The Relationship between the Estimation and Computation Abilities of Year 7 Students

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This paper reports on part of a larger study to compare students' computation and estimation skills. Year 7 students in the Perth Metropolitan area were given a computation test and an estimation test of matched items. The performance on computation was 10 percentage points higher than on estimation. It was clear that students' computations tended to be undertaken mechanically rather than meaningfully. In particular, students were weak at fractions and decimals and showed significant misconceptions. It is recommended that much more time be devoted to estimation as an essential and integrated process in the mathematics classroom.

Estimation is considered to be a significant topic in school mathematics, as confirmed in various curriculum documents (Australian Education Council, 1991; Curriculum Council, 1998; NCTM, 2000). Yet, school mathematics textbooks seem to give it little attention. In conjunction with this situation, it is a truism that much more attention is paid to computation than to estimation in mathematics classrooms. Estimation is a process rather than content knowledge in the mathematics curriculum, and it applies to several strands; but in this paper we will focus on the number strand. It is an aspect of a larger study reported in Dolma (2002).

The significance of estimation and its place in the learning and using of mathematics has been highlighted by many respected mathematics educators. Trafton (1986) stressed that "building a strong computational estimation strand into school mathematics programs must be a top priority for curriculum developers" (p. 16). Usiskin (1986) argued that "the uses of estimation fit the ideals of mathematics, namely, clarity in thinking and discourse, facility in dealing with problems, and consistency in the application of procedures" (p. 2). Then students would come to view mathematics as a distinct way of thinking, rather than as a collection of unconnected rules. The growing importance of estimation in a technological society is well recognised. Reys (1992) suggested that "over 80% of all mathematical applications call for estimation, rather than exact computation" (p. 142). But school mathematics is very much focussed on computation in the number strand.

In the past decade or so, estimation has come to be seen as a very significant component of number sense (Sowder, 1992). During this period there has also been a growing emphasis on the importance of mental computation. Estimating requires mental computation, thinking, and making sense of the computation – the process cannot rely on rules or mechanical procedures. Northcote and McIntosh (1999) found that adults' everyday mathematics consists mostly of mental computations; and of course many of these involve estimations. Reys and Yang (1998) found that sixth- and eighth-grade students in Taiwan were much more successful on written computation than on number sense. Interestingly, most of their number sense test items required estimation, thus stressing the close link between estimation and number sense. McIntosh, Reys, Reys, Bana, and Farrell (1997*)* in their international study of number sense found that students in Years 3, 5, 7, and 9 scored low on number sense, where many of the items involved estimation.

Aim of the Study

The close relationship between estimation and number sense is generally well established. At the same time there appears to be a significant gap between students' number sense and their computation ability. It could be argued that number sense involves more factors than estimation. We thus decided to use estimation ability *per se* to make a direct comparison with computation skill in an Australian setting. Therefore, the aim of this study was to investigate the relationship between the estimation and computation skills of Year 7 students in a number of Perth Metropolitan schools

Methodology

Subjects

The sample consisted of 77 students from three heterogenous Year 7 classes – one class from each of three typical primary schools in the Perth Metropolitan area of Western Australia, where students do not commence secondary school until Year 8.

Instruments

The researchers developed a 15-item computation test and a parallel estimation test with identical computation items. The major sources for these items were the instrument used by McIntosh et al (1997) in their international study of number sense, and that used by Reys and Yang (1997) for Taiwanese students A pilot study was conducted to refine the instruments. Table 1 provides two examples to illustrate the style of the instruments,

Table 1
Examples of Matching Estimation and Computation Items

Estimation	Computation
Without calculating an exact answer, circle the best estimate for: 0.5 x 840 a. 840 ÷ 2 b. 5 x 840 c. 5 x 8400 d. 0.50 x 84	Calculate: 0.5 x 840
Without calculating the exact answer, circle the best estimate for: $^3/_4 + ^1/_2$ a. 1 b. 3 c. 4 d. 6	Calculate: $^3/_4 + ^1/_2$

and how the items were matched for computation and estimation. All estimation items had identical stems as shown in Table 1, and were in multiple-choice format to help guard against students actually calculating precisely before determining an estimate. A common stem – *Calculate:* – was also used throughout the computation test. All fifteen items are shown in Table 2. A semi-structured interview schedule was developed for use in a follow-up to the tests with a sample of 12 students (four from each school). Modifications to this schedule were made as appropriate for individuals, based on their test results and on their responses during interview.

Procedure

All students were tested in the same week in the second school term. To avoid the intrusion of the researchers, the class teachers administered the tests by following a set protocol. The estimation test was given first, with a time limit of 30 seconds per item to minimise any precise calculations. To further support this aspect, students were not permitted to write anything on the test paper, other than their responses. After a short break, the computation test was given, with a three-minute time limit for each item. Students were instructed that the computations could be done using any method/s of their choice, but calculators were not permitted. Two points were awarded for each test item. Estimation items were allotted two points if correct, and zero if incorrect. Computation items were allotted zero if incorrect, one point if partially correct, and two points if entirely correct.

Students were selected for interview according to their performances on the tests. Four students were chosen from the sample year 7 class at each of the three schools. The interview group consisted of a male and a female student who performed above average, and a male and a female who scored below average; but students who scored near the ends of the performance spectrum were not selected. This process was followed so that each sample would be reasonably representative of the class. The interviews were undertaken within a week of the test administrations in order to assist student recalls of their performances. Each interviewee was presented with his or her estimation and computation tests, and was questioned on the strategies used and the reasoning behind them. The interview was audio-taped and field notes taken.

Results and Discussion

The percentage scores on all 15 matching pairs of estimation and computation items are given in Table 2. The mean scores show that students scored 10 points higher on computation than on estimation, with 51 and 41 percent respectively. Both scores were relatively low, but this was at least partly due to the fact that the test items constituted a Year 7 test and were administered in the early part of the year, but they are commensurate with the results of the McIntosh et al (1997) number sense test with similar items where students in Perth schools tested a little later in the year averaged 52 percent (p. 75). In only four of the 15 items did students do better at estimation that at computation. In all other 11 items performance on computation was much higher than on estimation. In fact, in eight of these 11 cases the differences ranged from 16 to 33 percentage points. Thus it was often the case that students who were successful at the computation seemed unable to judge the reasonableness or otherwise of their answer.

Table 2
Percentage Scores on Matching Estimation and Computation Items in Year 7 (N = 77*)*

Topics & Items	Estimation	Computation
Whole Numbers		
Addition		
1) 9965 + 8972 + 8138 + 8090	68	84
Subtraction		
2) 312 – 119	48	81
Multiplication		
3) 18 x 19	52	47
4) 51 x 48	44	60
Division		
5) 598 ÷ 9	55	46
Decimals		
Addition		
6) 590. 43 + 312.5	88	77
7) 96.7 + 147.4 + 62.75 + 36.8	48	74
Subtraction		
8) 0.72 – 0.009	26	44
Multiplication		
9) 0.5 x 840	25	56
10) 87 x 0.09	17	38
Division		
11) 54 ÷ 0.09	20	27
Fractions		
Addition		
12) $^3/_4 + ^1/_2$	33	43
Subtraction		
13) $^7/_8 – ^3/_4$	17	36
Multiplication		
14) $^1/_4$ of 798	40	27
15) $^5/_8$ of 512	14	21
Mean Scores	41	51

These results suggest that the Year 7 students in the selected schools tended to undertake computation in a mechanical, rather than a meaningful fashion. The teachers in these classes devoted very little time to estimation, focusing more on the traditional computation algorithms, despite the fact that real-life mathematics consists mostly of estimation and mental computation. It is quite likely that this is rather a common pattern in mathematics classrooms. Estimation is a process that requires understanding and is a significant component of number sense. It is not something that can be undertaken in a mechanical fashion. Computation on the other hand – particularly traditional written computation – can be performed by remembering rules or procedures, and without meaningful connections.

It is interesting to examine the results in some of the individual items. For example in Item 8 shown in Table 3, the percentage of correct responses for estimation was 26. The distribution among the four alternatives could well be classed as being randomly generated. It was clear from this and other items that students have great difficulty with decimals. One interviewee illustrated this as seen below:

I: You were correct with the written computation.

S: Yes, I can usually do take-aways.

I: Good. Now what about your estimate?

S: Well, I have trouble with decimals. I said nine from 72 is 60 something, so I thought it must be a or b. I put b, but I dunno.

Table 3

Percentages of Estimation Choices for 0.72 – 0.009

Alternatives	% Chosen
a. 0.06	19
b. 0.6	21
c. 0.07	34
d. 0.7*	26

Note: *Correct response

Table 4 shows the results of the estimation for Item 10. Only 17 percent of students were correct, but 36 percent computed the product correctly. The result of greatest concern is that almost two-thirds of Year 7 students considered that the product of 87 and 0.09 would be more than 87. The notion that multiplication results in a larger number, which is true for whole numbers, is not seen by students to vary with fractions and decimals where one factor is less than one. The interview excerpt below illustrates the point.

I: Tell me how you knew it was *b*.

S: Timesing always gives a bigger number so it must be bigger than 87.

I: OK, so what about *d*?

S: Er…well it is not timesing by a very big number.

Table 4
Percentages of Estimation Choices for 87 x *0.09*

Alternatives	% Chosen
a. A little less than 87	18
b. A little more than 87	44
c. A lot less than 87*	17
d. A lot more than 87	20

Note: *Correct response

Table 5 shows the results of students' estimations for $^{7}/_{8} – ^{3}/_{4}$, where 36 percent were correct in the matching computation item. As with decimals, students had considerable difficulty with the fraction items. Only 17 percent chose the best estimate. Student misconceptions about fractions are highlighted by the fact that 43 percent chose four as their best estimate of the difference between $^{7}/_{8}$ and $^{3}/_{4}$. It seems that for fractions, many students see them as two numbers rather than as one. In this example the difference between the numerators, and also between the denominators, is four; and this was the estimate chosen. The interview sample below well illustrates this occurrence.

I: How did you know the difference was about four?

S: Three from seven is four and four from eight is four, so that's how I picked it.

I: Now, let's see how you tried work it out properly. Tell me what you did.

S: Er… we've done fractions in class but I couldn't remember how we were supposed to do it.

Table 5
Percentages of Estimation Choices for $^{7}/_{8} – ^{3}/_{4}$

Alternatives	% Chosen
a. 0*	17
b. 1	31
c. 3	9
d. 4	43

Note: *Correct response

Although quarters and halves are the most common fractions in use, Year 7 students had considerable difficulty with these fractions. In the item for $^{3}/_{4} + ^{1}/_{2}$, 43 percent were successful with the calculation, and only 33 percent chose the best estimate, as shown in Table 6. Again, it is clear from the results that students have little understanding of the fractions or the operation. Almost half the students chose four or six as the best estimate, and these are the sums of the numerators and denominators respectively.

Table 6
Percentages of Estimation Choices for $^3/_4 + ^1/_2$

Alternatives	% Chosen
a. 1*	33
b. 3	18
c. 4	27
d. 6	22

Note: *Correct response

Conclusions and Implications

Year 7 students are much better at computations than at estimating. This is most likely due to the disparate amount of time spent on each of these aspects in the mathematics classroom. It is clear that computations are often undertaken by following rules rather being performed in a meaningful way. The importance of estimation is unquestioned. One of the main reasons its advocates put forward is that it provides a benchmark by which to judge the reasonableness of results. There is plenty of evidence that many students undertake computations, particularly with calculators, and do not question their veracity by making estimates (Swan, 2002). However, the importance of estimation also lies in its capacity to make number work meaningful for students. It requires thinking and therefore is a very significant aspect of number sense and of working mathematically. It is also clear from this study that students have considerable difficulty with even the simplest fractions and decimals, and that they harbour significant misconceptions about them.

Much more time needs to be devoted to estimation in mathematics classrooms. Estimation is a process rather than a topic and should be dealt with as such in mathematics classrooms. Thus it should not be treated in isolation. Rather it should be integrated into all topics where it is relevant. For example, 'estimating before calculating' and 'estimating before measuring' should be habitual when students are engaged in these mathematics topics. A greater emphasis on mental rather than written computation is an avenue that is recommended for mastering essential skills in estimation. This will assist the learning and doing of mathematics to become more meaningful.

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