An Exploration into Growing Patterns with Young Australian Indigenous Students

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This paper presents the results from an initial lesson in a series of design experiments focusing on young Indigenous students' understandings of growing patterns. Indigenous students in Year 2 and 3 (n=16) participated in pre lesson activities and a 45 minute lesson on growing patterns. Tentative findings from this study suggest that; (a) Year 2 and 3 Indigenous students are capable of working with growing patterns; (b) contextual artefacts assisted with communication; and (c) gesture played an important two-fold role in the lessons and communication of the mathematics experienced.

This paper explores Year 2 and Year 3 Indigenous students' initial understandings of growing patterns prior to their formal introduction at school. It is conjectured that Indigenous students have an affinity with the notion of pattern (Matthews, Cooper & Baturo, 2007) as an understanding of patterning underpins aspects of Aboriginal culture. An example of this is the construction of their kinship system, Matthews, Cooper & Baturo (2007, p.250) claim, the reason for this is that "their culture contains components that are pattern-based and which may lead to strong abilities to see pattern and structure". Currently, there is little research into how Indigenous students engage with western mathematical patterns, and whether their perceived affinity with pattern assists them in this engagement.

While it is acknowledged that algebraic thinking is the basis of higher levels of mathematics, there remains a persistent belief that young students are not capable of engaging in this type of mathematics (Carraher, Schliemann, Brizuela & Earnest 2006). Subsequent to this notion, teachers do not present complex mathematical tasks, as they feel young students are not ready to learn this type of mathematics and there is a tendency to focus on simple numbers and shapes within lessons (Sun Lee & Ginsburg, 2009). It can be argued that teachers who have engaged in these types of lower contextual teaching actions have limited the accessibile knoweledge capacity of students by not providing a challenging learning environment which provides opportunity to extend mathematical thinking. In Indigenous contexts, teachers have often provided lower contextual lessons based on skill and drill limiting students access to higher order mathematics (Baturo, Cooper, Michaelson, & Stevenson, 2008; Jorgensen, Grootenboer, Niesche, & Lerman, 2010). Additionally, studies have highlighted that educators have little faith in Indigenous students' mathematical ability (Matthews, Watego, Cooper & Baturo, 2005). There is a developing prospective statement that, in formal education settings with Indigenous students, little algebraic thinking is being developed.

Fundamental to the development of algebraic thinking and concepts, processes and knowledge of mathematics, is the ability to recognise patterns (Cooper & Warren, 2008; Mulligan & Mitchelmore, 2009; Papic, 2007). Research has highlighted that young students can recognise from a range of pattern contexts, the mathematical structure of the patterns. For example, students can identify the structure of repeating patterns as multiplicative, and the structure of growing patterns as functions (Blanton and Kaput, 2004; Cooper & Warren, 2011). In addition, Papic (2007) found that one year after students had engaged in an intervention focussing on creating and interpreting a range of mathematical patterns, these young students where achieving at higher levels in mathematics when compared to students

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who had not engaged with these experiences. Early patterning experiences assist students to engage in 'seeing the structure of mathematics', and students experiencing difficulty in learning mathematics do not always recognise pattern and structure (Mulligan, Mitchelmore & Prescott, 2005).

Visual growing patterns are predominately the initial experience students encounter when introduced to formal algebra (Warren & Cooper, 2008). Growing patterns are characterised by the relationship between elements which increase or decrease by a constant difference. Students in the early years experience growing patterns through activities such as coping, continuing, and extending patterns. Eventually, there is a need for the student to see the relationship between the pattern and their position (stage), which can be termed a generalisation. The ability to generalise a growing pattern enables students to predict the pattern beyond terms that are provided (i.e., 10^{th} position, 25^{th} position, 100^{th} position, nth position). As students engage in this function, they begin to explore the concept of co-variation as they reconsider growing patterns as functions, rather than recessive terms. Often this involves students generating a visual representation (drawing) usually involving a mathematical abstraction, this is then recorded in a table, and the relationship is identified between the two data sets (Warren & Cooper, 2008). Research has shown how this often leads to recursive thinking and the relationship between the growing pattern and the term is not identified.

This study aims to explore the following questions:

1. What is young Indigenous students' understanding of growing patterns?
2. How do young Indigenous students communicate this understanding?

Theoretical Framework

The theoretical frameworks underpinning this study are semiotics and Indigenous research perspectives. The learning of mathematics is two-fold; it involves the interpretation of signs, and the construction of mathematical meanings through communication with others (Saenz-Ludlow, 2007). The theoretical perspective of semiotics will be utilised as a lens within this study to interpret the interactions between teacher and students, and between students and context. It also assists in the selection of the types of materials used to represent growing patterns and how they are used in a classroom context.

In conjunction with semiotics, enacting the secondary theoretical perspective of Indigenous research perspectives, places a focus upon building relationships with students in order to facilitate the learning. It is essential to build a trusting environment for the researcher and students to share knowledge, particularly in the light of cultural variances. Acknowledging Indigenous research perspectives frames the research with an emphasis on empowering the participants, and thus facilitating the free transfer of knowledge and reducing the likelihood of the students being inhibited by the presence of the researcher. At the cultural interface, the researcher is conscious of building relationships so that students, IEWs and the researcher can partake in a meaningful exchange. This creates shared space is where empowerment can occur (Denzin & Lincoln, 2008). The implication of this decolonised approach dictates that the study must be viewed within the bounds of the individual community in which the research takes place and not generalised to the broader Indigenous population. In addition, throughout all aspects of the data collection and analysis an Indigenous perspective was provided by continual consultations from two Indigenous Education Officers.

Research Context

Given that the principal component in the study is young Indigenous students, it is essential to analyse students who are demographically located in an Indigenous community. The design research was conducted in one Year 2/3 classroom of an urban Indigenous school in North Queensland. The purpose for selecting the school was that there had already been a relationship established with the students and teachers in this community, as the school is a part of a longitudinal mathematical research project *RoleM* (Representations and oral language experiences in Mathematics) conducted by the second author.

Method

Participants

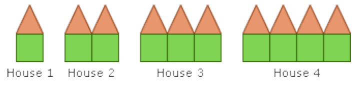
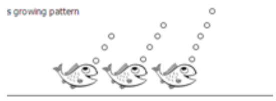
The students were from an Indigenous College situated in an urban town setting, in Queensland, Australia. Sixteen students (mean age 8.5 years) participated in teaching episodes. The students were in a mixed year level class with seven students in year 2 (4 girls and 3 boys) and nine students in year 3 (6 girls and 3 boys). All students were of Aboriginal or Torres Strait Island descent.

Data Gathering Techniques and Procedures

The methodological approach taken for the study was that of design experiment (Cobb, Confrey, Lehrer, & Schauble, 2010). The design aimed to produce both theoretical analyses and instructional innovations (Cobb, et. al., 2010) with one variation; one of the researchers acted as teacher. During the lessons tasks were modified according to the classroom dialogue and interactions. The design experiment consisted of three teaching episodes; each episode included three 45minute lessons on growing patterns. The teaching episodes took place over a six-month period and they focused on opportunities for the students to draw on their own contextual knowledge to explore growing patterns for the first time. This paper reports on the first lesson of teaching episode 1.

At the commencement of each teaching episode students were given pre lesson activities to determine their current understandings of patterning. All questions were read aloud to the students. Table 1 displays the questions asked to students prior to the commencement of teaching episode 1.

Table 1.
Questions Presented to Students Prior to the Commencement of Teaching Episode 1

	Questions presented to students	
1.	Draw your own pattern.	
2.	Use stickers to create your own pattern.	
3.	Copy this growing pattern.	
4.	Continue this growing pattern.	
5a.	Draw your own growing pattern.	
5b.	Explain how your pattern is growing.	

The first lesson of teaching episode 1 was approximately 45 minutes. Lessons were video recorded to capture interactions between the students and researcher/teacher. Two video cameras were used to capture the lesson, with one focusing on the researcher and the other on the students. During the lesson the researcher and supervisor acted as participant observers. At the conclusion of each lesson the researcher and supervisor reflected on observations to ensure that common perspective of the teaching instructions that occurred and the student responses. Additionally, the researcher interviewed the Indigenous Education Officer (IEO) to ascertain additional information that may not be prevalent to the research due to cultural difference.

Focus of Lesson 1 Teaching Episode 1

The initial part of the lesson reviewed the patterns drawn by the students on the pre lesson activity. Discussion with the students followed, examining the difference between the patterns drawn, particularly the difference between repeating patterns and growing patterns. Students were asked 'how was their pattern growing?' and were given opportunities to explain this. Many students provided examples from the environment to describe their initial ideas about growing patterns from the pre-lesson activities. Students were then given the opportunity to copy, extend, and identify the rule of a simple growing pattern using concrete items. The growing patterns provided were non-numeric and drew on environmental contexts. Growing patterns selected for the lesson were visually explicit, meaning that the link between the pattern and the term were easily accessible to students. An example of this was the relationship between kangaroo tails and kangaroo feet. Similar to the example given in past research studies between puppy dog tails and ears to explore co-variational thinking (Blanton & Kaput, 2004).

Data Analysis

The data were analysed in a four-fold process. Firstly, the initial video footage was transcribed to capture students' verbal responses and for noting emerging themes. Secondly, the evolving data were reanalysed using the construct of semiotic bundles (nodes consisting of signs, gestures, and language exemplifying student learning). In the analysis an emphasis was placed on students' physical gestures including their manipulation of the concrete objects and their body language. Thirdly, following this analysis the video footage was reviewed with the two Indigenous education officers (IEO). The Indigenous education officers watched the interview and provided feedback about the cultural signs that were displayed within the video. Their input was recorded and then transcribed to match the identified gestures and cultural signs used by the students. Fourthly, given this feedback with regard to cultural signs, the videos were reanalysed with an emphasis placed on students' physical gestures including their manipulation of the concrete objects and their body language.

Results

Responses to the Questions Presented to Students Prior to Lesson 1

All students participated in the pre lesson questions (n=16). Table 2 presents the frequency of correct responses for questions presented in the pre lesson task. For question 4, six students correctly continued the pattern to the left and three students continued the pattern to the left and to the right (continued both ways).

The 11 students who completed question 1 all drew repeating patterns (i.e., aabbaabb; abababab). Additionally, students exhibited more accuracy in copying growing patterns (question 3) than in continuing growing patterns (question 4) beyond the term given. When asked to continue the pattern, those students who responses were considered incorrect, tended to copy the pattern rather than continue the pattern. Four students created a mathematical growing pattern for question 5. The other students who answered the question provided either a repeating pattern (n= 5) or a picture pattern representing growing in an environmental context (n=4). Three students provided no response.

Table 2.
Frequency of Correct Student Responses to Pre-lesson Questions

Question	Frequency (n=16)
1. Draw your own pattern	11
2. Create a repeating pattern using the stickers	12
3. Copy this growing pattern	14
4. Continue this growing pattern	6 (3 both ways)
5. Create a growing pattern	4

Figure 1 displays examples of the three different types of patterns presented by students in question 5.

Repeating pattern | Picture Pattern (environmental) | Growing Pattern (mathematical)

Figure 1. Examples of patterns presented by students for question 5.

Students were then asked how their patterns were growing. Students responses were categorised into three areas; (a) spatial response – '*It's getting bigger and bigger'* or *'it's getting higher and higher'*; (b) numerical response – '*It is growing with numbers'*; (c) environmental response – '*It is growing with the sun, shade and water'*. This final response was often linked to pictorial patterns as that displayed in figure 1.

Excerpts from Lesson 1: Kangaroo Tails and Feet

During the lesson students were asked if they could predict further terms (if I had 10 kangaroo tails how many feet would I have and identify the relationship between the kangaroo tails and the number of feet. Below is an example of a student predicting beyond the pattern presented to them.

R: If I had 10 kangaroo tails, how many feet would there be?
S3: 20 (called out)
R1: How do you know that?
S3: I just added the same number.
R1: What do you mean?
S3: There were ten tails so I just added another 10 to get the answer.

Additionally, some students were beginning to identify the pattern rule. Below is an excerpt from the lesson.

R: So if I know how many tails I have, how do I work out how many feet there are?

S1: You're doubling it.

R: You're doubling what?

S1: The tails

Discussion with the Indigenous Education Officers

A discussion followed at the end of the lesson with the Indigenous Education Officers (IEO). Both the researcher and the IEOs watched the video recording of the lesson and interactions of the students. Themes that emerged from this discussion were: (a) Students could identify patterns easier when they were using contextual concrete items; (b) students gesture often when discussing the mathematics as they may not have the 'western mathematical language' to explain the concept; and (c) cultural factors contribute to communication in the lesson.

Discussion and Implications

The analysis of the initial teaching episodes has provided three prospective findings about young Indigenous students initial understandings of growing patterns and how they communicate these understandings: (a) students displayed capacity to copy, continue, create, and identify simple growing pattern rules; (b) contextual artefacts assisted with communication; (c) gesture played an important two-fold role in the lessons and communication of the mathematics experienced.

Firstly, students showed aptitude in copying, continuing and creating growing patterns prior to formal teaching. After the first lesson on growing patterns students were beginning to identify rules for simple growing patterns. The setting up of the activities allowed the students to see the structure of the pattern. It was essential that the signs of both variables (pattern and position) were embedded in the visual pattern (Warren, 2005). The use of concrete artefacts represented the growing pattern in such a way that students could attend to both signs and as demonstrated by student 1 begins use co-variation. Using the kangaroo tail and feet assisted this student to begin to see the relationship between the two variables as both position (tail) and pattern (feet) were explicit. Purportedly, by using explicit variables in concrete materials, students were facilitated in making connections with co-variation.

Research has highlighted that young students are indeed capable of thinking functionally (Blanton & Kaput, 2004), however the processes that assist students to 'notice' the relationship between the two variables is still unknown. The recognition of the two variables has been challenging particularly with many older students only identifying the recursive relationship with growing patterns (e.g., Lannin, 2005; Radford, 2006). Further analysis in this study is needed to determine if the explicit nature of the two variables will assist the students to make links to co-variation, identify rules, and generalise simple growing patterns.

Secondly, it became apparent that the students responded positively when the artefacts utilised in the learning activities were related to the students' local environment. The use of contextualised patterns provided opportunity to discuss the pattern in terms of language which was already accessible for students, such as in the case of kangaroo ears and tails. This too was evident through the discussion with the IEOs. The opportunity for the students to engage with the concrete artefact assisted students to discuss the structure of the pattern.

The students manipulated the concrete items and used them during their explanations. Engaging with concrete contextual artefacts acted as a medium between the mathematical structure and the students thinking as they began exploring abstract thinking.

Thirdly, there was a predication toward gesture during the initial lesson on growing pattern. The gesture was two-fold; gesture as embodiment of the task and gesture working with language for communication of mathematical concepts. Gesture as the embodiment of the task requires the students to interact with the artefacts. Both the researcher and the IEO agreed upon the observation that the interaction with the concrete items assisted students to think about the growing pattern. Past research has indicated that the use of the body experiences is seen as strongly related to cognition (Lakoff & Núñez, 2000). This challenges the belief of mathematics that is seen as objective, abstract and disembodied. From a semiotic perspective these physical processes helped students to objectify the task. Secondarily, the use of gesture as an adjunct to language for communicating ideas about growing patterns was observed. Many students used gesture to supplement the language used in communicating their ideas about growing patterns. This theme was consistent with literature suggesting that gesture and language play significant roles in the learning of new mathematical concepts. The relationship between the two has been described as 'unsplittable' (McNeill, 1992). It was apparent that at times during the lesson, some language was not accessible to the students or the students' home language was at odds with the mathematics (Goldin-Meadow, 2002). Researchers have concluded when language is not apparent or is mismatched to home language students will use gesture to assist conversation (Goldin-Meadow, 2002). Additionally, gesture may be the first place students display a new thought (Goldin-Meadow, 2002). It was apparent that both of these concepts were the consistent with students in this study. Understanding student gesture in Indigenous contexts and giving opportunities for that is paramount. Teachers must be cognisant of using gesture and providing opportunity for gesture

In conclusion, this paper represents the initial stage of a larger study and further analysis of the subsequent lessons, teaching episodes and interviews will assist in exploring more definitive conclusions. This paper is exploring a phenomenon as opposed to creating a defining stance on Indigenous students. While this study provides some insight into how young Indigenous students engaged in growing pattern tasks, how they reach generalisation remains a pertinent area for analysis.

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