Nature of an *Attitudes toward Learning Mathematics* Questionnaire

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Students' attitudes toward mathematics and its learning have been subject to numerous studies in the past six decades. These studies treat such attitudes as both desirable learning outcomes and correlates of mathematics achievement. Many Likert-type attitude scales have been devised to measure significant constructs underlying mathematics-related attitudes, such as confidence, anxiety, and utility of mathematics. The psychometric properties of these attitude scales may be culture and age dependent. As part of a research project called Singapore Mathematics Assessment and Pedagogy Project (SMAPP), an effort was made to devise and validate an attitude toward learning mathematics scale that can be used with lower secondary school students in Singapore. This paper explains the use of exploratory and confirmatory factor analyses to reduce an initial 57-item questionnaire to one with 24 items that cover these six dimensions: *Checking solutions, Confidence, Enjoyment, Use of IT in mathematics learning, Multiple solutions,* and *Usefulness of mathematics*. The data comprise responses from about 890 Secondary 1 (Grade 7) students in 2010, who took the 57-item questionnaire, and another 850 students who took the 24-item questionnaire in 2011. The nature of the final questionnaire is discussed. This effort contributes to the continual effort to devise validated attitude scales that are suitable for different cultures and student groups.

Introduction

Positive attitudes are desirable learning outcomes for most school subjects, including Mathematics. They are also important and positive correlates of mathematics achievement. Numerous studies, including large international comparative studies, such as the Trends in Mathematics and Science Study (TIMSS), have included student attitude as a significant variable of interest (House, 2006; Mullis et all, 2000; Schreiber, 2002; Van den Broeck, Opdenakker, & Van Damme, 2005), because, according to the National Mathematics Advisory Panel (2008) of the United States, "Children's goals and beliefs about learning are related to their mathematics performance" (p. xx). These studies have used Likert-type attitude scales to measure significant constructs underlying mathematics-related attitudes, such as confidence, anxiety, and utility of mathematics. Well-known scales developed by researchers such as the Aiken, Fennema, Sandman, and Sherman, have been modified and used extensively in many studies (e.g., Fennema & Sherman, 1976; Fogarty, Cretchley, Harman, Ellerton, & Konki, 2001; Tapia & Marsh, 2004), and comprehensive reviews of this research area have been provided by researchers over the past thirty years, such as Kulm (1980), Leder (1992), Ma and Kishor (1997), Maasz and Schlöglmann (2009), McLeod (1992), Sriraman (2008), Zan, Brown, Evans, and Hannula (2006).

The psychometric properties of attitude scales toward learning may be culture dependent because the practices of teaching and learning are strongly influenced by cultural milieu, which Schmidt et al. (1996) referred to the pedagogical flow of different countries. Furthermore, the responses gathered from students using such scales also depend on their age. As part of a research project called Singapore Mathematics Assessment and Pedagogy Project (SMAPP), an effort was made to devise and validate an attitude toward learning mathematics scale that can be used specially with lower secondary school students (Grade 7, age 13+) in Singapore. This part of the project attempts to address the culture and age factors of using a Likert-type attitude scale with school students. This questionnaire was

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used to measure the attitudes of participating students towards mathematics and its learning before and after the SMAPP intervention. Hence, the research question of this paper is: What were the psychometric properties of the SMAPP *Attitudes toward Learning Mathematics* (ALM) questionnaire? The following sections will describe the design and validation process that was undertaken to answer this research question.

Procedure and Results

Many researchers, including those mentioned above, have pointed out that *attitude* is an ambiguous construct with many shades of meanings overlapping the affective and cognitive domains. However, even a cursory examination of published attitude scales will have identified items used to describe emotions, beliefs, anxiety, confidence, and so forth. Thus, for this project, we have taken a working definition of *attitudes* to encompass items commonly found in these published scales that we feel are within the learning experiences of lower secondary students.

Step 1: Build on a Previous Project

The initial questionnaire was built from one used in a previously funded project called the *Mathematics Assessment Project* (MAP). This earlier project investigated the use of four types of alternative assessment strategies in mathematics learning at primary and secondary levels and included a questionnaire about students' attitudes toward mathematics and its learning (Fan & Zhu, 2008). The 2010 version of ALM consisted of 57 items, 12 of them were from the MAP project. These 57 items were classified under three general categories: (1) Attitudes towards Mathematics; (2) Mathematical Thinking and Reasoning; and (3) Others. The questionnaire was a 9-point scale: 1 (Disagree totally); 2 (Disagree a lot); 3 (Disagree); 4 (Disagree a little); 5 (Neither Disagree nor agree); 6 (Agree a little); 7 (Agree); 8 (Agree a lot); 9 (Agree totally). This 9-point scale allows for greater variation in opinions, hence may be closer to an interval scale, than the more common 5-point scale, but it has the disadvantage that some students may not be able to make such fine distinct judgments about their beliefs. The students took about 15 minutes to answer the questionnaire on-line.

Step 2: Constructs Underlying the 57 Items

The next step was to identify the underlying constructs of the 57 items beyond the three broad categories mentioned above. Two datasets were collected in 2010: (a) a "pre" dataset obtained from 893 Secondary 1 Express[1] students from 5 schools in February; (b) a "post" dataset from the same students with reduced size (*n* = 751; 84%) due to absentees in October.

The Cronbach's alpha for the whole questionnaire, after reverse scoring of negative item, was 0.967 for the pre-data and 0.963 for post-data. Although these values were acceptable, our aim was to design a shorter questionnaire that still reflected the underlying constructs of ALM. This shorter version should have clear psychometric properties and it will save administration time for the students and teachers who may use it in their own research in the future. To achieve this, exploratory factor analysis (EFA) was used.

Both datasets were subject to Principal Component Analysis (PCA) with Varimax rotation. Items with factor loadings less than 0.4 in magnitude or free loadings or cross loadings were mostly ignored. With this procedure, both the pre-data (57.0% of total

[1] Secondary school students in Singapore attend one of three courses: *Normal (Technical), Normal (Academic),* or *Express.* Students in the *Express* course have average to high ability performance.

variance) and post-data (56.5%) seemed to cover six similar factors: *Checking solutions, Confidence, Enjoyment, Use of IT in mathematics learning, Multiple solutions,* and *Usefulness of mathematics.* Some factors (e.g., *Multiple solutions*) were more well-defined than others (e.g., *Confidence*), and some items were loaded on different factors in the two solutions. Given the theoretical links between some of the factors such as confidence and enjoyment, this result is not unexpected for a long questionnaire.

Step 3: Construction of the Shorter Version

The construction of the shorter version of ALM was guided by the above EFA results and more importantly theoretical meanings to be attached to the retained constructs. Briefly, the theoretical ideas are as follows:

1. The *Checking solutions* scale was based on the looking back feature of the Polya's problem solving framework, which is used extensively in Singapore mathematics teaching.
2. The *Confidence* scale covered students' self-concept of their ability to do the mathematics.
3. The *Enjoyment* scale examined the degree to which students enjoyed studying mathematics.
4. The *Use of IT* scale dealt with to what extent students believed that IT could support their learning of mathematics, and this scale was the weakest one in the original questionnaire.
5. The *Multiple solutions* scale dealt with students' propensity to look for multiple solutions, a trait about flexibility in problem solving that is emphasized in the mathematics education literature but may not be widely practiced in Singapore classes.
6. The *Usefulness of mathematics* scale was a ubiquitous one covering students' beliefs about the usefulness and relevance of mathematics to their daily life; this aspect was the main motivation for SMAPP.

Four items were to be used to measure each scale, giving a total of 24 items. Fourteen original items were retained based on the factor solutions and ten new ones had to be created where the original items were deemed to be inappropriate. The Cronbach's alphas of the six scales using the retained items were re-computed using both datasets, with results shown in Table 1. These were within commonly reported values about attitude constructs.

Table 1
Cronbach's Alphas of 24 items (2010 data)

Scales	No. of Retained Items	Pre α	Post α	No. of New Items
Checking solutions	3	0.777	0.816	1
Confidence	3	0.799	0.728	1
Enjoyment	3	0.887	0.830	1
IT and mathematics learning	1	NA	NA	3
Multiple solutions	2	0.782	0.810	2
Usefulness of mathematics	2	0.752	0.786	2

The 24 items are shown in Table 2. The related items for each scale were placed at intervals of six; for example, the *Check* items appeared as items 1, 7, 13, and 19. For convenience, the scales to which the items belong are shown in Table 2 but they did not appear in the actual questionnaire. Both positive and negative items were used. The same 9-point scale was retained to allow for comparison of responses in the 2010 and 2011 data.

Step 4: Analysis of Data of February 2011 using the Short Version

This short version was administered to 849 and 854 Secondary 1 Express students from eight schools in February and October. Only results of the February 2011 data are reported here.

As shown in Table 3, the *Enjoyment* scale was the most reliable one, while the *Multiple* scale was the least reliable. The reason could be that the items in the *Enjoyment* scale were readily understood by the students, but they may not have adequate learning experience about finding multiple solutions in problem solving to give consistent responses to the items in the *Multiple* scale. The alphas of the other four scales were also not as high as those reported in Table 1. This suggests that the selection of items for these scales based on earlier factor solutions and the design of new items at Step 3 needs to be further fine-tuned.

Table 3 also shows that *Confidence* and *Enjoyment* had the strongest correlation, and this reinforces the notion that both constructs are highly related. Beliefs about use of IT to learn mathematics were weakly related to all the other scales, and its highest correlation with *Multiple* (.279) suggests that IT could be an environment to encourage students to seek for multiple solutions. Further research is needed to verify these observations.

Step 5: Confirmatory Factor Analysis (CFA)

Confirmatory Factor Analysis (CFA) was conducted using AMOS 19 to check whether the hypothesized six-factor structure of ALM has empirical support. The tested model initially consisted of six factors with four indicators per factor. After several rounds of modification, it was found that acceptable model fit could be achieved only with item 24 deleted from the analysis. This item was newly added to form this short version, and it seems to differ from the other items by being very specific about certain aspects of the usefulness of mathematics. Whether or not this item format is the cause of its misfit in the model requires further investigation. Hence, the final model works for 23 items, with only three items for the *Usefulness* factor.

Table 2
Items in the Attitudes toward Mathematics Learning (ALM): Short Version

Item No.	Scales	Items
1.	Check	When I know I have made a mistake in solving a problem, I will try to find out why.
2.	Confidence	I am good at using mathematics to solve real-life problems.
3.	Enjoy	I enjoy doing mathematics.
4*.	IT	I do not like to use the computer to learn mathematics.
5*.	Multiple	I do not like to think of other ways to solve the same problem.
6.	Usefulness	Mathematics is important.
7.	Check	After I have solved a problem, I will go through the solution again and check if I have made any mistakes.
8.	Confidence	I am confident in solving mathematics problems.
9*.	Enjoy	I find mathematics boring.
10.	IT	I can learn mathematics from playing computer games.
11.	Multiple	I often figure out different ways to solve mathematics problems.
12.	Usefulness	I think mathematics is useful in solving real world problems.
13*.	Check	Once I have worked out an answer to a problem, I do not check my answer.
14.	Confidence	I find mathematics easy.
15.	Enjoy	Overall, I have good feelings about mathematics.
16.	IT	IT (Information Technology) has been helpful to my mathematics learning.
17.	Multiple	I try to understand the different solutions given by my classmates.
18*.	Usefulness	I think mathematics is useful only for tests.
19.	Check	After I have solved a problem, I will ask myself if the answer makes sense to the given problem.
20*.	Confidence	I am not good at giving reasons in mathematics.
21.	Enjoy	Solving mathematics problems is fun to me.
22.	IT	Mathematics software (e.g., graphing) helps me to learn mathematics.
23.	Multiple	After I have solved a problem, I will look for other methods to solve it.
24.	Usefulness	Mathematics helps me to understand reports and advertisements about prices, sale, percentages etc.

Note: * Negative items.

Table 3
Cronbach's Alphas and Correlations of ALM Scales: Short Version (n = 849)

	Check	Confidence	Enjoyment	IT	Multiple	Usefulness
Check	1	.458**	.517**	.176**	.583**	.524**
Confidence		1	.709**	.177**	.491**	.526**
Enjoyment			1	.114**	.512**	.590**
IT				1	.279**	.209**
Multiple					1	.509**
Usefulness						1
Cronbach's α	0.714	0.695	0.898	0.693	0.633	0.711
Mean	6.66	5.21	6.15	6.03	5.93	6.91
SD	1.38	1.43	1.92	1.60	1.32	1.36

Note. **Correlation is significant at the 0.01 level (2-tailed).

Following the suggestions by Meyers, Gamst, and Guarino (2006) and others, we examined four model fit measures. The chi-square had a value of 465.972 (df = 184, N =

849), $p < 0.01$, and this indicates a mismatch between the proposed model and the observed data. However, the sole use of this index in judging the overall fit of the model is not appropriate because of the sensitivity of the chi-square to large sample, which is the case here. The three other indices indicate good model fit: NFI = .953 (acceptable values are .95 or above), CFI = .971 (similar to NFI), and RMSEA = .04 (acceptable values are .08 or below). In short, the CFA showed that the six-factor model was a well-fitting model, with the deletion of item 24.

The standardized regression weights for this solution are summarized in Table 4. These weights are useful in deciding to what extent the items are related to the underlying purported latent factors (Brown, 2006, p. 131). Although the six-factor structure of ALM was basically confirmed, items 5, 13, and 20 had coefficients much lower than those of the other items in the same factors. They were all negative items, and some lower secondary students may have difficulty agreeing or disagreeing with negative items that demand complex thinking processes. This observation need to be further examined.

Table 4
Standardized Regression Weights

			Estimate				Estimate
1	<---	Check	.624	*4	<---	IT	.548
7	<---	Check	.696	10	<---	IT	.552
*13	<---	Check	.392	16	<---	IT	.804
19	<---	Check	.664	22	<---	IT	.671
2	<---	Confidence	.575	*5	<---	Multiple	.205
8	<---	Confidence	.882	11	<---	Multiple	.842
14	<---	Confidence	.781	17	<---	Multiple	.606
*20	<---	Confidence	.238	23	<---	Multiple	.555
3	<---	Enjoyment	.894	6	<---	Usefulness	.794
*9	<---	Enjoyment	.664	12	<---	Usefulness	.772
15	<---	Enjoyment	.887	*18	<---	Usefulness	.524
21	<---	Enjoyment	.842				

Note: * Negative items (recoded).

Discussion and Concluding Remarks

In the above sections, we have described a rather elaborate process of developing a short version of an “attitudes towards learning mathematics” questionnaire that our project SMAPP had used to measure changes of the attitudes of lower secondary students during the implementation of the project. The practical value of the above validation is that the ALM questionnaire is a relatively short one with some known psychometric properties, and it will be disseminated as a research tool to other secondary schools in Singapore, so that it can be used by school teachers in their own action research, currently widely promoted in Singapore, or by other researchers in Singapore or elsewhere. Its relatively short length suggests that ALM can be an efficient research tool for the busy teachers.

Given that there are many mathematics-related attitude items in the literature, writing and compiling items for a Likert-type attitude questionnaire should at first glance be relatively straightforward. However, the above efforts show that validating the psychometric properties of a questionnaire against the putative theoretical structure is far from an easy task; instead, it requires considerable statistical work and substantive interpretations. Even if a satisfactory solution is attained for the moment, new data will have to be collected from the intended target population and subject to further analysis so that better solutions can be found. The possibility of replication is a hallmark for scientific advancement, and this is especially important in the social sciences where replication can be difficult to arrange and is often not attempted. The next phase of our work is to replicate the above factorial structure in at least three ways.

The first way is to test the model with new data. The original data were not drawn randomly from the target population because participation in the SMAPP project was voluntary, even though the sample was large and came from different schools. Thus, findings about the attitudinal structure of ALM may not be considered as normative for other student groups and across countries. This is related to the issues mentioned at the beginning about the influence of age, culture, and ability on measurement of attitudes.

Secondly, it should be beneficial to examine to what extent the less acceptable items are due to item format (as noted above), language usage, or the ability of students to make meaningful judgment about their own thinking, namely, their metacognitive awareness. Interviews could be conducted with students to gain insights into how they might interpret the items. Examples discussed by Blunch (2008) show that respondents may have paid attention to key words in statements different from those the researchers have in mind, thus leading to unexpected CFA findings. Further conceptual considerations may lead to deletion or re-writing of some of these items.

The third approach is prompted by the pattern of correlations given in Table 3. The *Confidence* and *Enjoyment* scales had very strong correlation and they may be combined into a single factor. The *IT* scale with its low correlations with the other scales seems to stand alone. The other three scales (*Check, Multiple,* and *Usefulness*) form a third factor. A general attitude factor can be hypothesized as a second-order factor with no indicators of its own such that it serves as a common cause of the three first-order factors in a hierarchical CFA model, parallel to the g-factor in the structure of cognitive ability (Kline, 2011). This is an interesting hypothesis to be tested, and it might contribute to the scientific investigation of the nature of attitude about mathematics or other subjects.

To conclude, similar efforts to create efficient and validated questionnaires to measure mathematics-related attitudes among students have practical values and research implications. Obviously such efforts are never-ending and unexpected new insights are truly the intrinsic rewards for conducting these studies.

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