Gesture Types for Functions

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This paper reports on the different gesture types employed by twenty-three Year 10 students as they endeavoured to explain their understanding of rate of change associated with the functions resulting from two different computer simulations. These gestures also have application to revealing students' understanding of functions. However, interpretation of gesture is problematic but classification of gestures assisted in the analysis of the video-recorded interviews probing participants' conceptions of rate of change. This paper builds on the classifications reported in previous research. Five additional gesture types are presented, which provide insights into students' thinking about rate of change, and hence functions.

Kelly, Singer, Hicks, and Goldin-Meadow (2002) suggest that the combination of speech and gesture often provides greater insight into children's knowledge and understanding than either words or gestures alone. It was anticipated that the teenage participants of this study may have found it difficult to discuss an abstract mathematical concept (Reynolds & Reeve, 2002). Therefore, consideration of participants' non-verbal communication made available through video-recorded data collection, is vital in revealing the meaning behind their utterances. However, interpretation of gesture is problematic, so classification of gestures may assist in the analysis of this kind of data. Some work has already been done in classifying gestures, especially those related to mathematical concepts (McNeill, 1992; Edwards, 2005; Arzarello & Robutti, 2004; Rasmussen, Stephan & Allen, 2004). This paper extends that work, paying particular attention to the classification of gestures relating to functions.

This paper reports on the different gesture types employed by twenty-three participants as they grapple to explain their understanding of the functions resulting from two different computer simulations. The simulations provided a focus for discussion as the participants attempted to articulate their thinking about the functions and their representations: numeric; graphic; and symbolic. Participants were able to point to specific places on the screen to clarify their explanations. This data provides a rich source of function-related gestures but space considerations limits this discussion to only a few of the most important.

In the sections below previous gesture research is described; details of the interviews and the computer-based simulations are provided; and examples are presented to illustrate the gesture types identified.

Background

Feyereisen and de Lannoy (1991) defined gesture to be "any kind of movement performed during speaking" (p.4). Gestures "may provide a window onto knowledge that is not readily expressed in speech" (Alibali, Bassok, Solomon, Syc, & Goldin-Meadow, 1999, p. 327). Video provides access to this data not available in other forms of data collection. It enables the researcher to take advantage of participants' non-verbal communications, such as sound and images containing facial expressions, tone of voice and gestures, together giving insights into emotions and depth of understanding of concepts (Cope, 2000; Pea, 2006). Pea (2006) claims that video enables the collection of richer and more reliable data about complex social interactions, such as interviews.

In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), Mathematics education: Expanding horizons (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia). Singapore: MERGA.

A difficulty faced by the researcher is the interpretation of pauses in the audio record. Video enables the researcher to formulate interpretations, such as indications of uncertainty (Reynolds & Reeve, 2002), of these gaps in the audio record. So, participants' representational gestures along with significant pauses, give "a more complete picture of the students' actual understanding of the problem domain" (Reynolds & Reeve, 2002, p.457). Video is especially useful for fast moving and complex events since it can be replayed repeatedly, picking up subtle details and checking interpretations, to ensure accuracy of transcripts and analysis (Pea, 2006). This fine-grained analysis may disclose insights into students' understanding not otherwise available.

Goldin-Meadow (2004) suggests that teachers can and do interpret children's gestures, but do not always take them into account in their teaching. It is interesting to note that Iverson and Goldin-Meadow (1998) report that blind people spontaneously gesture in the same way as sighted people. This suggests that gesture performs the function of assisting gesturers to clarify their own thoughts, so gesture may provide the researcher a window on those thoughts. In this study, the participants are young adolescents who may not be able to express clearly their conceptions of mathematical ideas. Broaders, Wagner Cook, Mitchell, and Goldin-Meadow (2007) assert, "speakers' gestures can reveal knowledge that they have but cannot yet articulate" (p. 539), so a close examination of students' gestures may offer insights into students' conceptions not available from the transcriptions of their utterances, for example, pointing to a particular part of the screen of the computer-based simulations used in this study. When speech adequately conveys a speaker's meaning, gesture only plays a supportive role, but when gestures convey information which differs from the information provided by speech (Goldin-Meadow, 2000) interpretation of gestures becomes especially important. The researcher scrutinising video for students' conceptions needs to be alert for instances where gestures contradict speech, as well as instances where gesture elaborates on speech.

Attention to gesture is a growing research area in the domain of mathematics education research (See for example, Reynolds & Reeve, 2002; Rasmussen, Stephan & Allen, 2004; Arzarello & Robutti, 2004; Arzarello, Robutti, & Bazzini, 2005; Edwards, 2005; Williams & Wake, 2004). Arzarello et al. (2005) in their study of middle-years students' conceptions of "variable" and "function", claim that gestures "play an important role in interpreting the students learning processes" (p.64). Reynolds and Reeve (2002) investigated the significance of two students' gestures in the interpretation of speed-time graphs, and they assert that the expression of difficult concepts, such as functions, may be facilitated by the support of gestures, especially if the language required is specific to a particular domain, for example, mathematics.

Interpretation of gestures informs teachers' and researchers' understanding of students' thinking. However, gestures are sometimes difficult to interpret (Williams & Wake, 2004) but practice in interpretation of gesture can lead to a better assessment of participants' knowledge (Kelly et al., 2002; Gerofsky, 2010). Classification of gestures may assist a researcher's interpretation of participants' gestures. Parrill and Sweetser (2004) propose that

> the analyst's claims about gestural meaning can be explicitly laid out, and furthermore, that this enterprise can profit from the use of a framework … [and] can therefore be used to express both correspondences between physical forms in space (hands, e.g.) and meanings (ideas, e.g) (p.198).

McNeill (1992) proposes a general classification of four types of hand gestures: beat, deictic, iconic and metaphoric. Beat gestures reflect the tempo of speech or emphasise aspects of speech. Gestures are classified as deictic when the participant is pointing at a real item, indicating directions or referring to something previously discussed. Iconic gestures resemble physical phenomena, such as when hands are held to represent the shape of a ball, and are usually easy to interpret. Metaphoric gestures clearly have some meaning representing an abstract idea, but are more difficult to interpret. However, it may be that interpretation of gestures in specialised domains, such as mathematics, may require an extension of McNeill's classification to include domain specific gesture types.

Edwards (2005) proposes a revision of McNeil's iconic classification to include "iconic-physical", for iconic gestures resembling the physical phenomenon referred to in speech, and "iconic-symbolic", for iconic gestures referring to mathematical symbols or written processes. In addition, she conjectures that the special role of semiotics in mathematics may need further classifications to facilitate the interpretation of gestures used in explaining mathematical understanding. Arzarello and Robutti (2004) use the term "iconic-symbolic" in the same manner as Edwards, but extend the categorisation further to include "iconic-representational" gestures that are gestures related to graphs. Rasmussen et al. (2004) refer to a number of function gesture types in their study of classroom practices in a class studying first-order differential equations. Images in their paper clearly convey several gesture types and their interpretation in the context of the particular class. They mention gesture classifications for functions: "slope shifting gesture" (p.312); "moving rate gesture"; "slope hand gesture"; "pointing function gesture"; and "pointing moving slope gesture" (p.309).

Awareness of gesture types from previous research provided guidance in the interpretation of participants' gestures in data initially collected to reveal middle secondary students' conceptions of rate of change (Herbert & Pierce, 2009). This paper describes additional gesture types related to rate, and hence functions, evident in this data.

Method

Two computer-based simulations were prepared: one in JavaMathWorlds (JMW) simulating two characters walking (Figure 1) (Mathematics Education Researchers Group, 2004); and the other in Geometers' Sketchpad (GSP) (Key Curriculum Press, 2006) simulating two windows with blinds (Figure 2). These simulations were chosen to provide experientially real-world instances of functions with ease of access to the multiple mathematical representations of the functions: numeric; graphic; and symbolic. It was thought that these twenty-three Year 10 participants may respond differently in different contexts or representations so these two simulations offered contrasting real-world contexts in which functions may be found.

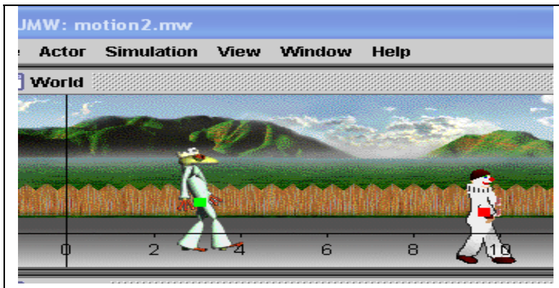


Figure 1. JMW screen with frog & clown walking

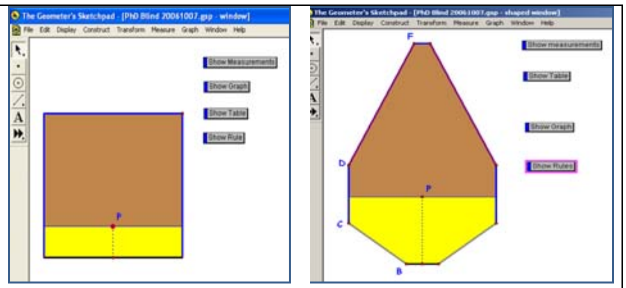


Figure 2. GSP screen of diagram of two blinds

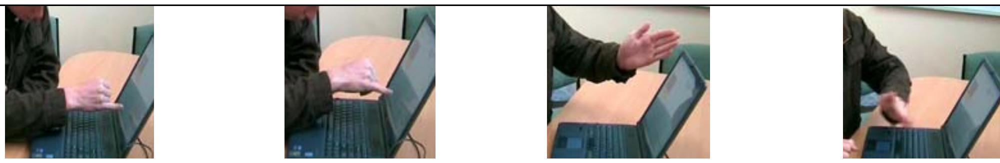
Each participant (pseudonyms used) was videoed as they responded to the interviewer who prompted them to discuss the functions seen in the simulations. They were encouraged to explain their reasoning and think aloud as they were presented with different representational forms of function: the simulation, table of values, graph and rule. The video of each interview was viewed repeatedly to identify gesture episodes which were then isolated into separate clips resulting in approximately twenty to thirty clips per video. Each clip was scrutinised to identify the gesture type shown.

Results

The gesture types noted in the videos combined McNeill's (1992) deictic and metaphoric categories with the refinements of the iconic classification of iconic-representational (Arzarello & Robutti, 2004) and iconic-physical and iconic-symbolic (Edwards, 2005). Also evident were gesture episodes which appeared to correspond to gestures identified by Rasmussen et al. (2004). In addition, further gesture types were observed and five have been described in detail below.

Relationship Gesture

Expression of the relationship between the variables connected in a function is fundamental to a deep understanding of functions. Several participants indicated their awareness of this relationship through the gestures they employed. A typical example of the *relationship gesture* can be seen in Figure 3, where Jo employs deictic gestures combined with a relationship gesture, pointing to particular places on the screen to clarify his explanation of his calculation of constant rate, and then moving his left hand in an upward followed by the horizontal movement of his right hand. This relationship gesture comprising two distinct movements indicates Jo's awareness of the relationship between the variables of area and height and is not just reading values off the graph.



Jo: Well it looks roughly like five, because there's five fives in twenty-five and to begin with [it's] ten and fifty, looks fairly even. There's about five marks in here where there's only one mark here. Well the rate would be five to one. [because] we've got a red mark here [points at (1,5) on the line] which is on one.	so that tells us there is five up here [moving hand vertically]	to every one across here [moving hand horizontally]

Figure 3. Gestures indicating awareness of relationship between the variables.

Imagined- formula Gesture

In Figure 4, Sue employs gestures to indicate the position of variables in the formula for speed. She is visualising the formula and showing her thinking with the gestures used. This indicates awareness of the need for both distance and time in the calculation of speed, and also expectation that a formula would be supplied to complete the calculation of rate. In this case the participant has remembered the correct formula to use and has experienced substituting values into it.

Sue: There's a formula for that [pause] yes we did that in science. Get those measurements and divide the distance by time. We've got time & distance, got velocity. Distance [points to indicate where distance occurs in the formula] over time [points to indicate where time occurs in the formula]. So 22 metres divided by 7 is [pause]. [uses calculator] he's going at 3.14 metres per second.

Figure 4. Gestures indicating awareness of the position of variables in formula for speed

Air- graphs Gesture

The *air-graphs gesture* type refers to drawing out the shape of graphs without using a pencil and paper. This can take several different forms. It may be tracing out the graph on a computer screen or drawing with a finger on a desk or drawing in the air. This type of gesture can be seen in Figure 5, where Mimi traces out the shape of the graph in a vertical plane in the air, helping her to explain her reasoning that the speed was not constant.

Mimi: He's not going at the same speed, because there's a sort of a slope sort of thing. It is not a straight line. He is just going slower rather than faster. yeah I just mean slower yeah he's just going … Well his speed changes, it is not the same the whole way through. He is going slow at first and then he is going sort of faster, at the top he is going more straighter.

Figure 5. Air-graph gesture

Imaginary- axis Gesture

The *imaginary-axis gesture* type refers to the use of an axis imagined on a table or in the air to support the student's thinking about the change in one of the variables. In Figure 6, Noni is looking at an automated version of the rectangular blind and considering the rate the blind is moving. Noni has been given a timer and the symbolic representation of the linear function.

Noni: OK what I'm trying to do see what the number comes [on the] timer. At the end, like, it took 25 seconds to get there from the bottom. It took 6 height from bottom. I am trying to make a connection [between] the time and that formula up there.

Figure 6. Imaginary-axis gesture

First Noni establishes the position of the origin by placing her left index finger on the edge of the table. Then she places the index finger on the table to indicate the first position considered when thinking about the graph, followed by a movement of this finger to further along the imaginary *x*-axis. This gesture episode suggests that Noni is thinking about the changes in the variable of time in an attempt to express her thinking about the function involved.

Table- difference Gesture

In Figure 7, Verity gestured repeatedly with a curved arch shape with thumb and first finger, sometimes used to indicate a small distance (Herbert & Pierce, 2007), but in this case indicating the difference between values in the table. She holds her fingers in an arched shaped and repeatedly moves it downward to emphasise the common difference coming down the table for a linear function compared to the changing difference for a non-linear function.

Verity: Like up to about there it would keep going up like 3.2, 3.2, 3.2, 3.2 [then] not as by as steady amount as it was before, it might go up by one or two. [so] instead of going up by 3.2. For every point five it might go up by two or one or one point five because, you can't keep going [the same] because it’s not square.

Figure 7. Table-difference gesture

Discussion

In addition to attending to the gesture types described by McNeil (1992), Edwards (2005), Arzarello and Robutti (2004) and Rasmussen et al. (2004) these new gesture types facilitated the interpretation of participants’ responses, a combination of verbal and gestures, and supported the analysis of phenomenographic interviews revealing middle secondary students’ conceptions of rate of change (Herbert & Pierce, 2009). These additional gesture types also have applicability when considering students’ understanding of functions in general.

Whilst the example of the relationship gesture type (Figure 3) shows a participant using one hand, other forms of the relationship gesture might involve the use of both hands to express the relationship (Herbert & Pierce, 2007). The intention of this gesture type classification is to identify gestures which indicate an awareness of a relationship between the function variables. Observation of a relationship gesture informs the teacher or researcher that there is a simultaneous awareness of the changes in two variables and the relationship between them. This is especially important when the student does not possess the words to explain this relationship but can demonstrate their understanding of the relationship by the gestures they employ (Broaders et al., 2007).

Instances of an imagined-formula gesture inform the teacher or researcher that the student understands the position of the variables in a formula and gives an indication of which formula is being imagined. It implies that there is an assumption that mathematics is formula driven and all one requires, when solving a mathematical problem, is the appropriate formula. Employment of this gesture type may suggest inadequate understanding of the concepts behind the derivation of the formula and further probing may be necessary to reveal the extent of a student's understanding.

The air-graphs gestures are useful in interpreting students' thinking and assessing their understanding of the shape of the graphic representation of a function. Attention paid to air-graphs may inform teachers and researchers of the depth of a student's understanding of the relationship between the representations of functions, for example, when the symbolic representation of a quadratic function is correctly matched with an appropriate air graph the observer can infer that the student is able to transfer some understandings about the function across representations.

Students may employ the table-difference gesture to help explain the manner in which the values in the table differ. This gesture type indicates awareness of the changes in at least one variable. However, if the focus is only on one column, this gesture type may suggest a lack of awareness of the relationship between the variables. A teacher may grasp this teachable moment to discuss the differences in the other column of the table and emphasise the relationship between the columns as a focus on individual columns may result in an inability to connect the columns and hence lack awareness of the relationship between the variables. Similarly, when students display an imaginary-axis gesture, it may imply that they are only focussing on one variable in the function relationship and may not be aware of the necessity to consider both variables, nor understand that the function describes the relationship between the variables.

Conclusion

Gesture classifications supported the detailed analysis of data provided by video-recorded interviews. This analysis afforded increased awareness of participants' understanding of the concepts related to functions embedded in the computer simulations, which provided stimulus for the participants to discuss their mathematical conceptions. These participants, in middle years of secondary schooling, did experience difficulty in expressing their understanding (Reynolds & Reeve, 2002). However, they demonstrated an extensive use of gestures to supplement their utterances (Kelly et al., 2002) in order to explain their thinking about rate of change, and hence functions. The computer simulations gave participants an opportunity to employ deictic gestures to indicate positions on the screen to support and expand their explanations of their mathematical conceptions.

This paper extends or elaborates on existing gesture classifications. The gesture episodes presented illustrate some useful gesture types which, in addition to gesture types described by McNeil (1992), Edwards (2005), Arzarello and Robutti (2004) and Rasmussen et al. (2004), provided insights into participants' thinking not available from their words alone.

Five new gesture types were identified and are presented in this paper. Two gesture types, air-graphs and imaginary-axis gesture types, could be considered as sub-categories of "iconic-representational" (Arzarello & Robutti, 2004) whilst imagined-formula gesture type may be a sub-category of "iconic-symbolic" (Edwards, 2005). The relationship and table-difference gesture types appear to be entirely new classifications.

The evidence presented in this paper highlights the importance of teachers and researchers attending to students' gestures to gain insights into their thinking. Further research is needed to clarify the interpretation of gesture related to mathematical notions.

Acknowledgements

This study is part of the RITEMATHS project, led by Kaye Stacey, Gloria Stillman and Robyn Pierce. The researchers thank the Australian Research Council, our six partner schools and Texas Instruments for their financial support of this project, and especially the participants.

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