Identification of Hierarchies of Student Learning about Percentages using Rasch Analysis

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A review of the research literature indicated that there were probable orders in which students develop understandings and skills for calculating with percentages. Such calculations might include using models to represent percentages, knowing fraction equivalents, selection of strategies to solve problems and determination of percentage change. To further describe the hierarchies, an assessment instrument was constructed and piloted before being refined and trialled with lower secondary students. Rasch model analysis was applied to the results.

A sound understanding of percentages is essential for students to develop the skills necessary for good adult numeracy. Percentages are used widely for reporting statistics to consumers, for example, wage increases, discounts on goods, interest rates on investments, and positions in sporting tables. For people to be able to make wise decisions about what constitutes a lower interest rate, a healthier product, a better buy, or a larger discount, it is essential that they have a good conceptual understanding of percentages and the skills to identify and use percentages correctly in estimations and calculations.

The purpose of this research was to identify hierarchies of understandings and skills associated with learning percentages, in which a concept higher in the hierarchy is considered more difficult to learn. The findings could help develop teachers' pedagogical content knowledge relating to percentages and inform teachers when planning programs of work. This knowledge is needed given that (a) many teachers of mathematics are teaching outside their area of expertise (McConney & Price, 2009), and (b) percentages is considered a challenging topic to teach (Chick & Baratta, 2011; White & Mitchelmore, 2005).

Literature Review

Teaching Sequence

A sequence for the teaching of percentages has been determined from the curriculum documents that currently inform the teaching of Mathematics in Western Australia. These are (a) the Curriculum Framework (Curriculum Council, 1998); (b) the advisory syllabus documents (Department of Education and Training, Western Australia, 2007); (c) the *Statements of Learning for Mathematics* (Ministerial Council on Education, Employment, Training and Youth Affairs [MCEETYA], 2006); and (d) the Australian Curriculum [K – 10] (ACARA, 2011). While the order, or teaching sequence, is relatively consistent across documents, it is very general and does not necessarily reflect the hierarchy of learning difficulty.

In this sequence, students are initially taught about percentages that have familiar fraction and decimal equivalents (e.g., 50%, 25%). They learn to use the equivalents (usually fractions) to calculate the percentage components of amounts and then determine

In V. Steinle, L. Ball & C. Bardini (Eds.), Mathematics education: Yesterday, today amd tomorrow (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia). Melbourne, VIC: MERGA.

the new amount resulting from the percentage change. They use their knowledge of equivalent fractions to express one quantity as a percentage of another and to determine percentage increase and decrease. As students learn more fractions, decimals and their equivalent percentages, they are able to use a greater variety of percentages for these types of calculations. Then, for percentages which do not have recognizable fraction equivalents, students are taught the conversion methods for changing percentages to fractions and decimals, as well as the algorithms to use for calculating percentage change and the amounts resulting from a percentage change. This is followed by learning about decimal and fractional multipliers for percentage change.

Learning Hierarchies

In learning hierarchies as described by Andrich (2002), students demonstrate increasing levels of competence in their learning of a concept. At each level of achievement, the student acquires new skills and understandings, is competent with the skills and understanding typical of achievement at the level below, and can demonstrate the skills from the lower level in a broader and deeper sense than they could previously.

Rasch modelling was proposed for the analysis of students' responses to the test items because a scale of competence in calculating with percentages can be produced and this allows levels of development to be identified. Rasch analysis places students and items on the same scale, provides ability estimates for each student and orders items by relative difficulty (Callingham & Bond, 2006). This scale of increasing order of item difficulty reflects developments in students' abilities and thus allows a developmental pathway to be identified. Rasch modelling has been used in other studies to identify hierarchies of learning in mathematics, including understandings and skills associated with equivalent fractions (Wong, 2010), linear equations (Linsell, 2009) and proportional reasoning (Watson, Callingham & Donne, 2001).

Some of the hierarchies associated with learning percentages, as suggested in the research literature, relate to understanding the concept of a percentage, knowledge of the fraction equivalents of percentages, use of different models to represent percentages, calculation with different types and sizes of numbers, solving particular types of questions, selection of strategies to perform calculations and determination of percentage change.

Concepts of percentages: Research by Baratta, Price, Stacey, Steinle and Gvozdenko (2010), Gay and Achiele (1997) and Van den Heuvel-Panhuizen (1994) reported student weakness in understanding the meaning of percentages. Students were not confident in knowing that 29 out of 100 is 29%, that 87% of 10 is less than 10 and that the percentages of the components of a mixture were maintained when only the total amount of the mixture was changed. No research into a learning hierarchy for the concept of percentage has been located but it is suggested that student understanding of the meaning of percentages develops as described. Students initially see percentages as whole numbers, then as common fractions because they are mostly taught to think of percentages as fractions (e.g., 50% means a half). As students consolidate their understanding they think of percentages as fractions with 100 as the denominator. With further experience the students see that percentages represent a proportion of a whole and they come to appreciate that the size of the part is totally dependent on the base, or whole amount. Such understanding may be necessary before an understanding of percentages greater than 100% is formed, and this might precede an understanding of the nature of percentages as scale factors or ratios.

Fraction equivalents: Results from studies by White, Wilson, Faragher and Mitchelmore (2007), Callingham and Watson (2004), and Gay and Achiele (1997), as well as data from an extensive review of related research by Parker and Leinhardt (1995), indicate that understandings and skills relative to percentages develop in an order consistent with the increasing complexity of, and likely familiarity with, common fractions. A possible order starts with students seeing 100% as the whole, 50% as a half and then 25% as a quarter. The students are then likely to be more successful with 10%, then with 75% and then with multiples of 10% before developing confidence with using percentages for which the equivalent simplified fractions are less familiar (e.g., thirds or eighths).

Data Collection

For each of the identified hierarchies, a set of items was created to reflect the suggested development of student understanding. Ideas for writing the questions came from the author's experience in the classroom over many years: writing tests for students, examining national and international tests as well as from using textbooks. A pilot test was conducted (n=232) to trial the questions which were further refined in response to student answers and to an analysis of the opinions of the teachers and students involved in the test

All tests were completed online during class time with students accessing the website where the test was located using either Ipads or computers and students did not have permission to access the calculator facility of their technology. The test for the study contained forty questions and was designed to be completed in forty minutes. There were two types of questions, short answer and multiple-choice. During the test, students were directed to the section appropriate for their year and the items for their completion related to the curriculum for their year level. The test was conducted at the end of the school year.

Year 7 students (n=211) were allocated questions 1-25, Year 8 students (n=152) were allocated questions 6-30 and for Year 9 students, (n=126) the allocated questions were numbers 10-35).There were 231 males and 256 females who participated in the study.

Results and Discussion

In the Rasch model analysis, each item on the test is allocated a number describing its location. These locations enable the items to be ordered according to their difficulty and in this assessment they ranged from -4.317 (easiest) to 3.953. The Person Separation Index was 0.91016; an indication of reliable item locations. The Person-Item Threshold Distribution showed good targeting; showing that the test items were well aligned to student ability. An examination of the Item Characteristic Curves showed that a high number of items fitted the proposed model. These measures suggest that the item locations generated can be used with confidence.

Student competence with fractions

An analysis of the results as shown in Figure 1, indicated that student competence with percentages for equivalent fractions was generally as expected. Item difficulties were compared for straight forward questions in which students were asked to state the percentage equivalents for common fractions, or, use percentages where knowledge of equivalent fractions is expected. As proposed, student competence with interpreting and using 100% and 50% was high (negative item locations) but there was no evidence to suggest that there was a difference between these two. When asked to write equivalent percentages for $\frac{34}{100}$ and $\frac{90}{100}$, the degree of difficulty was higher for $\frac{90}{100}$ (item location of -2.331) than for $\frac{34}{100}$ (location of -3.375). While both of these items were relatively easy the difference in difficulty between the two is high.

Confidence in using 25% was not as high as for 100% and 50% but significantly higher than for 10%. The item in which students had to express $\frac{6}{10}$ as a percentage was the easiest of the items involving 10 as a denominator or a percentage (item difficulty of -1.99) and it was interesting to note that students found this much easier than expressing $\frac{6}{100}$ as a percentage (item difficulty of -0.37).

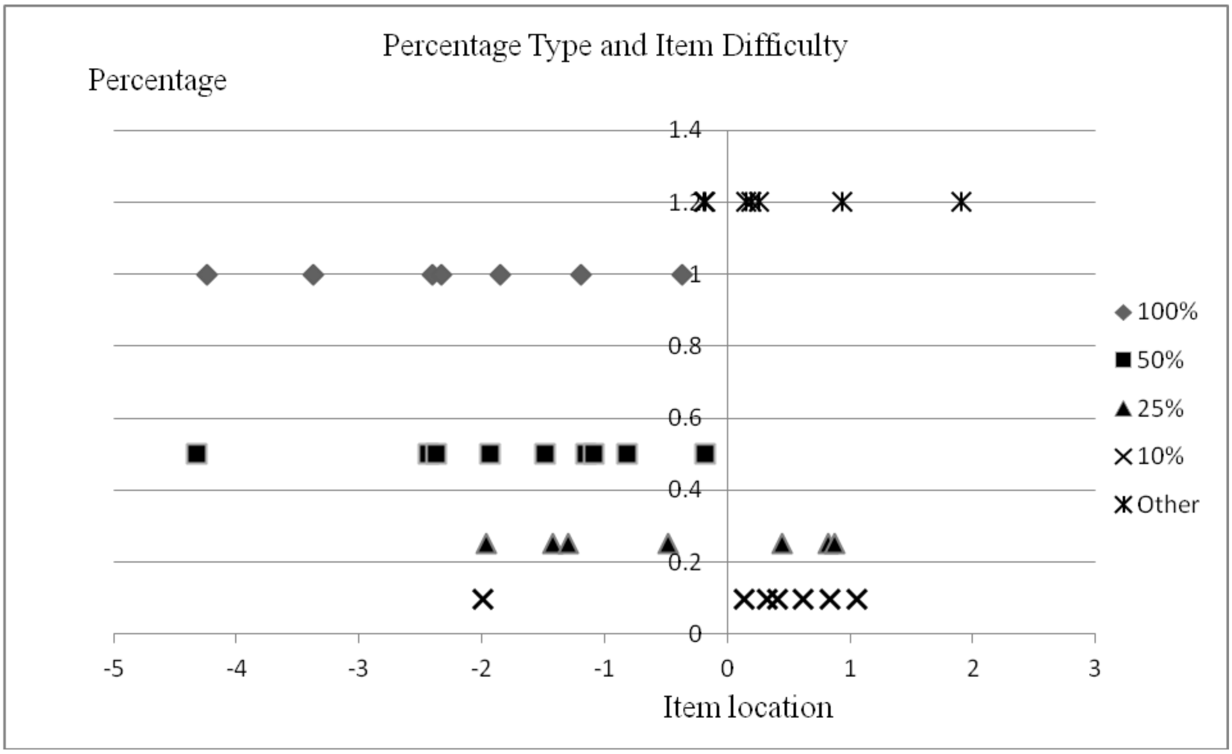


Figure 1. Item difficulty related to the nature of the percentage

Other fractions for which students had to give an equivalent percentage were, $\frac{1}{50}$, $\frac{4}{5}$, $\frac{3}{25}$, $\frac{17}{20}$ and $\frac{6}{15}$ and their level of difficulty is in the order written with $\frac{6}{15}$ having a much higher item location (1.91) than all the others.

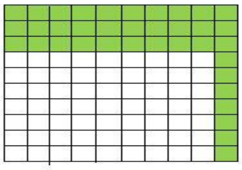
Student understanding of percentages

Data from questions where students were asked to recognise percentages, support the earlier conclusions that students are not confident in the knowledge and use of the percentage sign. In two multiple choice items presented to Year 7 students, the proportion of students selecting the correct response was not high. In Item 2, shown in Figure 2, 92% of students selected the correct answer and the first option was a popular distractor. For Item 3, shown in Figure 2, only 83% of students were correct, and of the 17% who were incorrect, more than 80% chose the second option.

Students in Years 8 and 9 were asked to nominate which fractions are greater than 100%. Results of these are shown in Table 1 and they indicate that, while the items were not difficult, only about 75% of these students had a clear understanding that fractions greater than one are equivalent to percentages over 100.

ITEM 2
In this diagram there are 100 small squares.
37 of them have been shaded.
What % of the diagram has been shaded?

- 37 out of 100
- 37%
- 63%
- 73%



ITEM 3
What percentage of $1.00 is 25c?

- 25 out of 100
- A quarter
- 25%
- 75%
- 100%

Figure 2. Items 2 and 3

Table 1
Items and Data for Fractions Greater than 100%

Item	46	50	47	48	49
Fraction	$\frac{9}{11}$	$\frac{15}{13}$	$\frac{3}{2}$	$\frac{1}{101}$	$\frac{150}{200}$
Item location	-1.249	-1.006	-0.769	-0.614	-0.149
Proportion correct	86%	84%	81%	80%	76%

For three of the more difficult items, as shown in Figure 3, students had to consider the base when determining the percentage. In these items the most popular incorrect answers (indicated by **) suggested that students used a different approach. Item 36 had a very high difficulty rating with approximately 50% of students in Years 8 and 9 correct on this item.

ITEM 16
If there were 50 students sitting on the lawn and 10 of them are boys, then the percentage of boys is

- 10% **
- 20%
- 25%
- 40%
- 50%

ITEM 35
In a class of 32 students, 8 students thought they had the wrong answer for the last question on their test. What % of students thought they were wrong

- 24% **
- 25%
- 32%
- 40%

ITEM 36
There were 90 students in the hall before recess and 99 students in the hall after recess. This is equal to an increase of:

- 9% **
- 10%
- 90%
- 99%

Figure 3. Items 16, 35 and 36

The proportional nature of percentages was one of the most difficult concepts for students to demonstrate on this test. These items in order of difficulty were Item 45 (location of 3.953, the highest on the test), Item 51 (location of 1.691), Item 33 (location of 0.431) and Item 32 (location of -0.01). These items are shown in Figure 4. For item 45, done by students in Years 8 and 9, only about 7% of the students had the correct answer.

ITEM 45

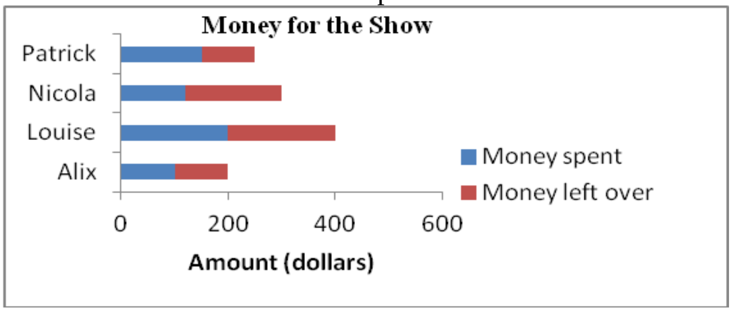
Philip and Damien each have a tub of the same brand and type of soft cheese.
Philip has the 400 gram tub and Damien has the 250 gram tub.
The percentage of protein in the 250 gram tub is 7.1%.
In Philip's tub (400 g), the % of protein is;

- About 7%
- About 12%
- About 14%
- More than 15%

ITEM 51

Louise, Alix, Patrick and Nicola had all saved up different amounts of money to spend at the show.
The graph shows the amounts they spent and the amounts left over. Who spent the highest % of money they had saved?

- Alix
- Louise
- Nicola
- Patrick
- Cannot tell from the data provided



ITEM 32

What can you say about 81% of 11?

- It is less than 11.
- It is equal to 11
- It is greater than 11
- I do not know if any of the three statements above are correct

ITEM 33

Compared to the yellow triangle, the height of the green triangle is closest to

- 10% of the height of the yellow triangle.
- 30% of the height of the yellow triangle.
- 50% of the height of the yellow triangle.
- 60% of the height of the yellow triangle.
- 120% of the height of the yellow triangle.

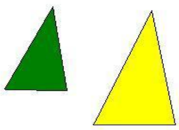


Figure 4. Items 45, 51, 32 and 33 (Note: Yellow triangle on the right in item 33)

Differences in student performances

Although the mean location of 1.166 for males was much higher than the mean location of 0.784 for females, no significant differences were reported between their performances on individual items. An examination of the Item Characteristic Curves (ICC) produced during Rasch modelling showed interesting deviations from this trend. For Items 13, 40, 41 and 42 where students were asked to calculate a common percentage of an amount, the observed mean values was higher for the girls than for boys in each class interval as shown in Figure 5. The trend for the females to show greater similarity of proportions of correct answers in the highest class intervals was also evident in the ICC for all of these items.

One would expect the mean location for each of the years to differ but there was only a small difference between the means for students in Years 8 and 9. The mean locations for Years 7, 8 and 9 were 0.559, 1.252 and 1.297 respectively. After Bonferroni adjustment, significant difference between the years was found for items 26, 27, 30 and 36. Further examination revealed a lack of growth from Year 7 to Year 9 in student knowledge of percentages for equivalent fractions and this is summarised in Table 2.

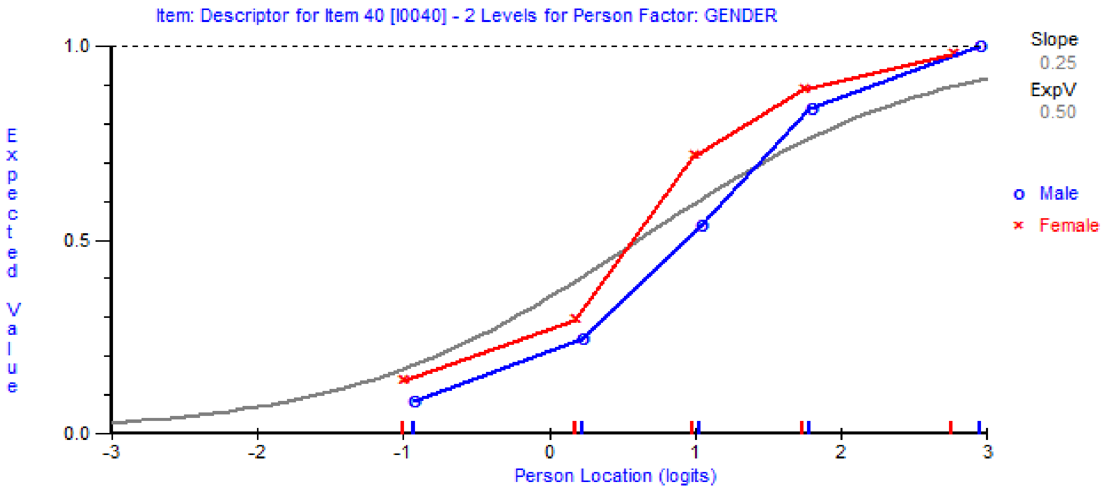


Figure 5. Observed mean values of correct responses.

Table 2
Proportion of students correct on percentage equivalents by year

Item	26	27	28	29	30
Fraction	$\frac{90}{100}$	$\frac{4}{5}$	$\frac{3}{25}$	$\frac{7}{20}$	$\frac{1}{50}$
Year 7	96%	66%	61%	58%	70%
Year 8	97%	85%	76%	76%	78%
Year 9	86%	67%	66%	66%	70%

Further Comments

Only some aspects of the different hierarchies for the development of student understandings and skills when using percentages have been presented. However, some implications for planning programs of learning and for classroom practice can be made. It may benefit students to have greater exposure to learning and calculating with equivalent fractions and particularly with those that can be simplified first. Teachers need to review and revise those fractions that are expected to be already part of the student tool kit in any particular year as one cannot assume that what has been taught has been remembered.

Simple tasks on the basic concepts of percentages could be presented at an earlier age. Questions relating to what constitutes the whole amount and is equivalent to 100% could be supplemented by more consideration of what less than 100% (e.g., 81% of 11, 60% of the height of the triangle) might look like. It was clear that students could very easily recognise half the area of a rectangle and questions requiring recognition of less than 50%, or more than 50% and many other percentages could support the development of student understanding in this area.

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