The Effect of Small-Group Game Play Activities on Number Sense Performance

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This study examines the effect of small-group game play before and after a Number Sense Test (NST). Fifty-seven 7-year old children were assigned into two conditions: (a) NST-Game Play, in which students sat for the NST before game play; or (b) Game Play-NST, in which students experienced game play before sitting for the NST. Findings suggested that Game Play-NST students outperformed the NST-Game Play students in number sense performance. Having Game Play-NST experience appeared to be more effective in enhancing students' performance in number sense.

Background

Number sense is one's general understanding of number and operations which includes the ability and inclination to flexibly use this understanding to make mathematical judgements and develop efficient strategies when handling numerical situations (McIntosh, Reys & Reys, 1992). The importance of the development of number sense is evident mathematics curricular documents such as the National Council of Teachers of Mathematics of America (NCTM, 2000), the Australian Council of Education (AEC, 1991) and the Singapore Ministry of Education (MOE, 2006).

Gersten and Chard (1999) has emphasized that the construct of number sense is essential in conceptual understanding. These students have a good sense of magnitude of numbers, observe number patterns and are able to recognise benchmark numbers (Case, 1998). On the other hand, children who do not have a good number sense tend to be inhibited in their learning of mathematics (Ekenstam, 1977) and will have persistent problems in many areas of mathematics especially in relation to conceptual understanding and application of procedural knowledge (Woodward & Baxter, 1997). In addition, a study by Griffin, Case and Siegler (1994) underscored the relationship between instruction in number sense activities and its effect in reducing failure in early mathematics.

Research has also brought to light the role of informal learning of number sense. Gersten and Chard (1999) assert that most children develop number sense through informal interactions with parents and siblings before entering kindergarten. Griffin et al. (1994) echoes similar thought, suggesting that number sense is informally acquired prior to formal school and that it is necessary for learning formal arithmetic in elementary grades. Howden (1989) has also cited on the intuitive nature of number sense and the way it gradually develops in learners by exploring numbers, visualising them in different context and relating them beyond traditional algorithms. Research has also indicated that prior knowledge is an essential variable in learning (Alexander, Kulikowich, & Schulze, 1992; Dochy, 1994). Glaser and De Corte (1992) describe prior knowledge as a "springboard for future learning" (p. 1). If students' intuitive, informal and prior knowledge are important resources, how do we activate such knowledge and leverage them for both teaching and assessment of number sense into the formal setting of mathematics in school?

The learning of mathematics as a product of social activity is widely accepted (Cobb, Gravenmeijer, Yackel, McLain & Whitenack, 1997; Sfard, 2001). Through social activity, students would be put into situations where they need to communicate and reason

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mathematics ideas with their peers. In lending support to social learning, the significance of context of play in the study of the learning process of mathematics in early years and primary schools has been highlighted by Edo, Planas and Badillo (2009) and Peters, (1998). They contend that classroom play situations tend to promote construction of mathematical knowledge, development of mathematical problem-solving strategies and enhancement of individual mathematical thinking. Such interactions in a game play setting make it possible for learners to communicate and appreciate the different viewpoints too.

"Number sense exhibits itself in various ways as the learner engages in mathematical thinking" (McIntosh et al., 1992, p. 3). Over the past decade or so, many researchers have assessed number sense through pencil-and-paper tests, interviews or individually administered tests, combination of both pencil-and-paper test and interview and even computerised test (Aunio, Ee, Lim, Hautomaki & Van Luit, 2004; Dunphy, 2006; Markovits & Sowder, 1994; Markovits & Pang, 2007; McIntosh, Reys, Reys, Bana & Farrell, 1997; Yang, Li & Li, 2008). The assessments of number sense cited may not have drawn upon the characterization of number sense that learners possess. The setting for the computerised test, though different from the pencil-and-paper test, is still one of a question-and-answer type where students read the questions and then choose or give their answers. Number sense being an intuitive feeling of numbers and its informal acquisition may not be uncovered or manifested in such assessments.

While there has been substantive research on leveraging children's intuitive ideas and knowledge for designing better teaching and learning (e.g., Kapur & Bielaczyc, 2012; Kapur, 2012), work on how assessment of learning could also be influenced by activation of such resources remains under-researched by comparison. Therefore, we designed this study to examine how assessment of number sense that students have already learnt over the school year can be influenced by game play activities that activate students' formal as well as intuitive priors about number sense. More specifically, we examine the effect of having students take number sense test (NST), followed by a Game Play activity (NST-Game Play condition) or engage in Game Play followed by NST (Game Play-NST condition) on their performance on the NST.

Methodology

Participants and Design

The sample of 57 primary 1 students (about 7 years old) consisted of the main ethnic groups in Singapore. They come mostly from the neighbouring residential and were from the middle to low Socio-Economic Status (SES) groups. Students were assigned to two groups to experience either the NST-Game Play condition (N=33) or the Game Play-NST condition, (N=24). Students' year-end mathematics scores were taken as a measure of mathematics ability. The year-end mathematics scores comprised an aggregate of several standardised pencil-and-paper tests and performance task assessments. Table 1 shows the mean and standard deviations of NST-Game Play students and Game Play-NST students A univariate analysis of variance, ANOVA, showed no significant effect of the experimental condition (NST-Game Play vs. Game Play-NST) on mathematics ability, $F(1,55) = 0.539$, $p = .466$, partial $\eta^2 = .010$.

Table 1
Descriptive Statistics of Math Scores by Experimental Condition.

Conditions	M	SD
NST-Game Play	86.08	9.92
Game Play-NST	88.13	11.10

NST-Game Play students sat for the NST test first followed by game play on the following day and the reverse sequence for Game Play-NST students. In both conditions, students sat for the pencil-and-paper NST. In the game play students were put into small groups of fours. In the game play, students recorded their responses after each round on the game sheets provided. The study was conducted towards the end of the school year term. Hence the students had completed the primary 1 mathematics syllabus.

Instrumentation

A Number Sense Test (NST), adapted from the Number Sense Item Bank (NSIB) by McIntosh et al. (1997), was used for this study. Twenty-one items were set based on five strands, namely; number concepts, computing and counting, equivalent expressions, multiple representations and effect of operations. The content and conceptual knowledge in the test items were consistent with the Singapore primary mathematics syllabus for primary 1. The items were either multiple-choice variety or blanks for students to fill in their own answers.

The game play comprised of two card games; Game 1-“Whose the Closest?” and Game 2-“Flip 4 and Subtract!”. Both games, adapted from Overholt, Holtz & Dickson (1999), were designed to match five of the NST items. Game 1 matches similar outcome as NST items 8 and 9 while Game 2 matches similar outcome as NST items 19, 20 and 21. The game play provides an informal setting that stands in sharp contrast to the NST. It provide opportunities for students to activate their intuitive, informal and formal number sense, hence affording them to a) explain, reason and justify their strategies, b) observe what other players do, c) listen and question others’ viewpoints, d) discover and generalise strategies, e) build on one another’s contribution and f) error correct themselves and that of the peers. A pilot study was conducted with a class of 28 primary 1 students with similar background as the study groups. The purpose of the pilot study was to test the difficulty and structure of the test items as well as the game play.

Results

The scores of the students for all the twenty-one items were calculated. Each correctly answered item was awarded one mark and zero mark for any partially correct answer or wrong answer. No half mark was awarded. The NST had scale reliability (Cronbach’s alpha) of .78.

NST Items Matching with Game Play

Analysis of the experimental condition performances on the matching NST items to the two games revealed that students in the Game Play-NST condition outperformed the NST-Game Play students. Figure 1 indicates that Game Play-NST students performed more than 1.5 times better than NST-Game Play students in NST items 8 and 9 (Game 1 NST items) and almost 2.5 times better than NST-Game Play students in items 19, 20 and 21 (Game 2 NST items). Having played the game first before sitting for the NST seemed to enable Game Play-NST students to translate the cognitive gains from the game play to the pencil-and-paper NST.

A multivariate analysis of covariance, MANCOVA, indicated statistically significant multivariate effects of: (i) the experimental condition (NST-Game Play vs. Game Play-NST) on NST scores by items that matched with the game play, $F(2,53) = 11.272$, $p < .001$, partial $\eta^2 = .298$; and (ii) mathematics ability on NST items that matched with the game play, $F(2,53) = 9.035$, $p < .001$, partial $\eta^2 = .254$. We can infer that performance on NST items that matched game play was significantly dependent on the sequence of the NST and game play. The ANCOVAs revealed statistically significant effect of the experimental condition (NST-Game Play vs. Game Play-NST) on NST items matching with Game 1 and NST items matching Game 2; (i) for NST items 8 and 9 that matched with Game 1 (Game 1 NST items), $F(2,54) = 10.582$, $P = .002$, partial $\eta^2 = .164$; and (ii) for NST items 19, 20 and 21that matched Game 2 (Game 2 NST items), $F(2,54) = 8.442$, $P = .005$, partial $\eta^2 = .135$.

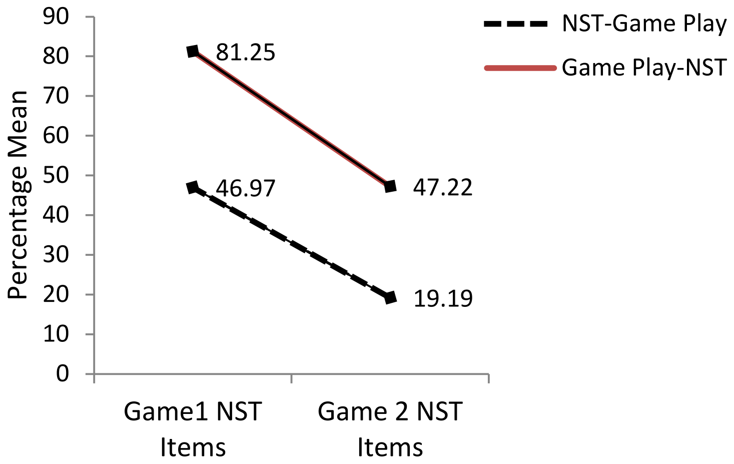

Figure 1. Mean percentage of experimental condition performance on NST items matching with game play

NST Items by Strands

The NST items were categorised into five strands. NST items 8 and 9 matched Game 1 were categorised in Strand 2 while NST items 19, 20 and 21 which matched to Game 2 were found in Strand 5. As seen in Figure 2, Game Play-NST Students did better than NST-Game Play Students, but greatest differences of 33.43% and 21.88% were observed in Strand 2 and Strand 5 respectively. The results suggest that having game play prior to NST had a positive effect on students' performance in NST.

A MANCOVA showed statistically significant multivariate effects of: (i) the experimental condition (NST-Game Play vs. Game Play-NST) on NST scores by strands, $F(5,50) = 6.553$, $P < .001$, partial $\eta^2 = .396$; and (ii) mathematics ability on NST scores by strands, $F(5,50) = 12.778$, $p < .001$, partial $\eta^2 = .561$. Therefore, we can infer that performance on the various strands in NST was significantly dependent on the sequence of the NST and game play.

The ANCOVAs revealed statistically significant effect of the experimental condition (NST-Game Play vs. Game Play-NST) on NST scores by strands: (i) for Strand 2 in which the two of the NST items matched with Game 1; $F(2,54) = 25.529$, $p < .001$ and partial $\eta^2 = .321$ and for (ii) for Strand 5 in which three of the NST items matched with Game 2; $F(2,54) = 6.513$, $p = .014$ and partial $\eta^2 = .108$.

Similarly, the ANCOVAs showed statistically significant effect of the students' mathematics ability on NST scores by strands: (i) for Strand 2, $F(2,54) = 8.915$, $p = .004$, partial $\eta^2 = .142$; and (ii) for Strand 5, $F(2,54) = 24.122$, $p < .001$, partial $\eta^2 = .309$.

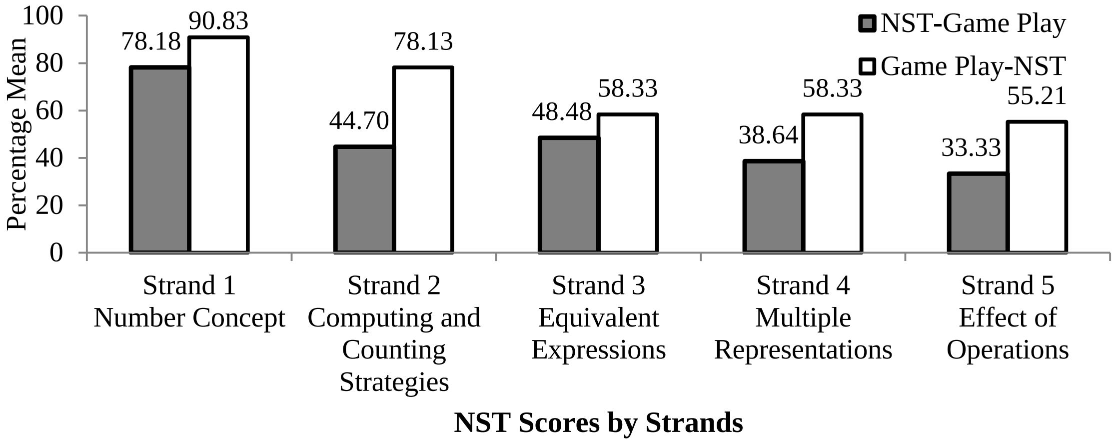

Figure 2. Mean percentage of experimental condition performance on NST scores by strands

Performance on Game Play

In the game play, one point was given for a completely correct response in the games and zero for a partially correct response or a wrong response. These scores were then converted to percentage and analysed. Table 2 shows the percentage score for NST-Game Play and Game Play-NST conditions correct responses in Game 1 and Game 2.

Table 2
Descriptive Statistics for Correct Response in Game 1& 2 by Experimental Conditions

	Conditions	M	SD
Percentage of Correct Response in Game 1[a]	NST-Game Play	76.77	20.28
	Game Play - NST	73.82	19.98
Percentage of Correct Response in Game 2[a]	NST-Game Play	76.28	30.25
	Game Play-NST	68.75	25.80

[a]For response to be considered correct in Game 1 and Game 2, students must at least score 2 out of 3 rounds correct or 3 out of 4 rounds correct or 4 out of 5 rounds correct in both games.

A MANCOVA revealed a significant multivariate effect of mathematics ability on composite game scores, $F(2,53) = 11.305$, $p < .001$, partial $\eta^2 = .299$. Composite game scores refer to the total scores for Game 1 and Game 2. However, there was no significant effect of the experimental condition (NST-Game Play vs. Game Play-NST) on composite game score, $F(2,53) = 1.484$, $p = .236$, partial $\eta^2 = .053$. From these results, we can infer that the performance on the game play was not significantly dependent on the sequence in which the students in NST-Game Play condition and Game Play-NST condition played the games. In other words, when the students had the game play had no significant effect on their performance on the two games. Though we observe a slight difference in the mean for NST-Game Play condition as compared to Game Play-NST condition, the difference was proven to be insignificant. For NST-Game Play students, having the game after taking NST test did not afford them to have a significant difference in their performance during the game play when compared Game Play-NST students who had the game play first before sitting for NST.

The ANCOVAs indicated statistically significant effects of mathematics ability on the experimental condition (NST-Game Play vs. Game Play-NST) students' scores in game play;

for Game 1, $F(2,54) = 6.973$, $p = .011$, partial $\eta^2 = .114$; and for Game 2, $F(2,54) = 14.775$, $p < .001$, partial $\eta^2 = .215$. However, the ANCOVAs revealed no statistically significant effects of the experimental condition (NST-Game Play vs. Game Play-NST) on their scores in the game play; for Game 1, $F(2,54) = .692$, $p = .409$, partial $\eta^2 = .013$; and for Game 2, $F(2,54) = 2.175$, $p = .146$, partial $\eta^2 = .039$. We can say that there was no main effect of the sequence of the game play which the students in the NST-Game Play and Game Play-NST conditions experienced on their performances in the two games; Game 1 and Game 2.

Discussion

This study was designed to find out how the different experiences students went through; NST-Game Play condition and Game Play-NST condition, effected their performance in number sense. We also aim to explore the possibility of how engaging students in game play could support the activation of prior and informal knowledge of number sense and hence bridge it to the formal assessment of number sense and vice versa.

The performance of the experimental condition on the NST test scores by strands presented significant differences in their mean scores. This suggests strong evidences that Game Play-NST students performed significantly better than their counterparts in the NST-Game Play condition. This was despite their matching mathematical ability. These results are aligned to a previous study conducted by Saxe & Guberman (1998) on mathematics learning in collective play with third and fourth graders. The post-test which they conducted after the Treasure Hunt game play on arithmetic problem with base-10 blocks, revealed that students who had game play produced more adequate solutions than students who were non-players. In addition, third graders who played with fourth graders demonstrated acquisition of strategic knowledge as they had many opportunities to construct such knowledge in their interactions with the older students. Similar situation could be inferred in the present study in which the small group came play could have facilitated and supported such learning and performance in the formal assessment – NST.

Further investigation on the NST by items that corresponded to the game play presented significant difference in the mean scores between the two conditions. What differentiated the two conditions was the sequence of the game play and NST. Hence it is apparent that the game play had contributed to the difference. Having played the game first before sitting for the NST seemed to enable Game Play-NST students to translate the cognitive gains from the interactions in the game play to the pencil-and-paper NST.

Another interesting result was that the differences in mean percentages for the correct responses for Game 1 and Game 2 in both conditions were found to be insignificant. This implies that the sequence of the game play did not contribute to the correct responses of the students' number sense in the games. In other words, having sat for the NST, which contained items with matching outcomes as those in Game 1 and Game 2, did not enable NST-Game Play students to translate it into the game play. Why did the students in the Game Play-NST performed better than students in the NST-Game Play in the NST? Why students in the NST-Game Play did not perform better in the game play after having sat for the NST test earlier? What were the kinds of processes that Game Play-NST students were engaged in that contributed to their better performance in the NST?

One explanation could be that the game play activity may have afforded a mechanism for activating students' intuitive and informal knowledge of number sense. The diversity of the students' cognitive level, prior knowledge and experience may have contributed largely to the group’s knowledge construction, giving opportunities for students to be engaged in reasoning, questioning, discovering and generalising (Kumpulainen & Wray, 2002). In addition, while sharing their views and perspectives, students can also build on each other’s contribution to re-construct new interpretations and views.

Another explanation could be error correction. During the game play, students were observed giving suggestions and correcting their peers on their choices of numbers on the cards. In situations where a child makes an error and a peer tries to help in clarifying his or her thinking, learning opportunity arises (Yackel, Cobb & Wood, 1991). Such learning of mathematics through errors has been pointed out by Labinowicz (1987) as a potential educational role which children’s errors can play. The child who is assisting his peer could have leverage on his prior knowledge of number sense. Forman & Cazden (1985) have highlighted in their research on the benefits of the acts of verbalisations or giving instructions to peers on the learner who is giving the instructions. In a game play, learning can also occur when children attempt to reach consensus. In such situation, the child needs to explain and justify his point of view or solution to others before the group is able to accept and agree to it

On the contrary, NST-Game Play students did not do better than Game Play-NST students in both games which are parallel to the NST items. This result demonstrates that pencil-and-paper setting does not necessarily afford the learner to explicitly capitalise on their informal or prior knowledge or intuition in number sense. Yet, after experiencing the formal assessment setting, NST-Game Play students were of the same footing as Game Play-NST students in the game play. They could not leverage the experience they had in the formal setting to help them perform better in the game setting. Number sense goes beyond the formal setting and it is insufficient to rely on formal setting in assessing students’ number sense.

Limitations

One of the limitations of this research study is the small sample size of 57 students from one school, and therefore not representative of the larger population. Nevertheless, the data and results in this study may be relevant to students, classes and schools similar to those participating in the study. Also, students in this study are traditionally exposed to learning mathematics through “drill and practice” and through written computation. Although most of the primary 1 teachers find the items in the number sense test interesting, they have also expressed their views that the items are not the typical mathematics the students do in the classroom or in school tests. Therefore, the students who participated in the study were not familiar with the types of items they encountered in the number sense test. Finally, the study lacks an analysis of the small-group interaction during the game play. Though a quantitative and qualitative approach is time consuming and requires extensive data collection and analysis, nevertheless analysis of the small-group interaction would provide a better understanding of the research study. The analyses of the interactions that occur among students during the game play would provide valuable data of in-depth information on the interactions and the difficulties teachers have in implementing such interactions in their mathematics classrooms.

Implications and Conclusion

Students bring along with them a myriad of priors and informal knowledge into the classroom and teachers could tap on these students’ priors and informal knowledge before even embarking on formal teaching. As evident in the results of the study, the informal setting of game play empowers students to use their prior knowledge, explicitly displaying what they know and what they do not. Through game play, the participatory structure of the setting allows for students to be engaged in an interaction to construct the knowledge scaffold by the teacher. Educators need to rethink their pedagogical practices to better understand how meanings and knowledge are could be constructed. The significant effect interaction has on students' learning and performance needs to be brought to the forefront in education especially to educators who miss on the broader goals of education and are results driven.

From this study, we could also draw implications on assessment. Activation of resource such as priors, intuitive and informal knowledge helped students to connect with the formal knowledge and ultimately pushes their performance higher. Formal assessment such as pencil-and-paper test can still play a role in teachers' daily practice in assessing students and may be a good platform to assess learners' competency and learning, but clearly it has its own limitations (Fan, 2011). The experience the students in NST-Game Play condition had in the formal assessment setting in NST in which they could not leverage that experience to help them perform better in the number sense in another setting (game play) further emphasised on limitation of such assessment. As evident in this study, small-group game play setting could afford learners to manifest what they know about number sense. Number sense goes beyond the formal setting and it is insufficient to depend on formal setting in assessing students' number sense. To rely solely on a pen-and-paper test to screen pupils into a programme or stream or to determine a child's level of competency may not be accurate. Other informal platforms such as clinical interviews should be explored.

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