# Students' Summaries of Mathematical Lectures: Comparing the Discourse of Students with the Discourse of Lectures

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This study focuses on a distinction between process- and object-oriented discourses when characterising the discourse of university students' summaries of lectures and examining connections between students' discourse and the discourse of lectures. Results show that students' discourse in general tends to be process-oriented, by their use of active verbs and little use of nominalisations. Students' summaries of process-oriented lectures also tend to be more process-oriented, but the differences between individual students are larger than differences caused by variations of the discourse in the lectures.

Learning, or at least one aspect of learning, can be seen as enculturation into a specific type of discourse. This process can occur through explicit teaching about properties of the discourse, for example as Mercer (1996) describes the teacher's role to give instructions about ways of questioning and arguing. However, the process can also occur through more implicit manners, by seeing a teacher's discourse as a model for students to follow, which for example could be one component when Rodd (2003, p. 15) describes lectures as important for "inducting students into their university's mathematical community".

In this paper, focus is on implicit aspects of mathematical discourse by examining lectures with varying properties of the discourse, which are not explicitly highlighted for the students, and studying students' summaries of these lectures in order to compare the discourse of students and the discourse of the lectures.

What are the central properties of mathematical discourse that students need to adapt to? This is a difficult question since it seems impossible to generally characterize mathematical discourse. For example, Burton and Morgan (2000) show that there is large diversity in the writing in mathematical research articles. The property of mathematical discourse of interest in this paper is a distinction between process- and object-oriented discourses. The reason to focus on aspects of processes and objects is that this distinction can be relevant in several different ways for mathematical discourse, in particular from the following two perspectives: First, when focusing on mathematical learning, aspects of processes and objects are central in several different frameworks (see Tall, Thomas, Davis, Gray, & Simpson, 2000), where a central aspect of learning is the process of objectification or reification (i.e., to create an object from something first seen as a process). Second, when focusing on the readability of texts, the aspect of nominalisation is sometimes discussed, that is, the use of nouns created from other types of words, as in the noun *description*, created from the verb *describe*. Empirical results sometimes show a negative effect on recall (Spyridakis & Isakson, 1998) and on reading speed (Bostian, 1983) for texts that have a more nominal style (i.e., include more nominalisations).

The first perspective above could be seen as focusing on content while the second on form, but it is not necessarily that easy to separate form from content. For example, as Morgan (1999, p. 56) notes concerning: (a) the use of passive voice, that it is "not solely a choice about form but is also a choice about whether to represent or to obscure the role of the agent in the process", and (b) nominalisations, that it is "not simply a different way of expressing the same meaning but actually allows the writer to say new things". In this paper,

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I do not focus on differentiating between form and content but I examine more generally properties of discourse, regarding aspects of processes and objects.

## Process- and Object-oriented Discourses

A framework of process- and object-oriented discourses (Österholm, 2011a) is used in this study as the basis for the creation of different lectures and for analysing students' summaries of the lectures. *Four aspects* are used that each describes differing degrees of process- and object-oriented discourse. These aspects are labelled and shortly described based on an imaginary shift from process-oriented towards object-oriented discourse:

1. *Direct nominalisation*: A verb is replaced with its corresponding noun, for example when "we use the same symbols" is replaced with "the usage of symbols is the same".
2. *Indirect nominalisation*: A verb is replaced with its corresponding adjective, for example when "the series converges" is replaced with "the series is convergent". This aspect is seen as a shift from process-orientation towards object-orientation since the adjective puts focus on a (static) property of an object.
3. *Voice*: An active verb is replaced with its passive counterpart, for example when "we can calculate this" is replaced with "this can be calculated". This aspect is seen as a shift from process-orientation towards object-orientation since voice is "a choice about whether to represent or to obscure the role of the agent in the process" (Morgan, 1999, p. 56).
4. *Structuralisation*: A process/procedural type of verb is replaced with a structural type of verb (i.e., a form of "to be"), for example when "we do the same as before" is replaced with "it is the same as before". This change makes an activity/event into a static situation/property, thus seen as a shift from process-orientation towards object-orientation.

It is acknowledged that there can be also other aspects that can be seen as making the discourse more or less process- or object-oriented, but this study focuses on these four aspects, and a plan is to include more aspects in future studies.

## Purpose

The study presented in this paper is one part of a pilot study that takes a student perspective on different types of mathematical presentations. The data collection procedure for the whole pilot study included several tasks and activities for students, with the purpose to function as a pilot study for several different foci, for example to examine students' quantitative evaluations of different types of presentations (Österholm, 2011a) and to examine their comments about oral and written presentations (Österholm, 2011b). However, for the purpose of the present paper, focus is on the discourse of students' descriptions of the content of oral presentations (i.e., lectures).

The main purpose of the study presented in this paper is to increase our knowledge about relationships between the mathematical discourse of students and the mathematical discourse of teaching situations, in particular lectures.

This topic is of interest for several reasons, some already highlighted in the introduction. A mismatch between the discourse of students and the discourse of teaching situations can be seen as a potential source of lack of communication and hindrance for students' learning, in particular if properties are not of mathematical necessity, since this can "be one of the sources of disaffection for some students" (Morgan, 1999, p. 55). This issue can be seen as a

generalisation of the notion of readability. Furthermore, it is of interest to examine if and how students adapt to the discourse in teaching situations, since such a (potential for) change of discourse can be seen as an essential part of learning.

More specifically, the present study focuses on the following two research questions:

1. When describing mathematical lectures, is the oral discourse of students mostly process-oriented or object-oriented?
2. Is the discourse of students' oral descriptions of mathematical lectures of the same kind, regarding process- or object-orientation, as the discourse of the lecture?

The first research question is of interest in relation to different types of perspectives and results: On the one hand, scientific discourse in general, but perhaps in particular mathematical discourse, seems to be more object-oriented (Bostian, 1983; Morgan, 1999). Perhaps students have adapted to such properties of discourse, in particular at university level, which is examined in this study. On the other hand, (a) there is evidence that students mainly need to focus on more procedural aspects in mathematics, through imitative reasoning, both in school (Palm, Boesen, & Lithner, 2011) and also at university level (Bergqvist, 2007), which could be seen as connected to process-orientation of discourse, and (b) oral discourse is generally less object-oriented than written discourse, at least when it comes to nominalisations (Einarsson, 1978). These two issues (i.e., a and b) could make it reasonable that students' discourse is more process-oriented in general.

The second research question is of interest in relation to some results I have previously presented (Österholm, 2011a). These results are based on analyses of different parts of the same data material used in the present study, and show that students seem to react to the differences in discourse examined here, regarding process- and object-orientation. In particular, when quantitatively evaluating different presentations, there is a tendency among the students that "process-oriented discourse was preferred for oral presentations, [while] object-oriented discourse was preferred for written presentations" (Österholm, 2011a, p. 367). Therefore, it is of interest to examine whether these types of differences in discourse also affect the students in other ways, in particular concerning their own discourse.

## Method

### *Data Collection*

A total of 22 Swedish university students participated in this study, all enrolled in the same calculus course. The students worked in pairs, and first one student watched and listened to a pre-recorded lecture (16-17 minutes long). The student could take notes, in order to imitate an ordinary lecture. After listening to the lecture, the student described the content of the lecture to the other student. Instructions were given that the student who describes the lecture should not show her/his notes but describe as much as possible orally. However, it was allowed to write symbols or draw pictures during the description.

Thereafter, the procedure was repeated but with changed roles, so that the student who first described a lecture then listened to the other student's description. The two lectures covered different content. One lecture was about sequences and included: connections between sequences and functions, different ways to describe sequences (with explicit and recursive formulas), convergence of sequences, and examples of calculating limits for sequences described with explicit and recursive formulas. The other lecture was about Maclaurin polynomials and included: the idea of polynomials as approximations to complicated functions, a derivation of the Maclaurin coefficients for a polynomial of degree five, and an example of calculating an approximate value of the constant *e* using a Maclaurin

polynomial for $e^x$. The content of both lectures were scheduled to be covered in the students' calculus course within a few weeks after participating in this study.

The wordings of all lectures were created by first recording and transcribing a spontaneous oral presentation of each topic. Two written versions of each topic were then created: one process-oriented and one object-oriented, where formulations were altered according to the four aspects described above. From these written versions, oral presentations were then recorded, which were used as the lectures in this study. The same person (the author) was being recorded for all different lectures.

### *Data Analysis*

Recordings of students' descriptions were transcribed for further analysis. The focus in this study is on the students' descriptions of the mathematical content presented in the lectures. Therefore, the following parts of the descriptions and conversations between students are excluded from analysis: (a) utterances from the student who are listening to the other student and (b) utterances that only address some kind of level of understanding (e.g., "I think this is the way it is supposed to be") or some practical aspect of either the lecture or own activity (e.g., "he described this really fast" or "let me check my notes").

Each verb, noun, and adjective is analysed by deciding if the word is in a form that is seen as process-oriented (then coded P) or object-oriented (then coded O) according any of the *four aspects*. For example, if the discourse includes the phrase "you can explain this by noting that", this instance of the word *explain* is coded with a P since a possible reformulation is "an explanation of this is noting that", which makes the phrase more object-oriented according to the aspect of direct nominalisation. In this analysis, it is essential that a reformulation is possible, within the specific context where the word is used, which changes the orientation according to a specific aspect, for it to be coded as either P or O. This demand of a potential reformulation is used in order to focus on a type of "choice" that is made between different ways to express oneself, between being more process- or object-oriented. This type of analysis reflects the process when creating the different lectures, where such reformulations were done. Thereby, a direct comparison between properties of students' discourse and of the discourse of the lectures is more reasonable.

A quantitative characterisation of a certain discourse is created by calculating the proportion of occasions coded with a P and with an O, corresponding to how process- and object-oriented the discourse is respectively. Based on this type of quantitative characterisation, each pre-recorded lecture used in this study is either 100% process-oriented or 100% object-oriented. By making this type of quantitative characterisation of students' discourses, it is possible to examine if the discourse is mostly process-oriented or object-oriented (research question 1) and also to compare the discourses between students who listened to different versions of the lectures (research question 2).

This type of characterisation is used in two different ways, in order to give as complete answers to the research questions as possible. First, all coded occasions (all four aspects) are included at once by creating a one-dimensional scale describing a discourse as more process-oriented or more object-oriented. Thereafter, the different aspects are distinguished, in order to examine which aspects are mostly affecting the process/object-orientation of students' discourse. However, it is not always possible to distinguish between all four aspects, since focus in the analysis is on whether a specific part of the discourse can be reformulated into becoming more object-oriented or more process-oriented, and in some situations such reformulations can be done in several ways, which are instances of different aspects. For example, in the example given above, with *explanation* as a direct nominalisation of *explain*, a reformulation could also have been "this can be explained by noting

that", which makes the phrase more object-oriented according to the aspect of voice, instead of according to the aspect of direct nominalisation. A phrase using the word *explain* can therefore not (always) be coded with a certain aspect, but only as process-oriented. However, it is always possible to code occasions that are more object-oriented with a certain aspect, since the four aspects involve different types of words for object-orientation (nouns, adjectives, verbs in passive form, and forms of *to be*). Furthermore, (some parts of) the aspect of structuralisation can always be distinguished from the other aspects. In particular, two very frequent words are instances of the aspect of structuralisation: forms of *to become* (process-oriented; "att bli" in Swedish) and forms of *to be* (object-oriented; "att vara" in Swedish). In fact, these are the only kind of instances coded as structuralisation in the present study. Therefore, in the present study, instances of structuralisation can always be distinguished from instances of the other aspects.

*Statistical analyses and issues of reliability.* It would be ideal to characterise the discourse of each individual student, and then to analyse the distribution of these characterisations, in particular when comparing different groups of students. However, due to the pilot study character of this study, the reliability of this type of analysis will be limited, for two reasons. First, students describe quite a short lecture, and their descriptions vary in length between approximately 5 to 15 minutes (and between 19 and 155 occasions are coded for each student). This variation makes at least the shorter descriptions difficult to characterise quantitatively in a reliable manner. Second, only 22 students participated, which can make at least comparisons between subgroups unreliable. Therefore, as a complement to the type of analysis that characterises individual students, an analysis is also performed where the characterisation is done at group level, focusing on groups of students who have listened to different lectures. This analysis consists of adding up all coded occasions within a group of students, and then calculating a quantitative characterisation of this group based on all included occasions. The power of this analysis is not dependent on the number of students but on the number of coded occasions, which are many more, and this type of analysis thus constitutes a better basis for reliable statistical analysis.

Non-parametric methods are used in all statistical analyses, where 0.05 is used as the level of statistical significance. See results section for description of specific methods.

## Results

The discourse of individual students tends to be more process-oriented than object-oriented, since all box plots in Figure 1 lie somewhat above 50%. A One-Sample Wilcoxon Signed Rank Test including all students (N = 22) also shows that the median of the fraction of process-oriented occasions (60.9%) is significantly different from 50% ($p = 0.000$). Mann-Whitney U Tests show that differences between groups of students who listened to different lectures are not statistically significant, neither when comparing lectures with different discourse ($p = 0.243$) nor lectures with different content ($p = 0.748$).

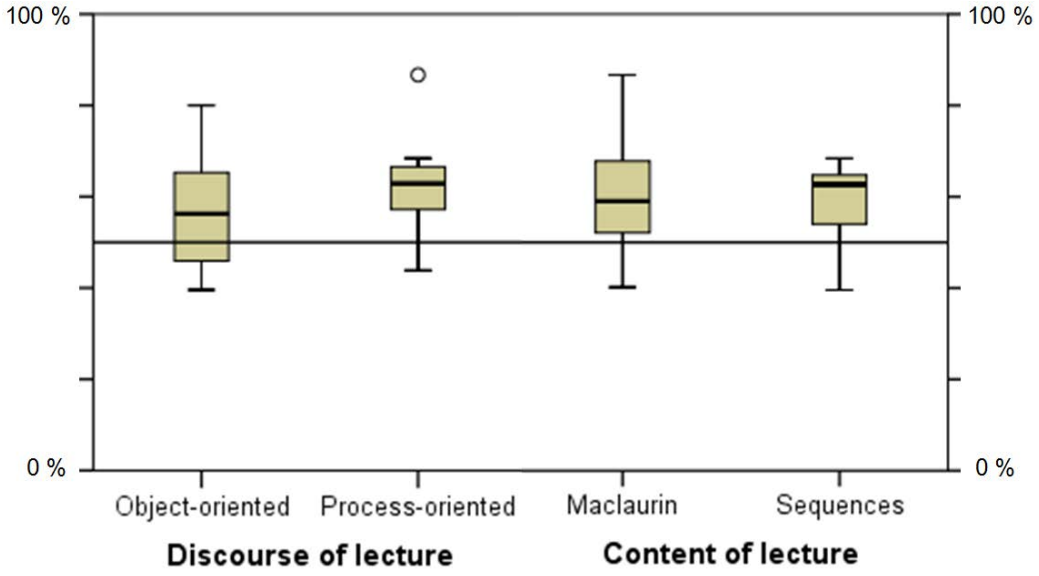


*Figure 1.* Fraction of process-oriented occasions in individual students' descriptions of different types of lectures.

When not distinguishing between different students, the results in Table 1 below also show that it is more common for the discourse of students to be process-oriented, since 58.2% of all coded occasions are process-oriented. A Binomial Test shows that this fraction is significantly different from 50% ($p$ = 0.000). Furthermore, Pearson Chi Square Tests show that descriptions of process-oriented lectures tend to be more process-oriented than descriptions of object-oriented lectures ($p$ = 0.002; 61.5% compared to 54.5%) while in general there is no significant difference in the discourse when students describe lectures with different content ($p$ = 0.458; 59% and 57.3%).

Table 1
*Type of Discourse in Groups of Students' Descriptions of Different Types of Lectures*

| | Occasions in students' descriptions | | Fraction of process-oriented occasions |
|---|---|---|---|
| Lecture | Object-oriented | Process-oriented | |
| Object-oriented | 406 (114 & 292) | 487 (414 & 73) | 54.5% (78.4% & 20.0%) |
| Process-oriented | 372 (94 & 278) | 595 (481 & 114) | 61.5% (83.7% & 29.1%) |
| Maclaurin | 397 (116 & 281) | 571 (449 & 122) | 59.0% (79.5% & 30.3%) |
| Sequences | 381 (92 & 289) | 511 (446 & 65) | 57.3% (82.9% & 18.4%) |
| All | 778 (208 & 570) | 1082 (895 & 187) | 58.2% (81.1% & 24.7%) |

*Note.* The two numbers in parentheses refer to occasions of different aspects of process- and object-orientation: the first number refers to the first three aspects and the second number refers to the fourth aspect.

Among all 778 object-oriented occasions, 93 are instances of the first aspect (direct nominalisation), 63 of the second aspect (indirect nominalisation), 52 of the third aspect (voice), and 570 of the fourth aspect (structuralisation). The main distinction can therefore be made between the first three aspects on the one hand and the fourth aspect on the other hand, for which data is given within parentheses in Table 1. Binomial Tests show that

occasions in the first three aspects are more process-oriented (81.1% is significantly different from 50%; $p = 0.000$) while occasions in the fourth aspect are more object-oriented (24.7% is significantly different from 50%; $p = 0.000$). When comparing different lectures, Pearson Chi Square Tests show that:

- Descriptions of process-oriented lectures tend to be more process-oriented than descriptions of object-oriented lectures, both for the first three aspects ($p = 0.026$; 83.7% compared to 78.4%) and also for the fourth aspect ($p = 0.004$; 29.1% compared to 20.0%).
- Descriptions of Maclaurin lectures tend to be more process-oriented than descriptions of lectures about sequences, but only for the fourth aspect ($p = 0.000$; 30.3% compared to 18.4%) while no significant difference exists for the first three aspects ($p = 0.145$; 79.5% and 82.9%).

## Conclusions

An answer to the first research question, about characterising students' discourse, includes two parts. First, students' discourse when describing the lectures is in general more process-oriented than object-oriented. However, the variation among individuals is fairly large, where the maximum difference between students is of magnitude 30-40% (see Figure 1) on the scale used in this study. Second, although there is a general tendency for students' discourse to be more process-oriented, there is a difference between different aspects of process- and object-orientation. In particular, the process-orientation of their discourse mainly comes from relatively little use of nominalisations (direct and indirect) and passive voice, while the aspect of structuralisation is more object-oriented in students' discourse, by mainly describing what *is* and not what *becomes*. These results are somewhat contradictory, since students' discourse seems to focus on what happens or what you do (by using mostly active verbs and less nominalisations) but at the same time does not seem to focus on what *becomes* from such events or activities but on what *is* already there. Perhaps this distinction comes from describing your own activities on the one hand (what is being done) and describing the mathematical content on the other hand (what is there and what is true[1]). A further analysis of the content of students' descriptions could test this hypothesis.

An answer to the second research question, about connections between students' discourse and properties of the lectures, includes two parts. First, students' discourse tends to follow the discourse of the lecture, regarding the property of process- or object-orientation, also when examining specific aspects of the orientation of discourse. However, this result only appears when not distinguishing between individual students and does not appear when characterising individual students, which might be due to unreliable measures as discussed in the methods section. In addition, the result does not show a *complete* shift in students' discourse when listening to lectures with different discourse but the shift involves 5-9% of the discourse (see Table 1) when examining groups of students and according to the scale used in this study (this is of similar magnitude as the difference between medians shown in Figure 1). Second, an unexpected difference between lectures with different content is observed, but not as clear as for lectures with different discourse. A significant difference is only observed for the aspect of structuralisation, where students' descriptions of the Maclaurin lecture tend to be more process-oriented when compared with the lecture

[1] This can also refer to a more relational, and not operational, perspective on mathematical symbols and concepts, which for example has been discussed and examined for the equal sign (e.g., see Kieran, 1981).

about sequences. Similarly as noted above, a further analysis of the content of students' descriptions could perhaps explain this difference.

In summary, the variation when looking at the process- and object-orientation of discourses of individuals (30-40%) is larger than the shift in students' discourse caused by lectures with different discourse (5-9%). A model of students' discourse can therefore be that the discourse of each individual is a more robust construct, acting as a type of baseline that in a specific situation (e.g., a lecture) can be adjusted to some degree based on the (dominant) discourse of that situation. It is then of interest to examine, more in-depth and longitudinally, situations where the dominant discourse is close to or far away from the discourse of a student, in order to find situations where a shift in discourse is not only an adjustment to a specific situation but if and how an individual's mathematical discourse (i.e., the baseline) also shifts. Using specific aspects of discourse as done in the present study can complement other research that has a similar focus but uses other methods of analysis (e.g., Sfard, 2008). It is also of interest to examine connections between changes in discourse and different measures of students' mathematical knowledge, in order to decide if and how specific properties of discourse, regarding process- and object-orientation, are connected to other aspects of students' mathematical knowledge and learning.

## References


Bergqvist, E. (2007). Types of reasoning required in university exams in mathematics. *Journal of Mathematical Behavior, 26*(4), 348-370.

Bostian, L. R. (1983). How active, passive and nominal styles affect readability of science writing. *Journalism Quarterly, 60*(4), 635-640, 670.

Burton, L., & Morgan, C. (2000). Mathematicians writing. *Journal for Research in Mathematics Education, 31*, 429-453.

Einarsson, J. (1978). *Talad och skriven svenska: Sociolingvistiska studier [Spoken and written Swedish: Sociolinguistic studies]*. Doctoral Dissertation. Lund, Sweden: Ekstrand.

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics, 12*, 317-326.

Mercer, N. (1996). The quality of talk in children's collaborative activity in the classroom. *Learning and Instruction, 6*(4), 359-377.

Morgan, C. (1999). Convention or necessity? The impersonal in mathematical writing. *Proceedings of the British Society for Research into Learning Mathematics, 19*(1), 55-60.

Palm, T., Boesen, J., & Lithner, J. (2011). Mathematical reasoning requirements in Swedish upper secondary level assessments. *Mathematical Thinking and Learning, 13*(3), 221-246.

Rodd, M. (2003). Witness as participation: The lecture theater as site for mathematical awe and wonder. *For the Learning of Mathematics, 23*(1), 15-21.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. New York: Cambridge University Press.

Spyridakis, J. H., & Isakson, C. S. (1998). Nominalizations vs. denominalizations: Do they influence what readers recall? *Journal of Technical Writing and Communication, 28*(2), 163-188.

Tall, D., Thomas, M., Davis, G., Gray, E., & Simpson, A. (2000). What is the object of the encapsulation of a process? *Journal of Mathematical Behavior, 18*(2), 223-241.

Österholm, M. (2011a). A framework for studying differences between process- and object-oriented discourse. In B. Ubuz (Ed.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, p. 367). Ankara, Turkey: PME.

Österholm, M. (2011b). What aspects of quality do students focus on when evaluating oral and written mathematical presentations? In J. Clark, B. Kissane, J. Mousley, T. Spencer & S. Thornton (Eds.), *Mathematics: Traditions and [New] Practices. Proceedings of the AAMT–MERGA conference held in Alice Springs, 3–7 July 2011* (pp. 590-598). Adelaide, Australia: AAMT and MERGA.