Predictive Validity of Numeracy Entry Requirements for University: Pre-service Teachers' Mathematics Knowledge and Attitudes

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This paper presents the findings of a study that assessed the numeracy competency of more than 200 students enrolled in pre-service primary teacher education. The Mathematics Thinking and Reasoning assessment consisted of 10 tasks that included 2-digit computations and proportional reasoning. Students rated their attitudes towards mathematics at primary and secondary school, and currently. Analysis of the data showed that university admission status was not related to students' scores on the assessment tasks. The correlation between meeting the University Entrance Numeracy Requirements and the total correct was very modest. The implications of these findings for ITE providers are presented.

Over the past few years, there has been increasing recognition among mathematics education researchers of the need for primary teachers to have sufficient mathematics subject matter knowledge (SMK), an essential component of pedagogical content knowledge (PCK) in mathematics (Ball, Hill & Bass, 2005; Ma, 1999; Moch, 2004; Rowland & Ruthven, 2011). Ball, Thames, and Phelps (2008) have built on Shulman's (1986) seminal work on teacher knowledge and provided a practice-based theory for "professionally oriented subject matter knowledge in mathematics" (p. 389). Their model further subdivides SMK into into three parts: Common Content Knowledge, Knowledge at the Mathematical Horizon, and Specialised Content Knowledge. Similarly PCK is broken down into Knowledge of Content and Students, Knowledge of Content and Teaching, and Knowledge of Curriculum.

Several researchers have investigated the connections between SMK and PCK (e.g., Askew, 2008; Ward & Thomas, 2008). Evidence indicates that there is no clear linear relationship between these two categories. Having tertiary level mathematics is not necessarily an advantage, although having limited understanding of mathematics may be a problem. Ward and Thomas (2008) found that teachers with low levels of SMK also had low levels of PCK. However, those with high levels of SMK had a range of scores on the measure of PCK. In other words, there were some teachers with high levels of SMK that had low levels of PCK. This evidence supports the claims of several writers (e.g., Askew 2008; Moch, 2004) that there is a certain threshold level of SMK that is necessary for good teaching, but being able to meet this requirement is not sufficient on its own.

In most pre-service teacher education programmes it is a requirement on entry that students provide evidence of having achieved a specified level of mathematics knowledge. Recent studies continue to reveal concerning gaps in prospective teachers' mathematical understandings (Livy & Vale, 2011; Zazkis, Leikin, & Jolfaee, 2011). Currently, New Zealand University Entrance (that includes 14 credits in mathematics at NCEA Level 1, normally completed in Year 11) is being taken as providing sufficient evidence of numeracy competency. From 2012, students over 20 years of age (with Special Admission to university) will be required by the Initial Teacher Education (ITE) provider "to meet comparable numeracy requirements as those entering with University Entrance" (NZTC, 2010). It should be noted that in New Zealand primary teachers are qualified to teach up to

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Year 8. In many countries, students at this level are taught by specialist mathematics teachers in the secondary school system.

Literature on the attitudes towards mathematics shows that it is important to consider how teachers and students feel about mathematics as well as looking at their mathematics achievement (McGinnis, Kramer, Shama, Graeber, Parker, & Watanabe, 2002; Southwell, White, Way, & Perry, 2005). It is thought that teachers' attitudes influence their classroom practices, and this has an impact on students' attitudes and learning. In a factor analysis of attitude questions, Southwell et al. (2005) identified two independent factors of insecurity and confidence contributing to attitudes towards mathematics and teaching mathematics. Recognition of the link between attitudes and beliefs about SMK and reform-based mathematics pedagogy in pre-service teacher education programs have led to initiatives to improve the attitudes and beliefs of prospective teachers and these have shown encouraging results (e.g., McGinnis et al., 2002).

This paper presents the findings of a study that assessed the numeracy competency of 248 students enrolled in a three-year pre-service primary teacher education program.

Method

The participants in the study included 248 undergraduate students enrolled in the first year of a three-year Bachelor of Teaching degree (see Table 1). The majority of students had been awarded University Entrance (61%), one fifth of the cohort had been given Special Admission (23%), just over one eighth had come from Other Tertiary institutions (14%), and the remainder had Discretionary Entry (3%). Numeracy Credits attained as part of other qualifications were noted on the files of those without University Entrance. This information showed that almost three-quarters of the cohort had the equivalent of 14 credits in Numeracy at Level 1 NCEA (73%), with just over one quarter having no evidence of Numeracy Credits (27%).

Table 1

Number and Percentage of Participants According to University Entry Status

Category of Entry to University	No. of students	%
University Entrance	150	60.5
Special Admission *	56	22.6
Other Tertiary Institutions *	34	13.7
Discretionary Entry *	8	3.2
Total	*248*	*100.0*

**NB: Of the students without UE, 30 were evaluated as having equivalent Numeracy Credits*

The Mathematics Thinking and Reasoning assessment consisted of 10 tasks that included 2-digit computations and proportional reasoning. Two of the tasks were at Level 3 of *The New Zealand Curriculum* (Ministry of Education, 2007) and seven tasks were at Level 4, corresponding to expected achievement levels for primary school students at the end of Year 8 (see Table 2). Students were asked to solve the tasks and show their thinking using words, numbers, and/or pictures/diagrams. Answers were marked as correct or incorrect, and information about the thinking used to solve the problem was also coded. Overall, total scores ranged from 1 to 10. Aggregated scores were calculated for the two Level 3 tasks and the seven Level 4 tasks. The Statistical Package for Social Sciences (SPSS) was used to analyse the quantitative data. The frequencies of correct responses and the most common incorrect responses were noted.

After completing the mathematics tasks, students were given a four-point Likert type rating scale ranging from "Really Like/d" to "Really Dislike/d" mathematics, "at primary school", "at secondary school", and "now". They were also invited to make comments in a box provided to give further insights to their experiences and attitudes towards mathematics.

Results

Students' performance was analysed as a function of University Entrance status and also whether or not they had the 14 Numeracy Credits at Level 1 NCEA (or the equivalent) required as part of University Entrance. The percentages of students who successfully completed each task according to their Entry and Numeracy status is presented in Table 2. Overall, students were more successful on tasks involving whole numbers and addition of decimal fractions. The most challenging tasks were those related to fractional quantities and proportional reasoning.

Table 2
Percentages of Students According to Entry and Numeracy Status Who Could Do Each Task

#	Question	Overall n=248	UE n=150	no UE n=98	NC n=180	No NC n=68
1	Tama has 64 stickers. He uses 27 on the first day of school. How many does he have left?	95	95	96	94	99
2	John needs $403 to buy a stereo. He has saved $297. How much money does he still need?	86	86	85	84	88
3	Sue used 8.3 metres of red material and 2.57 metres of blue material to make costumes for the play. How much material did she use altogether?	88	91	84	91	82
4	Ana bought 4.3 metres of rope to make skipping ropes, but only used 2.89 metres. How much rope was left over?	62	66	55	65	54
5	If 18 packets each hold 24 felt pens, how many pens is that altogether?	64	65	62	62	68
6	If 56 plums are shared among 14 people, how many plums will each person get?	92	92	91	92	90
7	Tama and Karen buy two pizzas. Tama eats 3/4 of one pizza while Karen eats 7/8 of the other one. How much pizza do they eat altogether?	32	37	22	33	27
8	If Ben got 72 out of a possible total of 90 marks, what percentage was that?	28	31	24	29	27
9	Jo spent $60 on stationery. She got one-third off the original price, because she was a teacher. What was the original price?	62	65	56	63	57

Students with University Entrance did slightly better than those without University Entrance on most tasks. On average, students with University Entrance attained an average total score half a mark higher than those without (M = 6.75 and M = 6.24 for those with and without UE, respectively). The biggest differences were evident on the task involving subtraction of decimals with regrouping (66% vs. 55%), addition of common fractions (37% vs. 22%), and converting fractions to percentages (31% vs. 24%).

Those with Numeracy Credits outperformed those without Numeracy Credits for six tasks, while the reverse pattern was found for three tasks. The average score for those with Numeracy Credits was slightly higher than for those without (6.62 vs. 6.37). The difference in mean total score was only one quarter of a mark, on average. The largest differences were evident on the task involving two-digit multiplication (65% vs. 54%) and subtraction of decimals with regrouping (91% vs. 82%).

Statistical analysis was completed to calculate the correlation between the total number of correct answers and University Entry status (using dummy variables, with 1 assigned to students with UE or with Numeracy Credits, and 0 assigned to those without). The correlation coefficient (Spearman's rho) for the relationship between UE status and number of correct answers was 0.129, while the corresponding value for the relationship between Numeracy Credits and number of correct answers was 0.066, extremely modest values at best. University Entry Status and Numeracy Credits were strongly correlated ($r = 0.760$).

Misconceptions

An analysis of common incorrect responses was completed for the cohort on tasks where at least two percent of students had given the same incorrect response (see Table 3). All of the common mistakes involved the Level 4 tasks, including adding and subtracting fractions and decimals, converting a fraction to a percentage, working out the whole from knowing a part, and multi-digit multiplication. For example some students when adding 8.3 and 2.57, treated the decimal part as whole numbers (3 + 57 = 60), later putting the decimal point back in the number (10.6 or 10.60). Subtraction of decimals was likewise challenging for some students who subtracted the smaller decimal part away from the larger part, confusing the subtrahend with the minuend (4.3 – 2.89 = 1.59). A common mistake (6% of those with UE vs. 4% of those without UE) when multiplying 18 x 24 was to multiply the tens (10 x 20 = 200) and multiply the ones (8 x 4 = 32), and then add these two partial products (200 + 32 = 232), disregarding the cross-products formed by multiplying each of the tens with each of the ones (10 x 4 = 40, 20 x 8 = 160) and adding these two partial products to the others to give an answer (total product) of 432.

Two-thirds of the students had problems adding $\frac{3}{4}$ and $\frac{7}{8}$. The most common mistake was to convert $\frac{3}{4}$ to $\frac{6}{8}$ then add both the numerators and denominators to get an answer of $\frac{13}{16}$. Almost as many of those with UE made that mistake as those without UE (15% vs. 19%). Some other students did not find an equivalent fraction for $\frac{3}{4}$, instead immediately adding both numerators and denominators to get an answer of $\frac{10}{12}$ (19% of those with UE; 26% of those without UE). These mistakes clearly show that students were simply executing a mis-learned procedure and were not paying attention to the meaning of the problem. Had the students used 'number sense', they would have realised that $\frac{3}{4}$ and $\frac{7}{8}$ are both close to one whole, so the answer had to be greater than one. Just over one-quarter of the students (28% overall; 31% of those with UE and 24% of those without UE) could convert 72 out of 90 to a percentage by noticing that 72 and 90 are both multiples of 9, so the fraction could easily be simplified to $\frac{8}{10}$ and from there converted to 80%. Alternatively they might have noticed that every nine marks was worth 10% and calculated how many groups of 9 are in 72, or used benchmarks such as one half (45 = 50%), one quarter (22.5 = 25%), and one fifth of one quarter (4.5 = 5%), then added the parts together to get 72 marks and 80% in total.

Another task that students found challenging was Question 9, asking them to find the original price given that $60 was two thirds of that price. A common mistake was to find one third of $60 and add it on, to give an answer of $80 (7% of those with UE; 13% of those without UE), instead of halving the $60 to find one third of the original price, then multiplying the amount by 3 to get a total of $90. A similar pattern was found for the comparison of those with and without Numeracy Credits.

Table 3
Percentages of Students Who Made Common Errors on Tasks

#	Question	Overall n=248	UE n=150	no UE n=98	NC n=180	No NC n=68
3	Sue used 8.3 metres of red material and 2.57 metres of blue material to make costumes for the play. How much material did she use altogether? (Correct Ans: 10.87)	**88**				
	Ans: 10.6	2	1	4	1	4
	Ans: 10.60	4	3	6	3	7
	Ans: 11	2	3	1	2	2
4	Ana bought 4.3 metres of rope to make skipping ropes, but only used 2.89 metres. How much rope was left over? (Correct Ans: 1.41m)	**62**				
	Ans: 1.14	3	2	5	3	4
	Ans: 1.59	10	8	12	10	9
	Ans: 2.59	4	6	2	6	0
5	If 18 packets each hold 24 felt pens, how many pens is that altogether? (Correct Ans: 432)	**64**				
	Ans: 108	4	4	4	4	3
	Ans: 216	2	2	2	2	2
	Ans: 232	5	6	4	7	2
7	Tama and Karen buy two pizzas. Tama eats $\frac{3}{4}$ of one pizza while Karen eats $\frac{7}{8}$ of the other one. How much pizza do they eat altogether? (Correct: 13/8 or 1 and 5/8)	**32**				
	Ans: 10/12	21	19	26	21	22
	Ans: 13/16	17	15	19	16	18
8	If Ben got 72 out of a possible total of 90 marks, what percentage was that? (Correct Ans: 80%)	**28**				
	Ans: 0.648	3	3	3	3	3
	Ans: 0.81	3	3	3	4	0
	Ans: 0.82	12	11	14	12	13
	Ans: 72/90	10	12	7	10	10
9	Jo spent $60 on stationery. She got one-third off the original price, because she was a teacher. What was the original price? (Correct Ans: $90)	**62**				
	Ans: 180	3	3	4	3	3
	Ans: 60	2	1	3	1	4
	Ans: 80	10	7	13	8	13

Threshold Level of Mathematics Knowledge (SMK)

Students were ranked according to their successful performance on Level 4 tasks. Only one student was unable to get any of the Level 4 tasks correct. That student had met the University Entrance requirements, including the 14 Numeracy Credits. Nine students were able to get only *one* response correct for NZC Level 4 tasks. Four of these students had University Entrance, three had been given Special Admission to university, and two had come from another tertiary institution. For three of the nine students, the one Level 4 task on which they experienced success was adding 8.3 and 2.57. One student was successful in subtracting 2.89 from 4.3. The remaining five students were able to work out that 56 divided by 14 was 4. The most common way of working this out was by drawing tallies to create physical representations, then counting groups of 14 to find the number of groups. None of these nine students could add $\frac{3}{4}$ and $\frac{7}{8}$, calculate 72/90 as a percentage, or work out the original price when 2/3 of it was $60.

Twenty-five students were able to get *two* responses correct on NZC Level 4 tasks. Nine of these students had University Entrance, eight had been given Special Admission to university, and eight had come from another tertiary institution. Sixteen were able to add the decimal quantities. Not one of the 25 students could subtract 2.89 from 4.3, and only three students could multiply 18 x 24. Two of the 25 students could add ¾ and 7/8. One student was successful in working out 72/90 as a percentage, and four worked out the original price when 2/3 of it was $60.

Forty-two students were able to get *three* responses correct on NZC Level 4 tasks. Just over half of these (n=24) had University Entrance, 13 had been given Special Admission, four came from another tertiary institution, one had been given Discretionary Entrance. Twelve students subtracted 2.89 from 4.3 correctly and 22 were able to multiply 18 x 24. Only five of the 47 students could add $\frac{3}{4}$ and $\frac{7}{8}$, and only one student could work out 72/90 as a percentage. Altogether almost one third (31%) of the students got fewer than half of the seven NZC Level 4 tasks correct. One quarter (25%) got *four* responses correct on NZC Level 4 tasks. It was interesting to note that only ten of the 62 students successfully solved the addition of fractions task.

Affective Responses

The percentages of students who disliked (or really disliked) mathematics at primary school, at secondary school, and currently are shown in Table 4. Students were most positive about mathematics at primary school (83% liked or really liked it). Less than two-thirds (64%) of the cohort was positive at the beginning of their pre-service primary teacher education course. Those students with UE were slightly more negative towards mathematics currently than those without (38% vs. 34% disliked or really disliked maths currently). On the other hand, fewer of those with UE were as negative about the mathematics they did at secondary school than those without UE (41% vs. 50% disliked or really disliked maths at secondary school).

Students with and without Numeracy Credits were very similar in their feelings about mathematics at primary school (17% vs. 18%), and currently (37% vs. 35%). However those without Numeracy Credits were substantially more negative about mathematics at secondary school than were those who had achieved the credits (53% vs. 41%).

Table 4
Percentages of Students Who Disliked (or really disliked) Mathematics

	Overall n=248	UE n=150	no UE n=98	NC n=180	No NC n=68
At primary school	17	17	17	17	18
At secondary school	44	41	50	41	53
Currently	36	38	34	37	35

Students' comments revealed information about the reasons for their ratings. It was evident that students believed that their teachers had influenced their attitude towards and success in mathematics.

> I had a primary teacher when I was ten that really knocked my confidence at maths. Since then I've found it extremely difficult and I often get confused – I use my fingers to count. (T030)

> At primary [school] my ability in maths was very dependent upon the teacher. With some I 'got it' and others not. (S050).

> My maths throughout my life started off great, but when it came to high school, teachers in the lower classes were too busy to teach or not that great a teacher. How can you or why would you want to try when you get that? I think with the right teacher and attitude I can really learn to be the best I can be. I would love help thanks. (T031)

Some students commented on the importance of links between mathematics and real life.

> I have never been a lover of maths. When I was introduced to maths at primary I could do the basics but not well. As I progressed through school I felt embarrassed I was not on the same level as my peers. When I started working, paying bills etc, I found maths was part of everyday life. Now I have a desire to learn and I want to improve my attitude towards maths which will hopefully improve my capability and ultimately my ability to teach maths as a subject. (T004).

> I found the way I use maths in day-to-day life is different to the way I was taught maths at secondary but probably more like the maths I was taught at primary school. Over the years I have developed my own way of working things out in my head. I find maths a challenge and enjoy the way it gets my mind thinking to find the solutions. (S025)

Discussion

Overall, the results were of considerable interest and concern. Although these undergraduates had three years in which to strengthen their mathematics, the current regulations only require them to do 72 hours of compulsory mathematics education papers. This part of the course is completed half-way through their programme, leaving eighteen months in which they may do no further mathematics before they become Provisionally Registered Teachers. Given the areas of weakness demonstrated here, it is questionable whether it will be possible to bring their performance up to an acceptable level within the 72-hour programme. Currently there is no assessment before students exit the programme to ensure they meet numeracy competency requirements. However, students have an opportunity to strengthen their understanding by enrolling for an optional mathematics education paper in the third year of their programme.

Analysis of the data showed that university admission status was not related to students' score on the assessment tasks. The correlation between meeting the University Entrance numeracy requirements and the total correct was very modest. Many of the lowest scorers (getting a total of 1 or 2 tasks correct) had been awarded University Entrance. By the same token, some of the students who got all ten tasks correct were those with Special Admission who had no evidence of meeting the University Entrance numeracy requirements. When the average score out of 10 was calculated for each category of admission status, those with the highest average score were those with Discretionary Entrance. Those with University Entrance were only marginally higher than those with Special Admission.

Most students used traditional algorithms to calculate their answers. This reflects a strong orientation towards procedures and rules, consistent with an instrumental approach to mathematics (Skemp, 2006). This is despite several decades of mathematics education reform that has called for greater emphasis on conceptual understanding and 'making sense' of the mathematics - Skemp calls this relational understanding. This is consistent with other findings showing that it is extremely difficult to change the ways that mathematics is taught and learned (Anthony & Hunter, 2005; Lamon, 2007). It should be noted that the Numeracy Credits for University Entrance are part of a standards-based assessment system that allows multiple opportunities for re-assessment. If students complete these requirements in Year 11, they may discontinue studying mathematics for their remaining two years of secondary education.

The findings of this study have important implications for ITE providers. If a certain threshold of discipline knowledge in mathematics is necessary for good teaching, then it is vital that institutions assess prospective teachers to ascertain the extent of that knowledge and identify particular areas that may need to be further strengthened. The findings suggest that the use of numeracy assessment tasks to reveal important misconceptions could be helpful in determining the extent to which students are likely to meet a threshold level of proficiency in mathematics.

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