands: Springer.
Ryan, J., & McCrae, B. (2005/6). Assessing pre-service teachers' mathematics subject knowledge. Mathematics Teacher Education and Development, 7, 72-89.
Ryan, J., & Williams, J. (2007). Mathsmaps for diagnostic assessment with pre-service teachers: Stories of mathematical knowledge. Research in Mathematics Education, 9(1), 95-109.
Ryan, J., & Williams, J. (2011). Teachers' stories of mathematical subject knowledge: Accounting for the unexpected. In T. Rowland & K. Ruthven (Eds.), Mathematical Knowledge in Teaching (pp. 251-271). Dordrecht, Netherlands: Springer.
Senk, S., Tatto, M., Reckase, M., Rowley, G., Peck, R., & Bankov, K. (2012). Knowledge of future primary teachers for teaching mathematics: an international comparative study. ZDM, 44(3), 307-324. doi: 10.1007/s11858-012-0400-7
Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. Educational Researcher, 27(2), 4-13.
Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. Educational Researcher,, 15(2), 4-14.
Skemp, R. (1976). Relational understanding and instrumental understanding. Mathematics Teacher Education and Development, 77, 20-26.
Stephens, M. (2003). Regulating the entry of teachers of mathematics into the profession: Challenges, new models, and glimpses into the future. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, F. K. S. Leung & S. Lamon (Eds.), Second International Handbook Of Mathematics Education (pp. 767-793). Dordrecht, Netherlands: Kluwer.
Tobias, S., & Itter, D. (2007). Mathematical backgrounds of pre-service teachers in rural Australia: A regional comparative study. Paper presented at the AARE.
van Hiele, P. M. (1986). Structure and insight. Orlando, FL: Academic Press.
Walshaw, M. (2012). Teacher knowledge as fundamental to effective teaching practice. Journal of Mathematics Teacher Education, 15(3), 181-185.
Young-Loveridge, J. (2010). A decade of reform in mathematics education: Results from 2009 and earlier years. In Ministry of Education (Ed.), Findings from the New Zealand Numeracy Development Projects 2009 (pp. 15-35). Wellington, New Zealand: Learning Media.