Mathematical Knowledge for Teaching of MERGA Members

Rosemary Callingham
University of Tasmania
<Rosemary.Callingham@utas.edu.au>

Kim Beswick
University of Tasmania
<Kim.Beswick@utas.edu.au>

Julie Clark
Flinders University
<julie.clark@flinders.edu.au>

Barry Kissane
Murdoch University
<kissane@murdoch.edu.au>

Pep Serow
University of New England
<pserow2@une.edu.au>

Steve Thornton
Charles Darwin University
<Stephen.Thornton@cdu.edu.au>

Although there has been considerable interest in teachers' knowledge for teaching mathematics for nearly a quarter of a century, little attention has been paid to the knowledge of mathematics teacher educators. The responses of 57 MERGA members to an online survey addressing beliefs about mathematics and its teaching, mathematics content knowledge and mathematics pedagogical content knowledge are reported. Teacher educators found the items addressing pedagogical content knowledge more difficult than mathematics content questions or endorsing beliefs, and the type of employment appeared to be a more important influence on outcomes than the level of mathematics studied.

Introduction

From the time that Shulman (1987) identified different aspects of teachers' specialised knowledge used in teaching, there has been considerable interest in describing and measuring specific aspects of this knowledge. The knowledge of those who teach pre-service teachers however, has been relatively unexplored (Jasman, Payne, Grundy & Del Borrello, 1998). In this paper the initial findings from a survey undertaken by 57 members of the Mathematics Education Group of Australasia (MERGA) members addressing aspects of their knowledge for teaching mathematics are reported.

Prior Research Base

As suggested, the focus of most studies about knowledge for teaching has been on mathematics teachers. Mewborn (2001) in her useful overview of studies of teachers' knowledge of mathematics indicated that measures such as the number and level of mathematics courses taken during training were not related to students' learning outcomes. Further, studies that focussed on how teachers' knowledge of mathematics influenced their teaching showed that the outcomes were far from straightforward. Effective teachers of primary mathematics are those having a deep connected understanding of the mathematical principles and concepts involved in the curriculum (Askew, Rhodes, Brown, Wiliam & Johnson, 1997; Ma, 1999). Such understanding however goes beyond mathematical content alone.

Shulman's (1987) notion of pedagogical content knowledge (PCK) stimulated considerable research within the mathematics education community into the nature of teachers' knowledge for teaching mathematics. Shulman described PCK as

> the blending of content and pedagogy into an understanding of how topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue. (1987, p. 8)

Although content knowledge in mathematics has been measured for many years attempts to develop measures of PCK are relatively new. Hill and her associates (Hill, Schilling & Ball, 2004) developed items to measure teachers' mathematical knowledge for teaching, which included aspects of pedagogical and content knowledge. Chick (2007) provided a useful

In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), Mathematics education: Expanding horizons (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia). Singapore: MERGA.

framework of categories of PCK in mathematics teaching derived from observation studies. Callingham et al (2011) developed an online instrument that addressed three aspects of teachers' knowledge: beliefs about mathematics and its teaching, mathematics content knowledge (MCK) and PCK. Beswick, Callingham & Watson (2011) showed that a similar instrument comprising beliefs, content knowledge and pedagogical content knowledge in mathematics created a single "thick" construct of teachers' knowledge for teaching mathematics. Internationally, the Teacher Education and Development Study in Mathematics (TEDS-M) (Tatto, Schwille, Senk, Ingvarson, Peck & Rowley, 2008) obtained outcome measures of mathematics content knowledge and mathematics pedagogical knowledge from pre-service primary teachers across 16 countries using a paper-based test, and also collected information about related teacher education courses. One result from this study was that opportunity to learn both mathematics and mathematics pedagogy was critical to program outcomes (Blömeke, Suhl, Kaiser, & Döhrmann, 2012). Baumert et al (2010) found that in German Year 10 classrooms both teachers' content knowledge and mathematics pedagogical knowledge were important, but that mathematics pedagogical knowledge had a greater influence on students' learning. Given this background, and the attempts to link teachers' mathematical knowledge to students' outcomes, it is surprising that so little work has considered the knowledge of mathematics teacher educators.

Background to the Current Study

The study grew out of a 2-year project, funded by the Australian Learning and Teaching Council (ALTC), that aimed to develop and use tools to provide an evidence base for improving mathematics education in universities (Callingham et al, 2011). Surveys had been developed for pre-service teachers and lecturers in order to investigate levels of both content and pedagogical content knowledge. A number of MERGA members expressed interest in examining the survey used with pre-service teachers, and indicated that they would also be willing to undertake a version of the lecturer survey. In addition to questions addressing beliefs about mathematics and its teaching, MCK, and mathematics PCK, a number of questions addressed background variables such as level and type of appointment, experience at teaching in both schools and at tertiary level and mathematics and educational backgrounds. The current study addresses the following research questions:

1. What is the mathematical knowledge for teaching of MERGA members who responded to the survey; and
2. What factors appear to influence this knowledge?

Method

The survey was sent out via the MERGA member list in the second half of 2011. Respondents were required to accept the ethics declaration about voluntary consent at the front of the survey. In order to meet both ethical standards and allow for professional interest, respondents who chose not to accept the declaration were not redirected out of the survey, as is common in online surveys. Instead, any respondents who did not accept the voluntary consent statement were removed from the data set prior to any analysis. A survey identical in content but slightly different in demographic information was provided to the seven universities participating in the project. The two data sets were combined for the purpose of this analysis, and it is assumed that lecturers in participating institutions were MERGA members or eligible for MERGA membership.

The survey comprised four main sections: background information, 10 items addressing beliefs about mathematics, 10 items focussed on MCK and 10 items that dealt with PCK. One open-ended question was included so that respondents could comment on aspects of the survey if they wished. Both MCK and PCK items included questions appropriate to primary and secondary mathematics. Respondents were asked whether they taught in pre-service primary or secondary courses, and were directed to the appropriate question sets as a result. Examples of primary and secondary MCK and PCK items are shown in Table 1.

Table 1.
Examples of MCK and PCK Items

Item category	Example [1]
MCK Primary	Steve buys a shirt that is discounted by 10% on the ticket. A sign on the rack stated, 'Discount by a further 15%'. This is the same as a discount of what percentage of the original price of the shirt? A) 12.5% B) 15% C) 17.5% D) 23.5% E) 25%
MCK Secondary	A student picks a value for x, and uses it in the function $f(x) = 2x^2$ to get an answer. He then picks a new value for x, which is 3 times his original choice. His new answer is … A) 3 times his old one B) 6 times his old one C) 9 times his old one D) 18 times his old one E) 36 times his old one F) Can't tell how the new answer relates to the old one
PCK Primary	Your class is exploring measurement concepts. Students make the following statements. Which one of these most urgently requires teacher intervention? A) Area is the space inside a shape. B) As the perimeter increases, the area always increases. C) Volume is the amount of space a shape takes up. D) Area is a measurement of the surface.
PCK Secondary	Below is a student's incorrect attempt to solve a pair of simultaneous equations: $3x - y = 7$ (1) $x + y = 9$ (2) $2x = -2$ $x = -1$ Substitute into (2): $-1 + y = 9$ $y = 10$ The student checks her solution via substitution into equation (1) and is surprised to see that it is does not make the equation true. Which of the following statements gives the most likely explanation for the student's error and the most appropriate next step for the teacher to take? A) The student tried to subtract equation (2) from equation (1) but "cancelled" the $–y$ and $+y$ to eliminate this variable. The teacher should tell the student to add the equations instead. B) The student tried to subtract equation (2) from equation (1) but "cancelled" the $–y$ and $+y$ to eliminate this variable. The teacher should ask the student to explain why she chose to subtract rather than add these equations. C) The student tried to subtract equation (2) from equation (1) but "cancelled" the $–y$ and $+y$ to eliminate this variable. The teacher should suggest that the student use the substitution method instead, by finding an expression for x or y from equation (2) and substituting this into equation (1). D) The student has incorrectly added equations (1) and (2). The teacher should ask the student to repeat this addition, ensuring she does it correctly.

[1]Note. Formatting removed for space reasons.

All 10 beliefs (BLF) items were presented to every person. MCK and PCK questions were presented randomly from the pool of items developed. Altogether 42 MCK questions and 31 PCK questions were presented. Some questions had multiple parts based on the same stimulus that were scored as separate items so the final number of items scored was greater than the number of questions presented. In both primary and secondary versions of the survey common questions were included so that the two surveys could be linked using Rasch measurement approaches (Bond & Fox, 2007). Among both MCK and PCK questions there were some that all respondents answered correctly. These questions were removed from the data set. Not every person responded to every item. The final data set for analysis is shown in Table 2.

Table 2.
Analysis Data Set

Scale	Number of respondents	Number of items	Number of link items
BLF	57	10	10
MCK	40	43	21
PCK	44	60	9

The data were analysed using Rasch measurement (Bond & Fox, 2007) and measures of performance in logits—person ability measures—were obtained for each of the three scales of BLF, MCK and PCK. These measures were imported into SPSS for further analysis.

Results

Characteristics of Respondents

A summary of the level of appointment and type of employment of the people who responded to the background information questions is shown in Table 3. The Continuing category tended to have the higher level (Level C and above) staff whereas most fixed-term contracts were at Level B. Those people at higher levels in the Casual category are likely to be retirees who are still active in the mathematics education community.

Table 3.
Summary of Respondents by Nature and Level of Appointment

	Level A	Level B	Level C	Level D	Level E	Total
Continuing	0	8	6	9	0	23
Fixed term	1	11	1	0	0	13
Casual	8	2	1	1	2	14

Over three-quarters of respondents (44/57, 77.2%) held postgraduate qualifications but a surprising proportion (17/57, 29.8%) had not formally studied mathematics beyond school level. Only one person indicated indigenous status and six people spoke a language other than English as their mother tongue. Nearly half of the respondents (26/57, 45.6%) indicated that they had been lecturing in the tertiary sector for 5 years or less, and 13 of these were on continuing appointments. Respondents were also experienced teachers with nearly 40 percent (22/56, 38.4%) having taught in schools for more than 15 years, and across every state and territory in Australia.

Beliefs, MCK and PCK Scales

All three scales showed good fit to the Rasch model with fit indices lying within commonly accepted values of 0.77 to 1.30, indicating that the items worked together to measure the target construct in a valid manner (Bond & Fox, 2007). There was a highly statistically significant correlation between PCK and MCK (Pearson correlation = .49, p = .001) and between MCK and BLF (Pearson correlation = .33, p = .024) but not between PCK and BLF (Pearson correlation = .17, p = .28) suggesting that beliefs about mathematics and its teaching have less association with approaches to pedagogy than with ability to do mathematics.

Performance of Respondents on the Three Scales

The PCK items were, in general more difficult for respondents than either the BLF or MCK items, with a lower mean logit score. In contrast to pre-service teachers, however, there was not a distinct decrease in performance from BLF to MCK to PCK (compare with Callingham et al, 2011). The most likely explanation is that mathematics educators in general found the MCK items very easy and also strongly endorsed the belief statements, whereas the pre-service teachers may not have reflected deeply about their beliefs but accepted the rhetoric, and found the MCK items harder, and the PCK items hardest of all. Figure 1 shows the distribution of mathematics educators' logit scores on the three scales. There was a highly statistically significant difference between the overall mean logit scores on PCK and MCK (n = 44, t = 20.14) indicating that despite the association between these two variables, the respondents did not perform in the same way on the two sets of items.

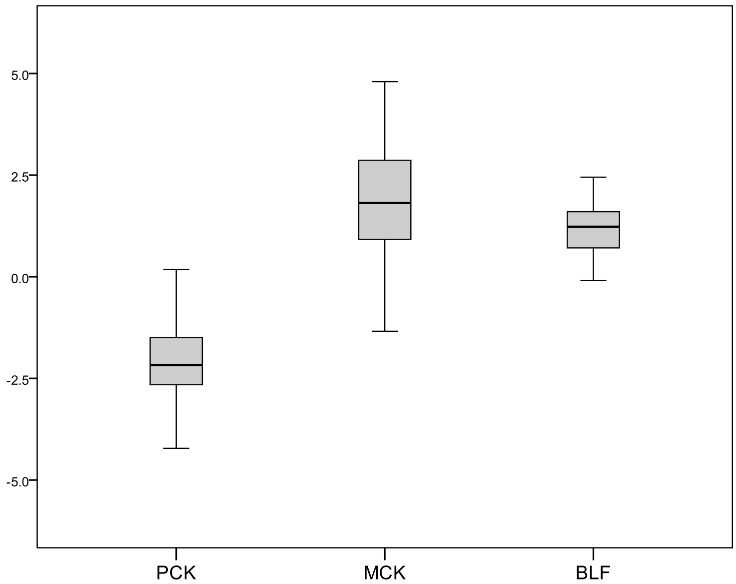

Figure 1. Distributions of mathematics educators' performance measures on scales of PCK, MCK and BELF.

Some interesting differences emerged when logit scores on each scale were analysed by type of appointment. PCK mean logit scores were lower than those on the BLF or MCK scales for all types of appointment but somewhat lower for Casual than other categories of appointment. In contrast, MCK mean logit scores were highest on the three scales for all types of appointment but somewhat higher for Casual than Continuing respondents. The findings are shown in Figure 2, including error bars. One-way ANOVA analysis indicated that there were no statistically significant differences between scores on any one scale between the three groups but with the small sample sizes this lack of significance is not surprising.

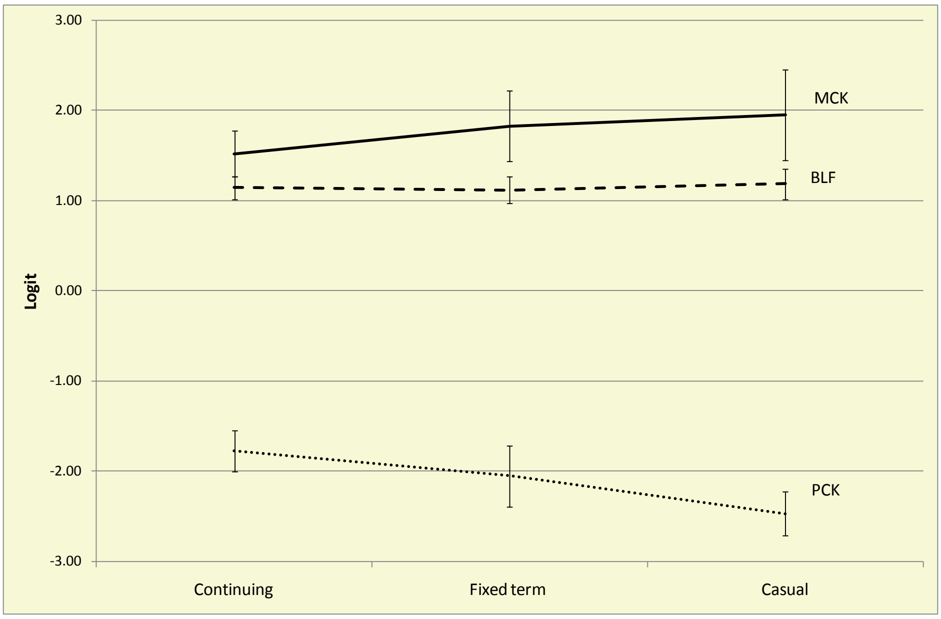

Figure 2. Mean logit scores on PCK, MCK and BLF by type of appointment.

A similar analysis was undertaken for the variable "years of tertiary teaching experience". Results are shown in Figure 3, including error bars. Of particular interest is the drop in the mean score on the BLF scale, suggesting that highly experienced mathematics educators were less likely to endorse strongly statements about the nature of mathematics and its teaching. The error bars on the BLF scale are also smaller than on the other two scales, suggesting less variation within respondents. PCK scores tended to be higher among experienced lecturers. Again none of the differences between the groups on any one scale was statistically significant using a one-way ANOVA.

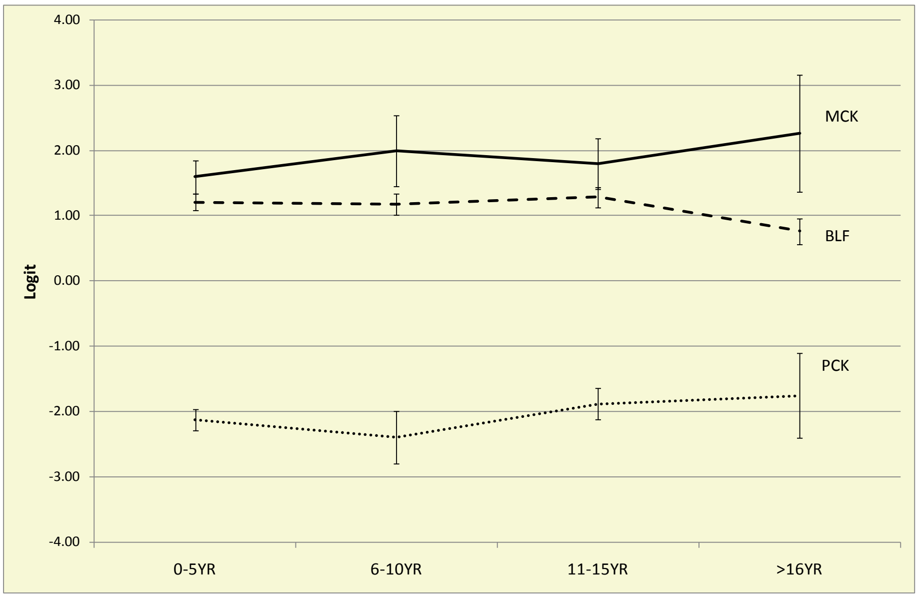

Figure 3. Mean scores on PCK, MCK and BELF by years of tertiary teaching experience.

Discussion

The initial findings from responses by mathematics educators to a survey instrument designed for pre-service teachers suggest some intriguing possibilities for further research. The instrument provided valid measures of mathematics educator knowledge but the three scales did not behave identically to those reported previously for pre-service teachers (Callingham et al, 2011). The results indicate that mathematics teacher educators had high levels of MCK, as would be expected. They did not endorse the beliefs items as easily as the pre-service teachers, possibly indicating a more nuanced understanding of mathematics and its teaching or the result of opportunities for reflection and discussion.

Mathematics educators found the PCK items the most challenging of all scales, as did the pre-service teachers. Although it is acknowledged that the PCK items do not cover the complexity of PCK (see Chick, 2011), it is argued that they do provide a crude measure of something more than MCK alone, especially given the highly significant difference between the performances on the two scales. In designing the PCK questions the ALTC project team engaged in deep discussion and heated argument about what aspects of PCK to incorporate and what kinds of choices to include in the selection of answers. At times it was impossible to choose a "correct" response, and for this reason some questions were scored with partial credit, allocating a 1 for a good response and a 2 for one that was deemed slightly better. As one respondent wrote in response to the open-ended question, "There were other options that I would have liked to be able to include in response to student's [sic] thinking." Mathematics educators are likely to have a range of views about what constitutes good mathematics pedagogy, but the finding that experienced mathematics educators had slightly higher performance on the PCK items than did less experienced colleagues may suggest that a level of agreement about PCK develops over time.

The finding that casual teaching staff had the highest level of MCK and the lowest level of PCK may indicate something about the types of people recruited to casual positions in universities. For a mathematics teacher educator position, the first requirement is to know some mathematics but this expectation alone may lead to many students being taught by teaching staff with limited understanding of the PCK needed to teach mathematics. For example, a good high school teacher recruited to take some primary mathematics courses in a teacher education program may have limited understanding about PCK in the primary classroom. The Casual group also included some Level D and E respondents who were highly experienced. It is tempting to surmise that without these people, the PCK knowledge of casual teaching staff might have been lower. The question of what tertiary mathematics teacher educators need to know in order to develop quality mathematics teachers—if you like PCK for teaching PCK—is emerging as an important issue.

The interactions of the BLF scale with the other two scales are worthy of more attention. There appears to be a strong association between beliefs and content knowledge but this is not so between beliefs and PCK. The idea that beliefs strongly influence how people teach may not hold for mathematics educators, but this can only be a conjecture at this stage. In addition, beliefs appear to be similar across all mathematics educators with only highly experienced lecturers showing any difference in the level of response.

This initial study of the knowledge of mathematics educators appears to break new ground. Larger studies and longitudinal measures are needed to identify the development of mathematics educators' knowledge about mathematics for teaching, and their own PCK for developing PCK in their students.

To conclude, one respondent wrote

> It is very difficult to teach something which you are neither competent in nor confident about - and I know that ends in a preposition but I am sure you know what I mean. Subject content knowledge has to be a co-requisite to pedagogical content knowledge for effective teaching.

The challenge for mathematics educators now is to identify what constitutes subject and pedagogical content knowledge in their specialised domain, and then to determine how this affects the outcomes for their students in terms of quality mathematics teaching.

Acknowledgments

This research was supported by the Australian Learning and Teaching Council Priority Projects grant number PP10-1638. In addition to the authors listed, Professor Merrilyn Goos and Assoc. Prof. Helen Chick are members of the project team and contributed significantly to the development and implementation of this study.

References

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., & Tsai Yi-Miau (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47*(1), 133-180.

Beswick, K., Callingham, R., & Watson, J. (2011). The nature and development of middle school mathematics teachers' knowledge. *Journal of Mathematics Teacher Education*. Available Online First at http://www.springerlink.com/content/th22781265818125/

Blömeke, S., Suhl, U., Kaiser, G., & Döhrmann, M. (2012). Family background, entry selectivity and opportunities to learn: What matters in primary teacher education? An international comparison of fifteen countries. *Teacher and Teacher Education*, *28*, 44-55.

Bond, T. G. & Fox, C. M. (2007). *Applying the Rasch model. Fundamental measurement in the human sciences* (2nd ed,). Mahwah, NJ: Lawrence Erlbaum.

Callingham, R., Beswick, K., Chick, H., Clark, J., Goos, M., Kissane, B., Serow, P., Thornton, S., & Tobias, S. (2011). Beginning teachers' mathematical knowledge: What is needed? In J. Clark, B. Kissane, J. Mousley, T. Spencer & S. Thornton (Eds.) *Mathematics: Traditions and [New] Practices* (Proceedings of the AAMT-MERGA Conference, July, pp. 900-907), Alice Springs, NT: AAMT-MERGA.

Chick, H. L. (2007). Teaching and learning by example. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, Vol. 1, pp. 3-21). Sydney: MERGA.

Chick, H. L. (2011). God-like educators in a fallen world. In J. Wright (Ed.) *Proceedings of the annual conference of the Australian Association for Research in Education, Hobart.* Available http://www.aare.edu.au/11pap/papers_pdf/aarefinal00667.pdf

Hill, H., Schilling, S., & Ball, D. (2004). Developing measures of teachers' mathematical knowledge for teaching. *Elementary School Journal, 105*, 11–30.

Jasman, A., Payne, L., Grundy, S., & Del Borrello, A. (1998). Uncovering the knowledge construction of university based teacher educators: what we learn from/with school based practitioners. In P. Jeffery (Ed.) *Proceedings of the annual conference of the Australian Association for Research in Education*, Adelaide. Available http://www.aare.edu.au/98pap/jas98305.htm

Ma, L. (1999*). Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* Mahwah: Lawrence Erlbaum Associates.

Mewborn, D. S. (2001). Teachers' content knowledge, teacher education, and their effects on the preparation of elementary teachers in the United States. *Mathematics Teacher Education and Development, 3*, 28–36.

Shulman, L. (1987) Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*, 1–22.

Tatto, M. T., Schwille, J., Senk, S. L., Ingvarson, L., Peck R., & Rowley, G. (2008). *Teacher education and development study in mathematics (TEDS-M): Conceptual framework. Policy, practice, and readiness to teach primary and secondary m.athematics.* East Lansing, MI: Teacher Education and Development International Study Center, College of Education, Michigan State University.