Supporting Teachers in Choosing and Using Challenging Mathematics Tasks

Peter Sullivan
Monash University
<peter.sullivan@monash.edu>

Doug Clarke
Australian Catholic University
<doug.clarke@acu.edu.au>

Debbie Michaels
Monash University
<deborah.michels@monash.edu>

Angela Mornane
Monash University
<angela.mornane@monash.edu>

Anne Roche
Australian Catholic University
<anne.roche@acu.edu.au>

The research reported here was motivated by curiosity about ways of suggesting tasks and activities to teachers that assist them in their planning and which allow them to see the purpose of the suggested tasks. A group of junior secondary teachers involved in a larger project worked through a set of tasks that sought to address the content they were about to teach, and then completed a survey about those tasks. A comparison group also completed the survey. There was consensus across the groups of teachers about the potential of the tasks, they felt they could implement them and that such tasks were preferable to comparable textbook exercises. There are implications for both teacher educators, and for resource developers.

Making Suggestions about Mathematics Tasks to Teachers

It can be assumed that teacher educators and others who seek to support teachers in improving the mathematics learning of their students focus on developing teachers' knowledge. This knowledge includes knowledge about curriculum, about pedagogy, about students and about mathematics. Teacher educators also seek to develop positive teachers' attitudes towards mathematics and its teaching, ways for teachers to elicit positive motivation in students and to foster in them an orientation to overcoming barriers and mastering content. The following is a report of an aspect of a larger project[1] that is focusing on the latter two of these aspects. The project is encouraging teachers to present students with challenging tasks and to describe the actions that teachers might take when students are working on those challenging tasks to maximise student persistence. We use the term *persistence* to describe the category of student actions that include concentrating, applying themselves, believing that they can succeed, and making an effort to learn. We term the tasks that are likely to foster such actions *challenging*, in that they allow the possibility of sustained thinking, decision making and some risk taking by the students.

The rationale for the project is based on the widespread generic advice that teachers should pose challenging tasks as a way of engaging students (see, e.g., City, Elmore, Fiarman, & Teitel, 2009). It is also founded on a belief that learning mathematics is not about students learning a large number of routines and small steps but about them connecting concepts, linking ideas together, planning multiple steps, and thinking about their mathematical

[1] The EPMC project is funded through an ARC Discovery Project (DP110101027) and is a collaboration between the authors and their institutions. The views expressed are those of the authors. We acknowledge the generous participation of the project schools, McKinnon Secondary College and St Peter the Apostle School, Hoppers Crossing.

In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), Mathematics education: Expanding horizons (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia). Singapore: MERGA.

knowledge in new ways (Hiebert & Carpenter, 1992).

One aspect of the project is focusing on the nature of support that teachers might need for implementing challenging tasks and in particular finding ways to offer teachers ideas about tasks that have the potential to be productively challenging.

A framework that guided our thinking about tasks was proposed by Stein, Grover, and Henningsen (1996) who argued that the consideration of classroom tasks by teachers goes from the …

Mathematical task as presented in instructional materials

> which, influenced by the teacher goals, their subject matter knowledge, and their knowledge of students, informs …

… mathematical task as set up by the teacher in the classroom

> which, influenced by classroom norms, task conditions, teacher instructional habits and dispositions, and students learning habits and dispositions, influences …

… mathematical task as implemented by students

> which creates the potential for …

… students' learning.

In this framework, the first step for the teacher is to identify some suitable tasks, whether from instructional materials or from some other source. As Sullivan, Doug Clarke and Barbara Clarke (2009) found, one of the major challenges that teachers experience is finding the time for detailed planning. Teachers have limited time for examining texts and other teacher resources, or trawling the internet for tasks and ideas, or even scouring their own files. It is likely to be helpful for teachers if support is offered in a format that assists them in identifying tasks with potential to foster student engagement and learning efficiently, or in the form of suggestions of tasks for which they see the purpose and potential.

While teachers may need support with expanding their pedagogical repertoire, and even in developing key aspects of their mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008), suggestions of suitable tasks and activities in a form in which the potential of the task is clear would seem helpful. The aspect of our project that is exploring the nature of that support is the focus of this paper.

A Model of Influences on Teacher Actions

The model of influences (Figure 1) that guides the design of the overall project was informed by Clark and Peterson (1986) and used successfully by Sullivan et al. (2009). It includes four sets of background variables: teacher knowledge; teacher attitudes, beliefs, and self-goals; situational and other constraints; and teachers' intentions. The first three of these influence each other and are presumed to inform decisions teachers make when selecting and using tasks.

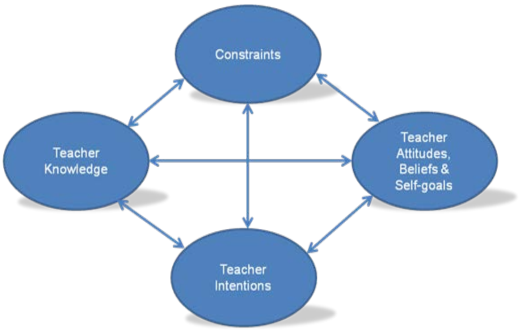

Figure 1. Influences on teacher actions

This model also informed the data collection for this aspect of the project. In the model, it is assumed that the teacher's knowledge influences the choice of task. This knowledge might be about the mathematics that is needed to appreciate the potential of the task (see, e.g., Ball & Bass, 2000; Chick, 2007) or about the pedagogical actions needed to implement the task (see Hill et al., 2008). This knowledge informs the articulation of clear mathematical goals for the students and the selection of tasks at the appropriate level of challenge.

The model further recognizes that a fundamental assumption is that teachers' beliefs, attitudes, and self-goals influence the extent to which they maintain the challenge in tasks that they pose (e.g., Middleton, 1999). Sullivan et al. (2009) reported that their project teachers seemed reluctant to pose challenging tasks for fear of negative student reactions. Stein et al. (1996) and Tzur (2008) also described similar responses from teachers when implementing tasks. Related factors that have been identified as influential are teachers' beliefs about mathematics and learning mathematics (Thompson, 1992) and their attitudes towards mathematics generally (Hannula, 2004). Another relevant factor is self-goals theory (Dweck, 2000) which refers to the extent to which teachers see their own and students' ability as incremental and amenable to improvement through effort, for example. These variables manifest themselves in the tasks posed and the forms of affirmation that teachers use.

The third aspect refers to constraints experienced by teachers when seeking to challenge students. We infer that there are powerful constraints since it appears that, in many Australian classes, the tasks used are often routine and unlikely to be challenging (see Hollingsworth, Locan, & McCrae, 2003). Some of the constraints on teachers include the diversity in students' cultural backgrounds, language fluency and readiness to learn (see Delpit, 1988), the ways in which the structure of classrooms and common responses of students to schooling restrict teacher choices (see Doyle, 1986), and that the skill levels of the students inhibit their participation in challenging tasks (see Stein & Lane, 1996). In addition, we anticipate that access to resources, available planning time and fear of negative student reactions also act as constraints on the teachers.

The above sets of variables inform teachers' intentions, meaning what they plan to do in their lessons. The research questions informing the data collection of this aspect of the project are as follows. With respect to the tasks suggested by us, do teachers:

i. see the mathematical and pedagogical potential?
ii. anticipate constraints in using these tasks?
iii. consider the tasks to have more potential than texts or other resources to which the teachers have access?
iv. need to be familiar with the implied pedagogies? and
v. have completed the task itself?

The intention is that the results can inform future support for, and advice to, mathematics teachers.

The Context and Methods of the Data Collection

In the overall project, as well as in this aspect, we are adopting a design research approach which "attempts to support arguments constructed around the results of active innovation and intervention in classrooms" (Kelly, 2003, p. 3). In particular, we are intervening to prompt the use of tasks that are more challenging than those commonly used by many teachers. We ensure that we are suggesting tasks that align with the teachers' current topics to avoid the constraint that teachers often experience due to what they see are the imperatives for content coverage. We encourage our project teachers to work through tasks prior to using them in their classrooms, and we do this in teacher learning sessions.

We present the tasks to the teachers in sequences, generally of increasing difficulty. This is partly because it is hard to anticipate what tasks will be challenging for a particular class, so the sequences allow teachers to make that decision. It is also partly because, for the tasks to be accessible yet challenging, some prior experiences are often needed. The earlier tasks in the sequences are not necessarily intended to be challenging. While we do make various pedagogical suggestions, the planning of the lessons is left to the teachers.

The teachers at our secondary level project school had suggested that they were about to teach the topic of surface area and volume to their year 8 classes. We proposed that the goals of the tasks that we suggested were to introduce students to: the nature of volume of prisms and cylinders (area of the end × length); surface area of prisms (as the total area of the faces); efficient methods of calculating volume and surface area of rectangular prisms; and ways in which surface area and volume are different. The particular sequence of tasks that was the basis of the data collection reported below was as follows:

Task 1: This is a rectangular prism made from cubes. What is the volume of this prism? What is the surface area? (a photo of a prism made from cubes was presented with the task)

Task 2: A set of 36 cubes is arranged to form a rectangular prism. What might be the dimensions of such prisms? What is the surface area of each of your prisms?

Task 3: The volume of a rectangular prism is 600 cm^3. What might be the surface area of the prism?

Task 4: A rectangular prism is made from cubes. It has a surface area of 22 square units. Describe what the rectangular prism might look like?

Task 5: The surface area of a closed rectangular prism is 94 sq cm. What might be the dimensions of the prism?

Task 6: (The students were presented with a photo of a "child' made with cubes) What is the volume and surface area of the child? Imagine an adult made with twice as many cubes. What is the volume and surface area of the adult? Imagine a giant made with three times as many cubes. What is the volume and surface area of the giant? What can you say about the ratio of surface area to volume is each case?

Task 7: Design a 3L milk container that will fit into the refrigerator

Task 8: What is the volume and surface area of a Toblerone box? I want to wrap up some gifts made up of Toblerones. What is the surface area and volume of a gift made out of 3 Toblerones? … out of 4 Toblerones? What other designs could you make? Find their surface area and volume.

Task 9: We can make open top boxes out of a sheet of paper by cutting squares from the corners and then folding up the sides. If a sheet of paper is 200 mm by 300 mm, what might be the volume of some boxes that we can create? (a relevant diagram was presented with the task)

Tasks 1 to 5 were intended to be a sequence using open-ended approaches to allow students choice in approach and level of complexity and to contrast surface area and volume. Tasks 6 to 9 were intended to be contextualised tasks that connect the study of surface area and volume (capacity) to students' experience. The overall mathematical focus should be clear to the project teachers since this was the topic they were about to teach, the tasks are simply and succinctly phrased, and it was intended that the sequence of the first five tasks would be clear to teachers, as would the practical potential of the latter tasks.

The project is intensive and so we are working with a small group of teachers. For the project teachers, we worked through each of these tasks in a teacher professional learning session, and then invited them to complete the relevant aspect of a survey after each task. To expand the number of responses, another group of junior secondary teachers who were not part of our project were also presented with the tasks (on a powerpoint) and the same survey questions. This comparison group did not work through the tasks before completing the survey but did have an opportunity to discuss them. Both groups were secondary mathematics teachers and seemed to have little difficulty with the required content.

Recognising that the project teachers may have been oriented to giving us the responses they anticipated we wanted to see, the other group of teachers were much less so. The tasks in the survey were not directly connected to the content of the professional learning sessions, and their anonymity would certainly have been protected in the larger group. The responses of this comparison can be taken as an accurate reflection of their opinions (as far as that is possible). The project teachers' responses can also be taken as an indication of their actual opinions in that the teachers subsequently choose some tasks for lessons that were observed by the project team indicating that they were comfortable enough with the tasks to use them in that public setting (see Sullivan, Clarke, Cheeseman, Michels, Middleton, Mornane, & Roche, 2011, for a report on those observations).

Note that the collection of responses such as those reported below is time consuming for teachers in that it required consideration of at least nine specific tasks, including working through them in the case of the project teachers, and for the teachers to respond to around 10 items for each of these tasks. Even though the total number of responses is small, it still represents a substantial and intensive data collection exercise.

Results

The results reported below the responses to the survey about the nine tasks presented in Figure 1. Table 1 presents the profile of responses to task 1 above of all teachers combined (31 teachers altogether), with the number of responses per category from the six project teachers in brackets. This table is presented to indicate the range of responses from both groups of teachers. The teachers were invited to respond to the prompts in the left hand column using the code 1 for strongly disagree, 2 for disagree, 3 for agree, and 4 for strongly agree.

Table 1

Responses of Teachers to Survey Prompts Related to Task 1 (above)

	SD	D	A	SA	Mean
a. I would use the task and I would let the students work out how to do it for themselves	0	3	12 (2)	10 (4)	3.3
b. The task would be organisationally difficult with my class	9 (3)	13 (2)	2 (1)	0	1.7
c. The task would provide experiences to develop important knowledge about these concepts	0	0	12	13 (6)	3.5
d. The task needs more information for the students before it could be used	7 (4)	13 (1)	4 (1)	1	2.0
e. The task would result in better learning about these concepts than the ones in the text book	0	1 (1)	17 (4)	6 (1)	3.2
f. I know a bunch of tasks on this topic that are more interesting than this one	2	22 (5)	1 (1)	0	2.0

Note that prompts a and c refer to research question *i*, prompts b and d seek data on research question *ii*, and prompts e and f address question *iii*. The overwhelming majority of the teachers felt that they would use this task as we intend, they felt able to use it, they recognized the potential of the task, did not feel more information was needed, that the task would be better than text exercises, and is more interesting than other similar tasks that they know.

Even though there are clear trends in the responses, there was also a spread indicating that teachers respond in different ways to suggestions of tasks and there is no such thing as a perfect task. In terms of providing suggestions, it is therefore important that teachers be offered options and choices.

In comparing the responses of the project teachers with the other teachers, there are some interesting results. Recognising that the number of project teachers is small, they were more likely to strongly agree that the task would provide important experiences for the students that the other teachers (the main figure in the table is the response of all teachers). The project teachers also were more likely strongly disagree with the proposition that the task needed more information for the students (two thirds of the project teachers vs one fifth of the others). It seems that having worked through the task in a professional learning session helps to convince teachers that there is enough information for the task to be completed and so resist the related student demands for hints that are not needed. The project teachers also seemed more likely to strongly agree that they would let the students work out how to do the problem by themselves (two thirds vs one third). This suggests that it is helpful for teachers to work through tasks before using them. It seems that having worked through the tasks and having the opportunity to discuss and share possible solutions with colleagues helped the teachers to see the potential in the tasks. It seems that working through challenging tasks with planning teams before using them in their classrooms will assist teachers in anticipating the potential of tasks.

Recognising that there were some slight differences in the strength of the responses of the two groups of teachers both to the prompts listed in Table 1 and in responses to the other tasks, the directions of the responses were the same. For simplicity and space reasons, responses of both groups to the other tasks are combined and means presented to indicate the overall direction of the responses.

Table 2 presents the mean of the responses to the same six prompts for the other tasks. Note that a mean of 2 indicates overall disagreement with the proposition, and less than 2 is stronger disagreement. A mean of 3 indicates overall agreement with the prompt and a score of more than 3 is stronger agreement. Between 2 and 3 can be considered balanced or neutral.

As with Table 1, it seems that for each of the tasks, the teachers would be willing to allow students to work through the tasks by themselves, they do not consider the organisational challenges to be significant, they recognise the potential of the tasks for developing conceptual knowledge, they think the information in the task is enough, and that the tasks offer better learning opportunities than those in texts or the ones they know already.

Table 2
Mean Responses of Both Groups of Teachers to Tasks 2 to 9 (from Figure 1)

	Task 2	Task 3	Task 4	Task 5	Task 6	Task 7	Task 8	Task 9
I would use the task and I would let the students work out how to do it for themselves	2.9	2.8	2.8	2.9	2.6	3.1	2.8	3.0
The task would be organisationally difficult with my class	1.9	2.0	2.2	2.0	2.0	2.0	1.9	1.9
The task would provide experiences to develop important knowledge about these concepts	3.2	3.4	3.3	3.5	3.4	3.3	3.3	3.4
The task needs more information for the students before it could be used	2.2	2.3	2.4	2.1	2.8	2.5	2.4	2.2
The task would result in better learning about these concepts than the ones in the text book	3.2	3.2	3.2	3.2	3.2	3.3	3.1	3.4
I know a bunch of tasks on this topic that are more interesting than this one	2.1	2.0	1.8	2.0	1.8	2.0	1.8	1.8

There is surprising consistency in the responses across the tasks. To take two examples, task 2 and task 6 are very different in what is required of teachers and students. While the former, although open-ended with a variety of possible responses, provides quite clear indication to students of what they should do, the latter requires substantial judgment and inference by the students, and presumably would be more difficult organisationally, if not mathematically. Noting that the more teachers felt task 6 needed more information, otherwise the teachers’ responses are similar to the tasks. It is hard to know whether the teachers consider the former more difficult pedagogically than we do, or whether they consider the latter to be less difficult for the students than we think it would be.

Conclusion

As part of a larger project that is suggesting examples of challenging tasks to teachers so that we can study the impact of those tasks on student learning, we explored the ways that teachers interpreted the tasks that we suggested. After working through a particular set of tasks as part of a professional learning session, our junior secondary project teachers completed a survey on aspects of their responses to those tasks. A comparative group of similar teachers also completed the survey but did not work through the tasks nor did they participate in professional learning on this content.

In terms of the research questions presented above, it seems that both groups of teachers saw the mathematical and pedagogical potential of the tasks, they did not feel constrained by the ideas, and they felt the tasks had more potential than those that they find in texts. While it is not critical that teachers have worked through tasks for them to see the potential in the tasks, this may help teachers to avoid providing too much information and may also allow deeper consideration of the related pedagogies.

We feel that succinct presentation of tasks fits with the time pressures that teachers are experiencing, and also that task suggestions need to align with what the teachers are planning to teach. There are clear advantages in offering tasks to teachers in sequences in that teachers can start the sequence with a task that is at the appropriate level of difficulty for their class. Perhaps making a decision on the starting point for the their sequence can encourage teachers to maintain the challenge in the task.

It also seems that while there were strong trends, there was also variability in the responses of the teachers, indicating that teachers have different responses to different tasks. It would be helpful for teacher educators and those who propose resources for teachers to be aware of the need for variety in tasks suggested for teacher use.

References

Ball, D., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83-104). Westport, CT: Ablex.

Chick, H. (2007). Teaching and learning by example. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Vol. 1, pp. 3-21). Adelaide: MERGA.

City, E. A., Elmore, R. F., Fiarman, S. E., & Teitel, L. (2009). *Instructional rounds in education.* Mass.: Harvard Educational Press.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought processes. In M. C. Wittrock (Ed.) *Handbook of research on teaching* (pp. 255–296). New York: Macmillan.

Delpit, L. (1988). The silenced dialogue: Power and pedagogy in educating other people's children. *Harvard Educational Review, 58*(3), 280–298.

Doyle, W. (1986). Classroom organisation and management. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (pp. 392–431). New York: Macmillan.

Dweck, C. S. (2000). *Self theories: Their role in motivation, personality, and development*. Philadelphia: Psychology Press.

Hannula, M. (2004). *Affect in mathematical thinking and learning.* Turku: Turun Yliopisto.

Hiebert, J., & Carpenter, T. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research in mathematics teaching and learning* (pp. 65-97). New York: Macmillan.

Hill, H., Ball, D., & Schilling, S. (2008). Unpacking pedagogical content knowledge: Conceptualising and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372-400.

Hollingsworth, H., Lokan, J., & McCrae, B. (2003). *Teaching mathematics in Australia: Results from the TIMSS video study* (TIMSS Australia Monograph No. 5). Camberwell, Victoria: Australian Council for Educational Research.

Kelly, A. (2003). Research as design. *Educational Researcher, 32*(1), 3-4.

Middleton**,** J. A. (1999). Curricular influences on the motivational beliefs and practice of two middle school mathematics teachers: A follow-up study. *Journal for Research in Mathematics Education*, *30*(3), 349-358.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455–488.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason and analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2*(1), 50–80.

Sullivan, P., Clarke, D, & Clarke, B. (2009). Converting mathematics tasks to learning opportunities: An important aspect of knowledge for mathematics teaching, *Mathematics Education Research Journal, 21(1), 85-105.*

Sullivan, P., Cheeseman, J., Michels, D., Mornane, A., Clarke, D., Roche, A., & Middleton, J. (2011). Challenging mathematics tasks: What they are and how to use them. In L. Bragg (Ed.), *Maths is multi-dimensional* (pp. 33-46). Melbourne: Mathematical Association of Victoria.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.

Tzur, R. (2008). A researcher perplexity: Why do mathematical tasks undergo metamorphosis in teacher hands? In O. Figuras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Proceedings of the 32nd Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol.1, pp. 139 - 147). Morelia: PME.