Mathematics as it Happens: Student-Centred Inquiry Learning

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This paper examines how mathematical understandings might emerge through student-centred inquiry. Data is drawn from a research project on student-centred curriculum integration (CI) that situated mathematics within authentic problem-solving contexts and involved students in collaboratively constructed curriculum. Participatory action research (PAR) was the methodology employed and mixed methods were used to collect data. The project took place in three primary school classrooms in New Zealand. The findings indicated that mathematics centred on real-life learning was highly engaging and that the measurement and geometric thinking explored went beyond New Zealand curriculum requirements.

Mathematics and statistics permeate everyday life - they are central to existing and contributing effectively in society. There is a diverse range of everyday practical situations where mathematical thinking is applied, while mathematical and statistical thinking are also applied in other learning areas (Ministry of Education, 2007). Often the application of mathematical knowledge is considered as something that is situated at the end of a learning process, as an application of learnt skills, but it should also be an aspect of the initial engagement (Freudenthal, 1968). Rather than commencing with certain abstractions or definitions to be applied later, some mathematics educators contend that the learning "must start with rich contexts that require mathematical organisation or, in other words, contexts that can be mathematised" (van den Heuvel-Panhuizen, 2010, p. 5). When learners engage in the investigation of a problem, interpretation influences the nature of that engagement. The perspectives that underpin those interpretations are conditioned by socio-cultural experiences (Gallagher, 1992) while the interpretations influence the ongoing understandings that emerge (Calder, 2011). Others contend that understanding emerges when students learn in ways that "engages one's identity on a meaningful trajectory and affords some ownership of meaning" (Wenger 1998, p. 270). Problem-solving contexts that relate to real-life situations enable this sense of ownership to evolve (Lowrie, 2004). It can also be argued that bringing student's naturalistic out-of-school experiences into more traditional settings has positive effects on problem solving. Meanwhile, Lowrie and Clancy (2003) maintained that student problem-solving was enhanced when traditional settings were complemented by student out-of-school experiences.

Studies have identified that students' motivation and persistence with tasks have increased when they engaged in authentic problem-solving situations (Nolan & McKinnon, 2003; Vars, 1997). Other researchers maintain that replicating situations in the classroom that resonate with out-of-school mathematical processes will promote mathematical thinking in real-life situations (e.g., Bonotto, 2002). Meanwhile, the learning environment assumes some primacy and will need to differ substantially from traditional settings for students to connect real-life and problem-solving contexts (De Corte, Verschaffel & Greer, 2000). They advocated that tasks need to be authentic, and as with real problems, that they be diverse and open, enabling multiple approaches and solutions. Others contend that problem solving should involve simulations of real-life community experiences where the mathematical thinking influences the solution of everyday problems (Lesh & Harel, 2003).

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Educators are also suggesting that student-centred inquiry, based on problems the students pose, utilise authentic learning contexts and lead to a strong sense of student ownership, enhanced student engagement and understanding, and motivation to learn (Beane, 1997, Brough, 2008, Dowden, 2010). The New Zealand curriculum advocates an inquiry approach and recently classroom environments have emerged that foster authentic inquiry to stimulate high–level thinking. Several new schools have been built and structured specifically to enhance student-centred inquiry learning. Student-centred curriculum integration (CI) encompasses similar pedagogical principles including student-negotiated curriculum and investigation of authentic problems.

Student-centred CI is a democratic teaching approach where relevant and meaningful contexts are central to curriculum design. Students pursue questions, issues or inquiries that are of genuine interest and curriculum is collaboratively co-constructed (Beane, 1997). The power-sharing pedagogy heightens student ownership and raises relevance for students, as they are fully involved throughout the learning process from the initial planning stage through to assessment. Subject area knowledge is employed in order to pursue pertinent issues of inquiry; for example, in mathematics children may explore strategies for calculating money or receive explicit teaching on place value and decimals to determine the costs of going on their class camp. Subject material is repositioned contextually and learning is strengthened, as children are motivated to acquire the skills and knowledge necessary to solve relevant problems. A substantive body of research supports the efficacy of student-centred CI with heightened achievement reported, student positivity and enhanced engagement (Beane, 1997, Brough, in press, Nolan & McKinnon, 2003, Vars, 1997). In addition, the nature of inquiries involves many skills that are not measurable through standardised tests such as the ability to negotiate, create knowledge, think creatively and critically, and work together for the common good. These kinds of competencies align with the New Zealand Curriculum (Ministry of Education, 2007). Contemporary integrative theorist James Beane (1997) advanced these notions further, theorising a curriculum design that used student-generated learning contexts, and involved students and teachers collaboratively constructing curriculum. Meanwhile, others have found that using digital pedagogical media for authentic mathematical inquiry enhances students' engagement (Salsovic, 2009; Calder, 2011).

This paper draws from a participatory action research (PAR) project that explored the implementation of the principles and practices of student-centred curriculum integration in the primary-school setting (Brough, in press). It examines the mathematics learning that was engaged with through the student-centred inquiry approach taken, and contends that the authentic engagement led to powerful learning opportunities.

Methodology

PAR was the methodology of choice, as its emancipatory research process was in keeping with the democratic pedagogy of student-centred CI. Hence, it enabled participants to explore a range of self-determined democratic inquiries. Mixed methods were used to collect data. These included: semi-structured interviews, focus group meetings, informal discussions (electronic and face to face), naturalistic observations, work samples, and photographs. The three teacher participants were early in their teaching careers, while the age range of the children spanned from 5 years of age to eleven. A case study approach was used to document this nine-month inquiry. The principles and practices that the participants explored included how to create more democratic learning environments, the use of relevant learning contexts and collaboratively constructed curriculum. While a broad range of disciplines were required to pursue the meaningful questions arising during the project, the focus of this paper is the mathematics that occurred. While two scenarios will be the discussed, the first is examined in more detail.

In all three schools, teachers were required to implement mathematics using the numeracy project. Children were grouped based on data attained during a diagnostic interview that assessed children's strategic and mathematical knowledge. The mathematics that evolved naturally as part of student-centred CI took place beyond these times through inquiries triggered by students' questions, classroom conversations, or incidents from within the classroom or the community.

Results and Discussion

The first scenario to be discussed arose as part of an open discussion that sought students' opinions on how they might contribute to decision-making and how their learning might be enhanced in class. After several iterations of discussion and review, a suggestion emerged that learning would be far more effective in a bigger classroom and that the children's small, prefabricated classroom should be extended. Predictably the children wanted to begin designing classroom extensions. Subsequently, conversations led to wanting to calculate the volume of their own classroom suggesting they could then compare it to the larger classroom next door. Over the subsequent week the propositions were explored using both measurement and geometrical thinking.

> Teacher (Toni): We started by talking about the space and how to use space in a classroom. We talked about how different shapes might work. They came up with exploring circles, rectangles, squares and other shapes, including irregular ones. I asked them to work in groups to consider different shaped classrooms. How would you make sure you could fit everything in? What gives you the most room for the size? What shapes worked best for doors and the furniture we have? They explored different shapes and drew sketch plans. One group even had a donut shape with the doors opening to the outside. When they wanted to open it into the middle they realised it wasn't going to work very well after all. They saw that there was no flow and the space would be too confined without a hallway off it.

From their initial conceptual drawings, the students soon determined the need to envisage their plans in 3-dimensional space.

> Toni: They drew floor plans and explored as 2D. They then set the plans out with multi-link cubes before exploding them up into their height. They built them up with the multilink cubes. They scaled them up with a centimetre cube block representing a cubic metre.

This led them back to the actual structure of their room and the problem of it being too small. They decided to measure the room. One group's discussion initiated an approach to measuring the room.

Sam	We need to know how big the room really is now.
Fran	How are we going to measure it?
Sam	Let's get a ruler and measure the length of the walls.
Tineke	And to the ceiling

One student was concerned with making sense of the information.

Sam	But what does that mean? How does that show us the space?
Tineke	Well we could find the area of the floor and see how much space we have to put stuff.

The nine and ten-year-old students were unfamiliar with the concept of measuring in square metres. The teacher challenged the class to work in small groups and make one square metre out of paper. This was used to estimate and measure different areas of the school. It helped the children gain an understanding of size and discover how length can be multiplied to determine area.

Toni: The students started with some measuring in groups. They found some lengths for the room, 10m long and 8m wide.

Toni talked about 2D and they identified things that were 2D. Someone suggested the carpet. What about the height of the carpet? The students considered making the 2D things into 3D.

Finn	We could turn the flat 2D stuff into things that you could sit on or put stuff on.
Sophie	Yeah sort of measure it up to 3D

The teacher prompted them.

Toni	How can we use this information? How do we make it into 3D?

The students continued:

Sam	What about making a cube like the multilink but bigger.
Ani	We could make paper cubes, like the multi cubes, scaling up.
Hine	We could roll up the paper to make the sides.

They make the cubes.

Figure 1. Students measuring with model of cubic metre.

They measure up the size of the room with the paper cubic metres. They rolled the cubes across the floor and marked off how long the classroom is. They started to measure up the wall as well to get the volume. Their classroom had a pitched roof, which presented an additional measuring challenge. They ran into a problem when they hit the sloped angled roof. The children were stuck as to how to solve this problem.

Eli	How are we going to measure that?

They tried things and made lots of suggestions:

Ben	We could bend down the sides.
Matt	Yeah, flatten the sides a bit.
Ash	Maybe we could make some other shapes.
Ben	People are measuring angles all the time. How are we going to get to it?
Chris	We could measure all around the outside.

They tried modelling it with bits of paper to show the profiles. Eventually the teacher gave them some scaffolding to change their approach a bit.

Toni	What about cutting the squares down the middle.

The teacher asked them to consider halving the apex triangle shape and seeing what they noticed to determine if that would help them to solve the problem. They went away in groups to discuss the challenge. Several groups cut up the triangle and worked out that if the two halves of the triangle were shifted this would make a rectangular shape which helped them to calculate the total linear area. They drew more squares and tried cutting them down the middle. They physically did it and moved the bits around. The children who generated this solution taught the other members of the class who were still grappling with the challenge.

The next-door class became interested in this activity and decided to measure their classroom using the same approach. The children made an initial estimation then working with the other class they determined the volume of both rooms in cubic metres. The children multiplied the length, width and height from their linear measurements and found the room next door was 11-12 cubic metres bigger. The teacher supplied additional explicit teaching to the children who required additional support to calculate the challenge. Another exploration that evolved from this was: How much water is there in the swimming pool? They decided to measure the sides to get the surface area. They then dropped a stick down into the bottom and marked off the wet mark to get the depth.

In the interim, students had been creating flow charts of the building process they anticipated would be required to bring their proposal to fruition. They also continued to work on the classroom extension plans that included adding on to either side of the building, or both, and second storey options. Recognising they would need help and advice the class arranged to consult with a builder and sought the expertise of a member of the city council who immediately offered to visit.

The councillor presented further mathematical challenges as building costs per metre were shared for single and two storey options. Additional costs were a building permit, architectural plans and any appliances required. The children inquired about wheel chair ramp regulations and sizes of windows to maximise sun and minimise heating costs. The councillor explained about heating cubic metres. The children raised the problem of a protected historic tree situated on one side of the classroom that led to a discussion on the length of roots and drip lines for trees. The children moved outside to estimate and step out the trees distance from the class. Subsequent to this visit the children calculated the costs of different options. They adapted their designs on the tree side of the classroom and reconsidered some of the more ambitious and costly designs.

While budgetary constraints prevented the extension becoming a reality, a large covered veranda has since been added to the classroom. The children reported that they enjoyed the project and the learning that took place. This context extended beyond the mathematics discussed including cross curricula links to literacy, technology and science. Student-centred CI provided an environment where the children were encouraged to make suggestions and consequently these children identified a learning context that they perceived to be highly relevant to their needs and learning. A high level of motivation and engagement was witnessed throughout the inquiry. In this instance, introducing children into considering volume extended children's mathematical knowledge further and helped children to understand how higher studs and roof apex can give the illusion of space and that additional cubic space does not always result in any additional floor area.

It is likely that the high level of student engagement in the classroom extension inquiry was a result of student involvement in determining the learning context. Student-centred CI was designed with middle-school students in mind (Beane, 1997). The questions or concerns the students have about themselves or their world are organised into similar areas of inquiry and pursued as learning themes. In the primary school setting, younger children are often more concerned with the here and now and consequently issues of immediate interest can be used as potential learning themes (Brough, in press). Themes may be initiated from small incidents that arise as part of daily programme or a community event that has attracted student interest. The second scenario to be discussed arose from such a situation when a drain digger arrived to create a large sump hole to rectify a drainage problem on the field. Spotting the arrival of a digger immediately captured the five-year-old children's attention. Students began asking questions and wanted to observe what was about to happen in the playground. Sensing an excellent learning opportunity the teacher escorted the class out to the field. The children began raising questions and making comments "How will they get the digger off the truck? How deep do you think the hole is? How will they make the hole bigger? How will they get the heavy pipe into the hole? If he digs anymore he will end up in space. Look the dirt is changing colour as he digs." In the initial phases of the project the teacher had focussed on establishing a classroom climate that was democratic, and empowering. She had actively sought student contribution and encouraged curiosity. The teacher achieved this by asking open-ended questions, not solving problems for children, and having high expectations of their independent research. Consequently, the children felt comfortable asking questions, making learning suggestions and solving problems for themselves.

The children took photographs of the digger making the hole and asked if they could talk to the driver so he could provide answers to their questions. The driver told the students the hole was to be four metres deep. He discussed what was happening and why, introduced the children to what porous meant and discussed the layers of the earth. While safety issues prevented students peering fully into the hole as they had requested, the children suggested a viable alternative was to ask the driver to photograph the hole. The children moved to the courts to work out how deep four metres is. The students made an initial estimation then decided they could use metre rulers to work out the problem. The students began measuring the four-metre depth but there were only three rulers, which presented a conundrum for the five-year-olds. Eventually, they determined they could reposition the first ruler to the end and consequently they successfully measured the holes depth. During the investigation one student had a moment of enlightenment when they announced: "Hey did you know these rulers are all the same length." The students lay down end on end and used a variety of other non-conventional measures such as their hands and body length to determine different ways to estimate then measure. They also poured water over various objects to understand what things were porous and what were not. The mathematics learning was invaluable in terms of applying problem-solving skills to a meaningful context and developing estimation and measurement skills, including using conventional and non-conventional measures.

Figure 2. 5-year-olds measuring 4m.

Conclusions

This paper examined mathematics as it emerged through student-initiated inquiries. During this inquiry the students had explored and effectively used two-dimensional representations of 3D objects, enhanced their understanding of area and volume, and made connections between the various measurement representations. The mathematical thinking and knowledge gained through the class extension investigation involved students learning how to apply multiplicative strategies for area and volume, and how to measure using metric units for length, area and volume. Further, the children needed to calculate various building costs that involved students having to apply additive and multiplicative strategies.

It was found that children were highly motivated and engaged with conceptual and mathematical understandings that extended well beyond curriculum year level guidelines, particularly in measurement and geometry. Links were made to other curriculum areas while the key competencies were also achieved including: thinking and problem solving, managing self, and participating and contributing (Ministry of Education, 2007).

Teachers in the project pursued issues of substance that arose on a daily basis and were negotiated alongside students. Teacher flexibility was essential in order for learning to be genuine, on the spot, and authentic. It required teachers to break away from more traditional structured mathematics programmes that commence with abstractions and instead use meaningful contexts that can be mathematised (van den Heuvel-Panhuizen, 2010). Teachers required democratic and empowering pedagogical understandings combined with excellent curriculum content knowledge and a high expectation of students.

In student-centred CI teaching takes place “just in time” as students require skills to solve particular problems. This enables learners to identify the purpose for the acquisition of skills. Teachers require excellent content knowledge to recognise the potential of children’s inquiries and to extend children into new subject material positioned within a meaningful learning context. While teachers of student-centred CI are required to assume a more facilitative and empowering approach to teaching, extending children’s mathematical thinking into areas they may not have considered is frequently required and explicit teaching is still vitally important.

This study was small in scale, and therefore it is difficult to draw generalised conclusions. Nevertheless, it serves to build upon emerging research on the benefits of contextualised learning in mathematics. It also demonstrates that content knowledge is not lost when using student-initiated learning contexts. Instead, it is repositioned within highly engaging learning contexts, while understanding is facilitated and enhanced.

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