Characteristics of Problem Posing of Grade 9 Students on Geometric Tasks

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This is an exploratory study into the individual problem-posing characteristics of 480 Grade 9 Singapore students who were novice problem posers working on two geometric tasks. The students were asked to pose a problem for their friends to solve. Analyses of solvable posed problems were based on the problem type, problem information, solution type and domain knowledge. With the open-ended task, the students tend to over-condition their problems and to produce more problems with implicit assumption. How the findings can contribute to research in problem posing in schools is discussed.

Introduction and Brief Review of Literature

Complementing the notion of mathematical problem solving is problem posing which often requires imaginative skills (Gonzales, 1998). For the present study, Silver and Cai's (1996) notion of problem posing as the "the generation of a new problem from a mathematical situation or experience" (p. 294) will be used. Various studies have pointed to the importance of students' mathematical problem posing. This is related to students' exploration in mathematics (Cai, 2003) and the teaching and learning of mathematics (Crespo, 2003). Bransford, Zech, Schwartz, and Vye (1996) noted that developing students' ability to formulate their own problems is important for developing the mathematical thinking needed to solve complex problems. Brown and Walter (1993) noted that problem-posing activities in classroom helped in lessening mathematics anxiety, in explicating misconceptions and in fostering group learning and that "we learn mathematics when we were actively engaged in creating not only the solution strategies but the problems that demand them" (p. 187). They also strongly endorsed it as one important element of mathematical proficiency. English (1995) noted that while there was considerable amount of studies on children's ability to solve problems, less work was done on children's ability to pose them. Pirie (2002) noted that in the last two decades, problem posing had only received "sporadic interest within the field of mathematics education" (p. 4). Problem-posing studies tended to revolve around students writing of story problems and with the students coming from the elementary grades. In Singapore, problem posing has also not been emphasized in the mathematics curriculum. One goal of the present exploratory study is to further contribute to the knowledge base in problem-posing research in geometric tasks.

Methodology

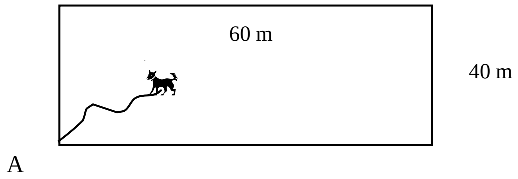
Participants and Problem-Posing Tasks

The subjects comprised 480 grade 9 students (222 males, 258 females) from three secondary schools in Singapore. The two tasks were chosen because they were within the curriculum. Students solved their posed problems. The first task involved the writing of a problem for their friends from a known answer: *"Write a problem so that the final answer is 60°."* This answer-specific task required the poser to form the initial state, the goal state and

In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), Mathematics education: Expanding horizons (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia). Singapore: MERGA.

the building of a context for the emerging problem. The task, piloted earlier by Chua and Yeap (2008), contained little context and so afforded itself for students to freely explore options of posing their problems. The second task focused on building and developing a problem from a given context and checked if there were variations in posing. It required the students to pose a problem for their friends based on the situation:

"A goat is inside a 60 m by 40 m rectangular fence in a farm. It is tied to a pole at A by a 30 m rope which could not be stretched."



Analysis

The problem-posing characteristics were inferred from the posed problems and the solutions. Students' solutions could show how they have conceptualized their problems since the solving and posing processes are complementary (Contreras, 2007). The solution strategies and the use of the mathematical domain knowledge could give insights into the posed problems (Cai, 2003). Drawing from work by English and Halford (1995), problems were first classified as either solvable or non-solvable. A solvable problem has a well-defined initial state, a goal state and an inherent solution path. Solvable problems were coded based on the problem types (*relational*, *direct recall*), problem information (*edit information*, *add object*, *over-conditioning*, *implicit assumption*), solution type (*multi step*, *use of algebra*) and domain knowledge (topics). The descriptions of these categories are given below. Binary descriptors were used to indicate the presence or absence of these attributes in each solvable problem. Three raters independently coded the responses with high degree of agreement with the researcher's codings. Crosstabs were then used to investigate the strength of association (*phi*) between problem types, problem information and solution type. A non-solvable problem (92 in Task 1, 98 in Task 2) was either *ambiguous* (no explicit goal state or were non-mathematical in nature) or it involved a *contradiction*. Non-solvable problems were not further analyzed. Missing entries occurred when there were no attempts made (18 in Task 1, 19 in Task 2). Table 1 shows the distribution of the problem types and characteristics.

Table 1

Frequency Table of Problem-Posing Characteristics of Solvable Problems

Problem Type	Problem Characteristics		Task 1(%^)	Task 2(%^)
Relational	i) Problem information	*edit information*	-	19.3
Task 1, n = 198		*add object*	-	25.6
Task 2, n = 176		*over-conditioning*	21.7	0
		implicit assumption	40.4	19.9
	ii) Solution type	*multi step*	97.0	94.3
		use of algebra	57.1	0
Direct recall	i) Problem information	*edit information*	-	9.1
Task 1, n = 172		*add object*	-	8.6
Task 2, n = 187		*over-conditioning*	29.7	0
		implicit assumption	22.1	10.2
	ii) Solution type	*multi step*	15.1	2.7
		use of algebra	53.5	0

^ as % within problem type

Classification by Problem Type

a) *Relational Problem*

In this study, a *relational problem* is a problem that involves a relation of at least two concepts in the solution path. Student F341's *relational* problem in Figure 1 involves *angles in a triangle* and *angle circle properties*.

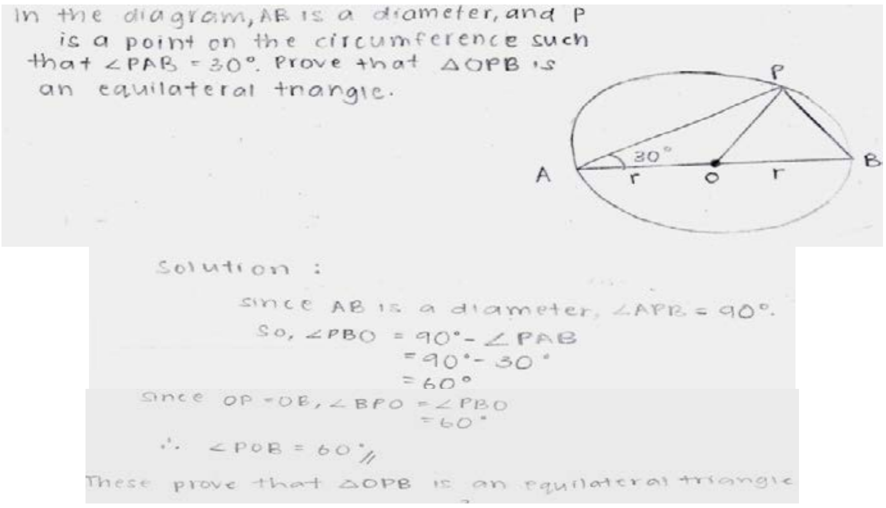


Figure 1. Student F341 posed problem and solution

b) *Direct Recall Problem*

A *direct recall* problem only has an assignment proposition in its solution path. Work by student T338 in Figure 2 is an example of a *direct recall* problem. It involves the *sum of angles at a point*. Such problems usually produce a direct link between the initial state and the goal state.

$360^0 – 300^0 = 60^0$

Figure 2. Student T338 posed problem and solution

Classification by Problem Information

a) *Edit Information*

Brown and Walter (1993, p. 23) proposed two problem-posing strategies. In "accepting the given", the poser poses a problem without changing what is given. The other strategy is "challenging" the given stimulus by making changes. Christou, Mousoulides, Pittalis and Pantazi (2005) referred to this "challenging" strategy as *editing* the information. Student W302 in Figure 3 *edited* the original condition that rope was not stretchable. *Edit information* was not used as a descriptor for Task 1 since there was insufficient context for editing.

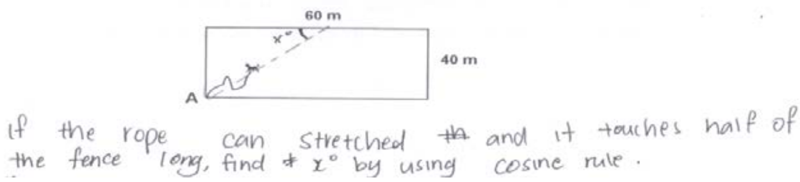
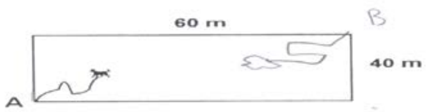


Figure 3. Student W302 posed problem and solution

b) *Add Object*

In Task 2, some students added objects or information into the initial state. Student N341 introduced another goat B into the initial state. This was analogous to the *selecting* process described in Christou *et al*.'s (2005) work when students added objects selectively to the initial state. The descriptor *add object* was only used in Task 2 since the nature of Task 1 had no contextual element to add to.



"*Another goat B was tied at B. If goat B could only be able to reach 25% of the grass in the farm and ate the grass, find the length of the rope. Leave your answer to 3 sig fig.*"

Figure 4. Student N341 posed problem and solution

c) *Over-Conditioning*

Over-conditioning occurs when there is extraneous information which does not contribute to the solution path and to the reaching of the goal state. In student T234's work in Figure 5, the angle 75^0 at point A is not needed for solving for *x*. *Over-conditioning* may suggest that the poser is not proficient in linking elements in the initial state to the goal state during posing. *Over-conditioning* was not found in posed problems in Task 2.

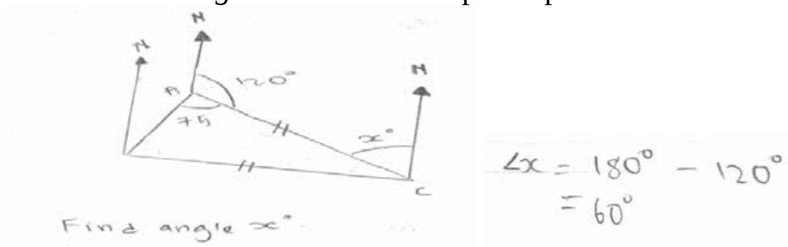


Figure 5. Student T234 posed problem and solution

d) *Implicit Assumption*

Implicit assumption occurs when students make unwritten assumptions about the given initial state. In student T309's work, an assumption was made about the triangle formed as being right angled and without which the problem could not be solved. *Implicit assumption* reflects flaws in the linkages between the initial and the goal states and the students' inability to fully articulate the initial conditions.

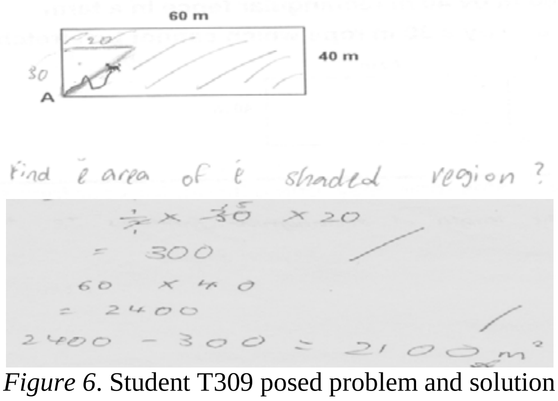
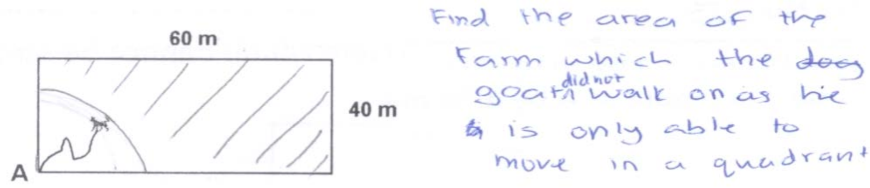


Figure 6. Student T309 posed problem and solution

Classification by Solution Type

a) *Multi Step*

A *multi step* problem involves more than one operation given by the student either as a repeated application of one operation or a combination of different operations in its solution path. Student N405's *multi step* solution in Figure 7 involves the *area of the sector* and the *area of the rectangle*. Since bridging the initial and the goal states requires the solver to build *multi step* in the solution path, *multi step* problems could be linked to problem complexity. But in the *single step* problem (non- *multi step*) by student T312 in Figure 7 there is only a need to state the answer by using the *interior angle* property of parallel lines.



[student N405]

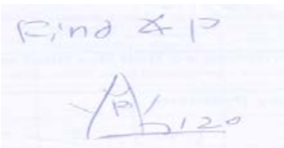
[student T312]

$180^0 - 120^0 = 60^0$

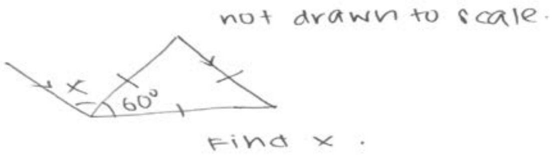
Figure 7. Students N405 and T312 posed problems

b) *Use of Algebra*

Algebraic variables as symbols in labelling points or vertices, sides and angles of figures were found in students' posed problems and their solutions.



. [student T425]



[student T502]

Figure 8. Students T425 and T502 posed problems

In Figure 8, students T425 and T502 had used *p* and *x* respectively to refer to "specific unknowns" (Kuchemann, 1981) in their posed problems.

Classification by Domain Knowledge

Posed problems reflected the extent of students' enactment of their learning of the related domain knowledge (Mamona-Downs and Downs, 2002). Table 2 shows students' use of the topics was dependent on the task type. Posed problems in Task 1 had the most number of geometric topics. More students used *Pythagoras Theorem* in Task 2. The topic on *angles in a triangle* was also the most common in Task 1. Students might have associated 60^0 with a triangle and were unwilling to go beyond this topic. Students also used the topics that they had learned in their earlier grades instead of the topics at their grade level to pose their problems across the tasks. One possible reason was their familiarity with these earlier topics.

Table 2

Table of Specifications of Topics Used

Main Geometric Domain Knowledge Used and Grade Level	Task 1 (%)	Task 2 (%)
1) Grade 7 Geometry topics		
i) Angles in a triangle	41.6	1.1
ii) Angles in straight line	20.3	0.6
iii) Alternate angles	15.7	0.6
iv) Corresponding angles	5.4	0
v) Vertically opposite angles	6.0	0
2) Grade 8 Geometry topics		
i) Pythagoras' theorem	3.0	18.5
3) Grade 9 Trigonometry topics		
i) Trigonometric ratios	14.6	9.6
ii) Angle circle properties	7.0	0
iii) Cosine / Sine Rule	4.3	2.8
iv) Angle of elevation / depression	3.0	4.1
v) Bearings	3.8	1.4

Association of Problem Type, Problem Information and Solution Type

For both tasks, problem type was found to be strongly associated with solution type. *Multi step* was strongly associated to *relational* problems across Task 1 and Task 2 with *phi* = .830 and *phi* = .918 respectively with p <.001. For both tasks, *multi step* was more common in *relational* problems than in *direct recall* problems. The involvement of multiple concepts within *relational* problems probably gave rise to more steps in the solution paths. The problem type and problem information (*implicit assumption*) were associated with *phi* = .196 and *phi* = .137 with p <.001 in Task 1 and Task 2 respectively. In each of the tasks, about 65% of problems with *implicit assumption* were *relational* problems. This perhaps showed students were not proficient in posing *relational* problems.

Implicit assumption was more common in Task 1. Students might have used *implicit assumption* in Task 1 (open-ended) because there were fewer structures to anchor the initial state and this might lead to mistakes in linking the initial and goal states. *Multi step* problems were more common in Task 1 (open).

In Task 2, the use of *edit* characterized the *direct recall* problems more than the *relational* problems. *Add object* was strongly associated with *relational* problems. In Task 2, some students formed *relational* problems by adding information or object to the initial state to create more complex problems. For *direct recall*, students just used *edit* to vary the given initial state. There were fewer *multi step relational* problems in Task 2 because of the lesser need to add context and hence the fewer occurrences of *multi step*. The absence of *over-conditioning* in Task 2 could be attributed to the more contextualized Task 2 and hence, the less need to add more 'information'. Figure 9 and Figure 10 show a summary of the key associations in Task 1 and Task 2 respectively. Table 3 shows a summary of the main characteristics of the posed problems.

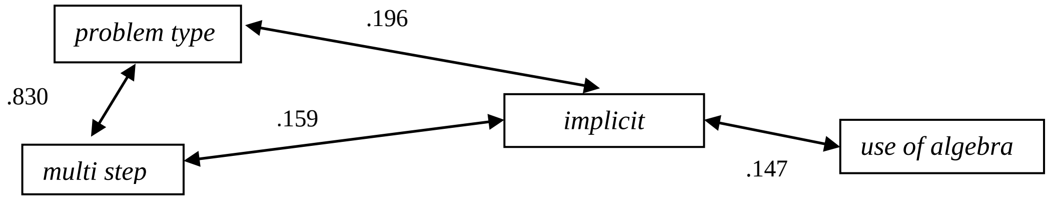


Figure 9. Summary of Associations in Task 1 ⟷ significant association (*phi* value)

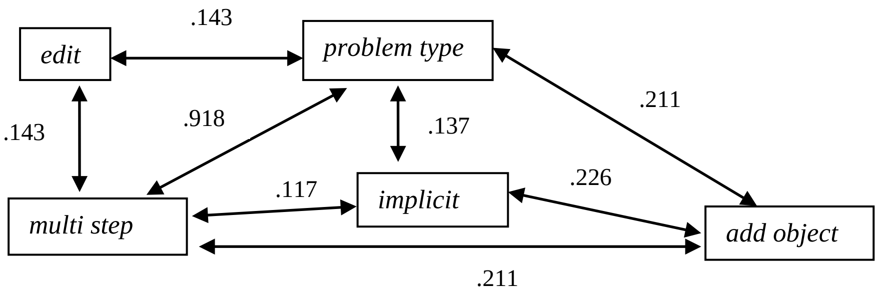


Figure 10. Summary of Associations in Task 2 ⟷ significant association (*phi* value)

Table 3
Main Characterization of Problem Types

Problem Type	Description	Task 1 (open)	Task 2 (contextualized)
Relational Problem	Problem Information	i) *over-conditioning* not associated with problem type and solution type ii) more likely to contain *implicit assumption*	i) *add object* more likely in *relational* ii) more likely to contain *implicit assumption* iii) *over conditioning* not present
	Solution Type	i) *multi step* likely to be present ii) *use of algebra* not associated to problem type	i) *multi step* likely to be present
Direct Recall Problem	Problem Information	i) *over conditioning* not associated with problem type and solution type	i) *over conditioning* not present
	Solution Type	i) fewer cases of *multi step* ii) *use of algebra* not associated to problem type	i) fewer cases of *multi step* ii) fewer cases of *use of algebra*

Conclusion

Most of the earlier work on problem posing largely involved arithmetic word problems, so the present study on geometric posing tasks contributes to the knowledge in problem-posing performance. The presence of unsolvable posed problems and the presence of *implicit assumption* in *relational* problems suggest that problem posing may not be a set of natural skills that students possess. This could be due to the less emphasis of such skills in the school mathematics curriculum. That students used more geometric topics in Task 1 suggests that open-ended tasks could elicit from students a wider spread of topics. Students' ability to link various topics in *relational* problems suggests that problem posing can also foster students' ability to see connection across topics. This ability to see connection is important for interpreting and solving mathematical problems. Like problem-solving heuristics, teachers may have to teach problem posing explicitly as part of the classroom instructional programme.

A suggestion for a research agenda would be to further examine problem-posing responses across task types and involving other domains in school mathematics like statistics or probability. Such work could be useful in the design of intervention studies to promote problem-posing skills. More work may be needed to examine gender and problem-posing performance. There is still the need for more follow-up studies on a bigger sample. The present study nevertheless contributed to knowledge in this area.

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Afternote:

This work was done during my attachment at the National Institute of Education, Singapore and that the views expressed here are entirely mine and not that of the Ministry of Education, Singapore.