# Mathematics Education as a Multicultural Field of Research and Practice: Outcomes and Challenges

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Mathematics education seen both as a field of research and as a field of practice is a multicultural field. For a while, this essential characteristic of the field has been underestimated but this is no longer the case. Both theoretical frameworks and empirical research help us today better approach this phenomenon, understand its impact on the field, reflect on research outcomes and the value of knowledge progressively built as well as on the challenges that we globally face as a community. In this lecture I would like to share with the audience my vision of this evolution and of the potential it offers for the field of mathematics education, relying on my personal experience as a researcher raised in a specific culture but also involved in many international collaborations, and on my engagement in ICMI, the International Commission on Mathematical Instruction, an institution which, for more than one century, has tried to contribute to the development of mathematics education through international exchanges and collaboration.

## Introduction

Mathematics education seen both as a field of research and as a field of practice is a multicultural field. For a while, this essential characteristic of the field has been underestimated but this is no longer the case. From the pioneering work of Ubiratan d'Ambrosio, ethnomathematics has developed as a specific domain in mathematics education, demonstrating the diversity and richness of the mathematical cultures which have accompanied and still accompany the development of human communities all over the world, and investigating how to make mathematics education benefit from this richness and diversity (D'Ambrosio, 2008). But the sensitivity to the cultural dimension of the field has extended far beyond the ethnomathematics community, thanks to the increasing influence of socio-cultural (Lerman, 2000), anthropological (Chevallard, 1992) and more recently socio-political approaches (Guttiérez, 2010), thanks also to the multiplication of research and innovative projects crossing cultural and educational contexts. As a researcher engaged in mathematics education for more than 30 years, I have experienced this evolution and observed how, progressively, it has impacted the field, research interests and practices. I have also experienced that there is a huge distance between the intellectual acknowledgement that mathematics education is a multicultural field and a practice which pays the necessary attention to contexts and cultures, which emancipates from predominant cultural views. In the first part of this text, I will review from this perspective ICMI history, showing how the process has been slow, even for such an institution created for fostering cultural exchanges and communications. I will then trace this progressive evolution in my own trajectory before evoking some recent projects which directly address this issue. I will end with some more global comments, balancing the problematic and positive dimensions of this cultural dependence.

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## ICMI: A Slow Progression towards True Internationalization

ICMI, the International Commission for Mathematical Instruction[1], was founded in 1908 at the International Congress of Mathematicians (ICM) in Rome, following an initial proposal made by the US mathematics educator David Eugene Smith in the journal *l'Enseignement Mathématique*, in 1905 (Menghini, Furinghetti, Giacardi & Arzarello, 2008). At that time of intense curricular reforms and debates regarding mathematics education, the ambition of the promoters of ICMI was clear: fostering international communication and exchange for stimulating reflection on sensitive issues such as:

- school organization, teaching methods and assessment modes;
- content issues, such as the relationships between geometry and algebra courses, the introduction of Calculus in secondary schools, the place to be given to applications with respect to pure mathematics, the differentiation of syllabi according to students' orientations;
- the preparation of teachers...

During the first years of its existence, under the Presidency of Felix Klein, the commission developed an intense activity. National subcommittees were created in 18 fully participating countries and 15 associated countries. These committees collected a huge amount of data, and produced more than 300 reports. This was certainly an impressive success but more in terms of data collection and descriptive texts than in terms of analysis and interpretation. Moreover, this was essentially a Western enterprise. Among the 18 fully participating countries, 16 were European countries, the two others being Russia and the USA. Asia was poorly represented with three countries only, China, India and Japan, all among the 15 associate members with no voting right. Moreover, until the Second World War, all the members of the ICMI Central Committees were from Europe and the USA.

After a period of reduced activity between the two World Wars, the Commission was re-established after the Second World War, becoming in 1952 a permanent sub-commission of the International Mathematical Union (IMU) charged with the conduct of the activities of IMU bearing on mathematical or scientific education. At that time, ICMI became more international with 27 member countries including India, Japan, Malaysia-Singapore, Pakistan, the election of the first non-Western member to its Executive Committee in 1954: Ram Behari from India, followed by Yasuo Akizuki from Japan and Aleksandr Danilovic Aleksandrov from USSR in 1958, and the development of actions in Latin-America, Africa and Asia. The first ICMI regional network: the *Comité Interamericano de Educación Matemática*, CIAEM, was established in 1962, at the initiative of ICMI President Marshall H. Stone after a conference held in Bogota in 1961. The links established at that time with UNESCO contributed to the strengthening of international exchanges and perspectives. Beyond the collaboration in the publication of the well-known series *New Trends in Mathematics Education* whose first volume was published in 1967, the support provided to the establishment of the first ICME (International Congress in Mathematics Education) in 1969 and to the CIAEM Conferences, ICMI and UNESCO co-operated for instance in the organization of two meetings held in Africa, one in Nairobi in 1974 on Language and Mathematics Teaching and one in Khartoum in 1978 on Developing Mathematics in Third World Countries.

With Hodgson's (2009) analysis, a new step was achieved when Shokichi Iyanaga became vice-President in 1971, and then President of ICMI in 1975, the General-Secretary being

[1] At that time, the commission was designated by CIEM, ICTM or IMUK according to the language used, French, English or German.

Yukiyosi Kawada[2]. That period coincided with the founding of the Southeast Asian Mathematical Society in 1972 and of national mathematical societies in Southeast Asia, these societies showing from the beginning evident concern for school mathematics and teacher education (Nebres 2009). This situation led to the organization of the first Southeast Asian Conference on Mathematics Education by the young Southeast Asian Mathematical Society with the support of ICMI in Manila in 1978, following a recommendation from Kawada. SEACME conferences were then held regularly every three years until 2002. In 1980s, when China rejoined IMU and ICMI, formal collaboration started to develop in East Asia beyond that existing in South East Asia with two ICMI – China Regional Conferences on Mathematics Education in Beijing in 1991 and then Shanghai in 1994, leading to the first EARCOME Conference in Korea in 1998, these conferences being held now every two years. Nebres pointed out that this situation

> introduced Southeast Asian mathematicians and mathematics educators to cultures with very successful approaches to mathematics education, which had significant differences from the approaches of the West. This was important as it invited our mathematics educators to ask whether we should not develop approaches more in conformity with our situations, rather than simply imitating trends from the developed countries. (2008, p.152)

East Asia was indeed emerging as a new center for mathematics education, due to the outstanding performance of its countries in international comparative studies such as SIMS and TIMSS (Robitaille & Garden, 1989), (Mullis & al., 1997). East Asian educational practices were valued and quickly became object of systematic study, collaborative projects began to develop with East Asian countries. The participation of East Asia in ICMI activities progressively increased leading in particular to:

- the organization of annual conferences of the ICMI Affiliated Study Group Psychology of Mathematics Education in East Asia: Tsukuba (Japan) in 1993, Hiroshima (Japan) in 2000, and Seoul (Korea) in 2007);
- the organization of an ICME congress for the first time in Asia: ICME-9 in Makuhari (Japan) in 2000;
- the launching of the East/West ICMI Study in 2000.

Extending ICMI outreach beyond the Western world and affluent countries was also a major ambition of Miguel de Guzmán, when he became President of ICMI in 1991. He was at the origin of the Solidarity Fund that permitted ICMI to take effective actions in Central America, Palestine, and some African countries, in the nineties. He was also at the helm of the Solidarity Task force held at ICME-8, Sevilla in 1996, which proposed a general policy of reserving a portion of the registrations fees for ICME congresses (typically 10%) to provide grants assisting the participation of delegates from non-affluent countries.

Under the presidency of Hyman Bass, Michèle Artigue and that of Bill Barton, extending ICMI outreach is still a major focus in the ICMI agenda (about 90 countries are today ICMI members), associated with the conviction that, beyond extending its outreach, ICMI had to find ways to give more exposure to the voices of those from outside the Western world and from the periphery on the international scene. Eventually, in 2000, ideas already expressed by Akizuki

> Oriental philosophies and religions are of a very different kind from those of the West. I can therefore imagine that there might also exist different modes of thinking even in mathematics. Thus I think that we should not limit ourselves to applying directly the methods which are currently considered in Europe and America to be the best, but should study mathematical instruction in Asia properly. Such a study might prove to be of interest and value for the West as well as for the East (1959).

[2] Since that time, East Asian countries have always been represented in the ICMI EC.

were implemented in the ICMI Study 13 mentioned above comparing mathematics education cultures from East Asian countries of Confucian tradition to those of Western countries (Leung, Graf & Lopez-Real, 2006). For the first time with ICMI Study 20 on multilinguistic issues in mathematics education, the responsibility of chairing an ICMI Study was given to two researchers from the periphery: Maria do Carmo from Brazil and Mamokgethi Setati from South Africa. For the first time also, in 2008, an ICME Congress took place in an emergent country: Mexico. Moreover, two more regional networks were created: EMF (*Espace Mathématique Francophone*) and AFRICME (*Africa Regional Conference of ICMI on Mathematics Education*). EMF is built on a notion of 'region' defined in linguistic rather than geographical terms, French being a common language among participants. It aims at supporting the participation to international exchanges of Francophone countries which suffer from the increasing domination of English as the international language of communication, and specific attention is paid to Africa. The sites selected for the conferences of this network which alternate developing and developed countries evidence this point: Grenoble, France (2000), Tozeur, Tunisia (2003), Sherebrook, Canada (2006), Dakar, Senegal (2009), Geneva, Switzerland (2012), Algeria (2015). The second regional structure is the African network, AFRICME, launched in 2005 for Anglophone countries of Africa. Using these new networks to facilitate contacts inside Africa, independently of the language, remains however a challenge to be overcome. The new program established in 2011 under Bill Barton's presidency, in collaboration with UNESCO: CANP (Capacity and Networking Program in the Mathematical Sciences) is the last outcome of this policy, involving the different communities in charge of teacher education in a given regional area, paying specific attention to regional context and needs, and fostering regional networking. This philosophy was perfectly reflected by the first realization in Bamako, Mali for Francophone sub-Saharan countries, in September 2011, paving the way for the next two planned in Costa Rica in 2012 and Cambodia in 2013.

There is no doubt that thanks to this evolution, ICMI has become progressively more and more sensitive to the cultural and social dimensions of mathematics education, and to the real challenges faced by mathematics education in a number of countries. At the ICMI Centennial Symposium in Rome, Nebres pointed out that

> The opening of ICMI to countries from Asia, Africa and Latin America also began to bring up concerns different from the central concerns coming from Europe and North America. Among the major concerns and issues that have entered the mathematics education discourse:
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> - Ethnomathematics, especially from Latin America and, in particular, the work of Ubiratan d'Ambrosio
> - Mathematics for All, coming in great part from UNESCO
> - The impact of society, economics and culture on mathematics education.
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> Voices from Latin America and from Africa have emerged in the ICMEs and other international and regional conferences. (2008, p. 152)

Despite all these efforts, the representation of the different cultures in ICMI activities remains today seriously unbalanced. This is even true, as pointed out in (Leung, 2008) for the Asiatic cultures mentioned above, despite the international interest they raise, and confirms that breaking with the prevalence of dominant voices and dominant cultures, which takes both visible and invisible forms, is not something easy to achieve. This is necessarily a long term enterprise, and a battle that we can never be sure to have definitively won. International institutions such as ICMI certainly can play an important role but, as individuals and members of more local communities, we have also to incorporate this sensitivity, and make it more than an intellectual idea. This leads me to the second part of this text.

## Reflecting on a Personal Trajectory

My career began at the time of the New Math period in the early seventies, and while preparing my doctorate in logic, I was asked by André Revuz, one of the professors at the Ecole Normale Supérieure in Paris to participate in the activities of the Institute of Research in Mathematics Education (IREM) recently created at University Paris 7 and headed by him. The IREMs[3] were and still are a very specific structure, where mathematicians, secondary teachers, teacher educators, researchers in mathematics education work together part time, developing experimentation and innovation, producing educational resources and organizing professional development activities for teachers. Some years later, André Revuz obtained the creation of an elementary experimental school attached to the IREM in the south of Paris, and with two other colleagues, I accepted to pilot the teaching of mathematics in this school. This creation was influenced by the on-going experience of Guy Brousseau at the COREM in Bordeaux (Brousseau, 2008), and we used and adapted many situations he had built there before developing our own constructions. This first experience was for me very rewarding. Everything seemed to work perfectly. Pupils were learning, teachers played their role, parents were happy… The emergent theory of didactical situations (Brousseau, 1997) provided an effective framework for the construction of situations, their management in the classroom, and the analysis of their outcomes, and our research, for instance on pupils' conceptions of the idea of circle contributed to its consolidation and development (Artigue & Robinet, 1982).

I could say nearly the same of my second important experience some years later when together with a group of physicists, we decided to create an experimental course for first year university students tightly connecting the teaching of mathematics and physics, introducing multi-disciplinary projects and computer laboratory sessions, which was very innovative at that time (1980). Everything worked fine except the planned common lecture on differentials, for which the mathematician and the physicist in charge were not able to conciliate their positions. With the physics educators of the team, we thus decided to take this theme of differential processes in mathematics and in physics as an object of research, combining historical and epistemological approaches, the study of didactical transposition in the two disciplines, the investigation of students' conceptions and difficulties, and the design and experimentation of teaching resources in the two disciplines. The project was also extended to integral processes with colleagues from Grenoble University (Artigue & al., 1988), (Artigue, Menigaux & Viennot, 1990). Once again, the theoretical frameworks we used combining the affordances of didactics of mathematics and physics efficiently supported our work, and conversely beyond its local results our research led to consolidate and refine them.

At that time, the early eighties, the French community of didacticians had decided to create specific institutions for supporting its collective work and development: a journal *Recherches en Didactique des Mathématiques*, a national seminar taking place two consecutive days three times per year, and a summer school of two weeks each alternative year. Thanks to these institutions, communication and exchanges were easy; capitalization of knowledge was systematically organized in a friendly atmosphere, which did not prevent fierce scientific debates. I had the feeling of being part of a community which was addressing important social issues, was creating efficient conceptual and methodological tools for addressing these issues in close connection with the terrain, and was accumulating results which constituted a coherent landscape. Interesting and challenging questions regularly emerged from research work, from the terrain, from curricular changes or technological advances, giving to the field an evident dynamics.

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[3] See the portal www.univ-**irem**.fr/

At that time, I already attended some international events but these had a marginal impact on my vision of the field, all the more as Spanish being my first foreign language, my understanding of English was very limited. The situation progressively changed for me in the late eighties, with the contribution to master and doctoral courses in Spain, the regular participation to PME and to its organization in 1989, my election at its international committee the next year, and a first collaborative project with Brazilian colleagues. But, looking retrospectively at this period, I would say that this internationalization was rather superficial. The courses I gave in foreign countries presented the French didactic culture. In PME I was exposed to other educational cultures but, at that time, essentially Western cultures, and regarding my first collaboration with Brazilian colleagues, I would confess that it did not escape the trap of cultural neo-colonialism.

Substantial changes occurred later. My increased engagement in ICMI activities (congresses and studies), and more especially my participation to the ICMI executive from 1998, played a crucial role. As mentioned above, we shared the ambition of extending ICMI outreach and of giving more voice to countries on the periphery, and in particular to African countries which, with the exception of South Africa, were poorly represented in ICMI. I learnt a lot from the privileged contacts I had with Jill Adler within the ICMI EC, and from the activities of the AFRICME and EMF regional networks. I also learnt a lot from the discussions around ICMI Study 13 launched in 2000 and from the resulting volume when it came out, and of course from my increasing participation in international research projects where the multiculturalism of the field was an essential issue, and where specific theoretical constructs and methodologies were used and developed for addressing it. All these experiences made an evolution possible, and the cultural nature of mathematics education something more for me than an intellectual claim.

Looking retrospectively at this journey, there is no doubt that this is a long and slow journey still unfinished. The fact that I began my career in a Western and developed country at a time when cultural issues and approaches were not on the top of the educational agenda, at a time when international exchanges and collaborations were not developed as they are today, certainly contributed to it. Moreover I conjecture that the strength of the didactic culture I was acculturated to and the home-grown nature of the theoretical frameworks progressively built in it were not facilitating conditions. Today, nevertheless, I find a powerful support for addressing the challenge of multiculturalism in this didactic culture and especially in the anthropological approach due to Chevallard (Chevallard, 1992), (Bosch & Gascón, 2006). In the next part, I will try to share this experience with the MERGA community, hoping that, despite the cultural distance, my discourse will make sense for it and my concerns will resonate at least partly with its preoccupations.

## A Situated Anthropological Perspective

As any sensitivity, sensitivity to the cultural dependence of mathematics education, must be supported by appropriate constructs and methodological tools for being productive. With the development of socio-cultural approaches, the field of mathematics education today offers a diversity of theoretical frameworks and constructs for such a purpose. As many French colleagues, due to my cultural environment, I have found a support in the Anthropological Theory of Didactics (ATD). In this theory initiated by Chevallard, indeed, an initial postulate is that human knowledge emerges from practices which are institutionally situated thus a fortiori culturally situated. What it means to know fractions or functions for instance is not a universal but depends on the norms and values of the institution at stake. Quite early, I have experienced the power of this approach for addressing transition issues, supervising Brigitte Grugeon's thesis (Grugeon, 1995) on the transition between vocational and general education. She tried to understand why motivated and talented students from vocational education generally failed when they were offered the possibility of integrating general education. In reaction to the common educational discourse attributing this failure to students' cognitive characteristics, she conjectured that the main obstacle faced by these students and their teachers was the gap between the algebraic cultures of vocational and general high schools, a gap all the more difficult to detect as the syllabuses at stake seemed rather close. The success of the didactic strategy she developed from this institutional analysis proved the power of this approach. She decided indeed to value the vocational students' algebraic culture based on formulas, to consolidate it and organize its transition towards the functional world, instead of offering them the usual and unsuccessful transition courses focusing on the manipulation of algebraic expressions and solving of equations, two genres of task emblematic of the entrance in the algebraic culture in general education.

Nevertheless, it was only recently that I used a similar approach for addressing cultural differences between countries. I did it first in collaboration with Carl Winslow in a meta-analysis of comparative studies (Artigue & Winslow, 2010) and then in a Brazilian-French comparative study (Alves Dias, Artigue, Campos, & Jahn, 2010). I will present and discuss the latter example first.

### *A Brazilian-French Comparative Study*

This comparative study was part of a bigger project involving two institutions in each country, addressing both the issue of teachers' representations and practices, and the design and appropriation by teachers of educational resources. One of its foci aimed at the collaborative development of digital resources for the secondary-tertiary transition and, due to its mathematical and educational importance, it was decided to focus on the concept of function. Our sensitivity to cultural issues led us to consider that a preliminary step should be the elucidation of the functional culture in the two countries and of the characteristics of the secondary-tertiary transition in that area. For this purpose, we decided to rely especially on the ATD notions of praxeology and of hierarchy of levels of co-determination which had proved to be productive in the meta-study mentioned above. I introduce these notions which are not necessarily familiar to the MERGA audience below.

*Praxeologies and hierarchy of levels of co-determination:* In ATD all human practices, thus mathematical and didactical practices, are modelled in terms of praxeologies. In its simplest form, a praxeology is pointwise and structured around a *type of task* (for instance, finding an algebraic expression for a quadratic function given by its graphical representation), a *technique* (a way, not necessarily algorithmic, for solving this task), a *technology* which is a discourse used to describe, explain and justify the technique, and a *theory* within which the technological discourse itself can be justified. A praxeology thus combines a practical part formed by the first two components and a theoretical and discursive part. A fundamental assumption is that any praxeology necessarily has such a discursive component even if this is difficult to access or embryonic, and that it is especially the case for school praxeologies. The progression of knowledge goes along with the progressive development and structuration of praxeologies. Pointwise praxeologies group into local praxeologies sharing a common technology and local praxeologies into regional praxeologies sharing the same piece of theory. Mathematical praxeologies differ from one institution to another, defining what it means to know such or such piece of mathematics in a given institution.

For understanding the *raison d'être* of a particular praxeology in a given institution, its current functioning and potential evolution, one must investigate the conditions and constraints governing its ecology. These may be situated at very different levels, which is the source of the hierarchy of levels of co-determination (Chevallard, 2002), structured into nine different levels which, ordered from the lowest to the highest level, are the following: subject, theme, sector, domain, discipline, pedagogy, school, society, civilization (cf. Figure 1). This last level must be understood as the level corresponding to conditions and constraints which transcend one given society. For instance, ICME Study 13 mentioned above has made clear the influence on East-Asia mathematics education of the Confucian culture, and no one would negate that today international evaluations such as TIMSS and PISA exert an increased influence on educational systems worldwide.

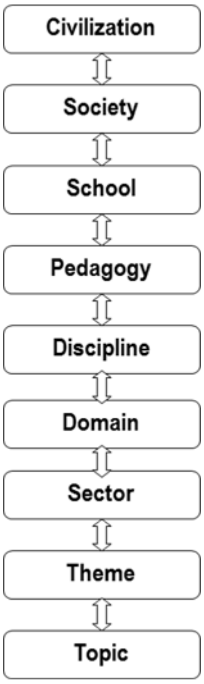


*Figure 1*. Hierarchy of levels of co-determination

For instance functions, a subject in grade 12 in France can be the variation of exponential functions, this subject being part of the theme of exponential functions, itself included in the sector of functions of one real variable, itself included in the domain of Analysis, a sub-domain of the mathematics discipline. But current teaching practices regarding this topic cannot be understood without considering influences situated at higher levels of codetermination, for instance, the insistence put by the CNP (Conseil National des Programmes) on the necessity of better connecting the teaching of scientific disciplines in France, in the nineties. This led to the introduction of exponential functions as solutions of differential equations modelling phenomena such as radioactive decay in the 2000 high school reform and influenced the teaching of the theme. Organizations, such as the one just described regarding exponential functions, are meaningful or not according to the educational context. Understanding the influence on the mathematical organizations observed in a given context of higher levels of co-determination is thus of crucial importance for making visible the constraints shaping mathematical and didactical praxeologies, for making these intelligible, and also for understanding what can be changed and how.

Coming back to the Brazilian-French study, for identifying the functional culture in the two countries, we tried to identify functional praxeologies and the conditions and constraints shaping these at different levels of co-determination. For that purpose, we collected a lot of data: curricular documents, textbooks, state or national examinations at the end of secondary school and also at university entrance in Brazil. I cannot here enter into the details of the analysis, but the resulting picture was that of two very different functional cultures, more than could be expected considering the common influences we are submitted to in this globalized world.

For instance, the traditional habitat of functions in senior secondary education in Brazil is the domain of algebra while in France it is calculus and analysis. This affects the curricular organization of subjects, themes and sectors related to functions, and the associated mathematical praxeologies in the two countries. In Brazilian secondary education, this algebraic tradition leads to give a particular importance to polynomial functions, and especially quadratic functions (theme level), and to the algebraic resolution of equations involving such functions (subject level). Coherently with this perspective, the introduction of parameters plays an essential role in the complexification of students' activity for a given praxeology and in access to generalization. In the French context, the study of functions in the two last years of high school is part of analysis (domain level). This contributes to the emphasis put on the study of the variation of functions including asymptotic behavior and the search for extrema (subject level), and also to exponential and trigonometric functions beyond polynomial functions (theme level). Effects of these differences in domains of inscription are even more visible when passing to the sector of sequences. For instance, in the Brazilian context, for the central theme of recurrent sequences a typical task is the determination of terms and reason for arithmetic and geometric sequences, on the basis of some partial information. The associated praxeologies are based on algebraic techniques and technology. Such a task is nearly nonexistent in France. A typical task is the study of the convergence of such sequences both qualitatively and quantitatively. The associated praxeologies use analytic techniques and technology, for instance theorems of convergence for monotonic sequences, the theorem ensuring that, if $f$ is a continuous function, the limit of a recurrent sequence $u_{n+l}=f(u_n)$ is necessarily a fixed point of $f$, or analytic techniques for overestimating the distance between two successive terms of the sequence or between the general term and the limit.

Such differences cannot not be understood without considering the highest levels of the hierarchy of co-determination. For instance, in France, differential and integral calculus has been part of the secondary curriculum since the 1902 reform whose ambition was to give scientific humanities a similar position to classical humanities (school and society levels). Achieving a better balance between quantitative and qualitative approaches towards convergence was an explicit aim of the 1982 reform rejecting the values of the New Math period, and its emergence was non independent on the facilities offered by technological advances (Artigue, 2011). In Brazil, the observed characteristics also reflect a tradition of algebraic teaching in secondary schools whose origin can be traced up to the turn of the nineteen century. It emphasizes transformation rules for algebraic expression and the solving of equations (Miorim et al., 1993). During the New Maths reform, the algebraic domain even gained in importance and set theoretical visions whose influence is still visible impacted the teaching of functions, but analysis remained a domain for tertiary education. The fact that mathematics education is not differentiated in Brazil at senior secondary level, contrary to France, that in France the scientific section is the "elite" section and that its syllabus influences those of other sections, certainly also contributes to this characteristic. The early influence of technological advances mentioned above for France did not have its counterpart in Brazil until recently, and one cannot doubt that higher levels of co-determination (society level) contribute to this difference. Even today, calculators are forbidden at most entrance tests to university in Brazil. Such a situation impacts lower levels, from the school one, and certainly contributes to the importance attached to the mastery of techniques for computations involving numbers or algebraic expressions. Another characteristic contributing to the stability of the algebraic Brazilian tradition is the importance traditionally attached to analytical geometry which also uses algebraic techniques (discipline level). This contrasts with the French situation where, due to a long term tradition, the teaching of geometry is more oriented towards synthetic geometry.

Our research also showed that, beyond these differences, the two traditions now have to cope with international perspectives promoting the values of an education for democracy and citizenship. Functional praxeologies are affected by this context in the two countries. In Brazil, this influence is especially visible in the tasks proposed at the national test ENEM taken at the end of high school, and in recent textbooks. Most of the ENEM tasks that we analyzed for instance, and all those involving functions, were linked to a real world context, and often societal or environmental issues. In France, this influence is also visible but it impacts less the teaching of functions at the end of high school, especially in scientific sections.

I will not enter into more details, but supported by these theoretical constructs, the efforts put at understanding these two functional cultures and their *raisons d'être*, deeply impacted our vision of the whole research project, and of the way we could envisage a productive collaboration in the production of educational resources for supporting the secondary / tertiary transition in the two countries. They revealed strengths and weaknesses in the two cultures, logics which were certainly different but equally coherent and could not be boiled over. Comparing with the way we had approached collaboration in the first project CAPES-COFECUB two decades before, this made a tremendous difference.

### *The ReMath Project*

The second example I would like to present is the European project Representing Mathematics with Technology, also known as ReMath. With this project, a new dimension enters the scene: the dependence of educational research itself on educational contexts and cultures. This project, led by Chronis Kynigos from Greece and mobilizing six teams from four European countries, has now officially ended but we are still working on the data collected and deepening our analyses[4]. Its ambition was to develop an integrated vision of the potential offered by digital representations for the learning of mathematics, and we considered that this goal could not be achieved without building appropriate connections between the different theoretical frameworks we were relying on. As made visible on figure 2 were the main theoretical frameworks used by ReMath teams are presented, there was an important theoretical diversity, which reflected the diversity of our respective research cultures. This diversity affected our vision of the representative potential of digital technologies for mathematical learning, and of the design and use of digital artifacts.

[4] Main deliverables, digital and educational resources produced in the frame of ReMath are accessible on the website: http://remath.cti.gr

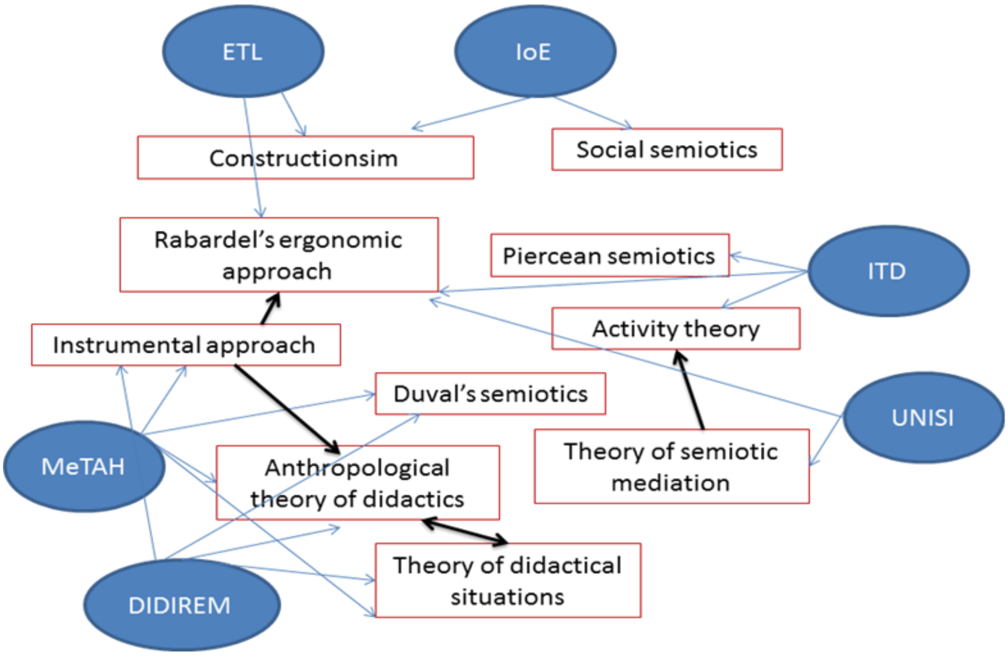


*Figure 2*. The initial theoretical landscape

In figure 2 above, the shaded ovals correspond to teams: ETL (Greece), IoE (UK), ITD and UNISI (Italy), MeTAH and DIDIREM (France). Note that this landscape already mentions connections, but these were internal to a given team or a given culture. The ambition of ReMath was to make networking progress beyond these cultural frontiers.

We had experienced in a previous collaborative work within the European network of excellence Kaleidoscope the limitations of strategies of theoretical networking based on mutual readings, exchanges and discussions, and learnt the importance of creating an appropriate *milieu*[5] for interaction. This is what we did, taking our research and development practices - within the language of ATD research and development praxeologies (Artigue, Bosch & Gascón, 2010) - as objects of study, investigating how our respective theoretical frameworks impacted them. This was not an easy task for several reasons:

- the fact that theoretical frameworks and constructs, when familiar, tend to be used in a naturalized way, the technological discourse accompanying their use progressively fading;
- the fact that theoretical frameworks and constructs are often used more as metaphors than as operational tools, which makes the exact role they play in research and development decisions more difficult to identify;
- the fact that many decisions escape theoretical control.

These characteristics make the relationship between the practical and theoretical block of research praxeologies problematic. For taking up this difficulty, we developed a specific language, that of didactical functionalities and concerns, and a specific methodology, that of cross-experimentation and cross-case studies that I detail below.

In ReMath, each team (with one exception) was in charge of developing a Dynamic Digital Artefact (DDA) which was innovative in terms of representations, and in charge of two

[5] The term *milieu* is used here with the technical meaning it has in the theory of didactical situations (Brousseau, 1997), that is to say the meaning of antagonist milieu offering researchers retroactions but also some resistance for preventing them to adapt and even distort theoretical frameworks and constructs in their desire of establishing connections.

experimentations, one with its own DDA and one with a DDA developed by another team from another country. The experimentation process was piloted by a very precise guide, from the design and a priori analysis of the experimentation, making explicit didactical functionalities and concerns, to its a posteriori analysis. Interactions between teams, and especially those experimenting the same DDA were also closely monitored as well as the cross-case studies.

Didactical functionality and concerns had been already introduced in TELMA with the ambition of approaching theoretical frameworks through their functionalities. They contributed to the creation of what Radford referring to what Lotman calls a *semiosphere*, that is

> an uneven multi-cultural space of meaning-making processes and understandings generated by individuals as they come to know and interact with each other (2008, p.318),

and what he claims to be a condition for developing productive theoretical networking.

More precisely, the notion of didactical functionality had been introduced for supporting the instrumental vision of the theoretical reflection concerning both the design of digital artifacts and the design and analysis of their use. For a given artifact, it aims at capturing its properties (or characteristics) and their modalities of employment, which may favour or enhance teaching/learning processes according to a specific educational aim. For that purpose, it is structured around three key elements:

1. a set of features / characteristics of the digital artifact
2. an educational aim
3. modalities of using the tool in a teaching/learning process referred to the chosen educational aim.

Artigue and Mariotti (2012) pointed out that

> Designers of educational artefacts have didactical functionalities in mind and these orientate design, but as evidenced by research users have also at this level a creative role and contribute to the didactical functionalities of artefacts through the modalities of use they envisage and implement. As, for both designers and users, the didactical functionalities identified are dependent on their theoretical culture, didactical functionalities can be used as windows on the instrumental dimension of theories. Hence, the importance attached to didactical functionalities in the ReMath methodology.

Beyond that, it was supposed that different teams involved in a common project as was the case for ReMath, even when living in different contexts and cultures and relying on different theoretical frameworks, more or less face similar problems and ambitions, and have shared sensitivities. The notion of concern was introduced for capturing these commonalities taken as a possible basis for communication and integration of perspectives. A list of key concerns was attached to each component of didactical functionalities, and the different teams were asked to rate these on a scale from 0 to 5, to make explicit the theoretical frameworks or constructs associated with if any, and to make clear if their use was metaphorical or operational. It was of course expected that the distribution of key concerns would be different from one team to another. Moreover it was expected that even when a particular concern would be given a similar importance by two teams, it would not necessarily be associated with the same theoretical constructs. Understanding these differences and their possible impact on design decisions was considered crucial for the success of the networking enterprise.

Another important ingredient of this methodology was that a common question for investigation through the cross-experimentation process had been fixed. It was the following:

> How can the representations identifiable in the DDAs be put in relationship with the achievement of specific educational goals?

The researchers of each team were asked to rephrase this question in their own language, and to complement it by questions of more specific interest for them. Thanks to this system of common and specific questions, we created in fact a new research praxeology on top of the research praxeologies proper to the different teams, and coordinated with these. This coordination was made in such a way that the praxeologies which became object of study were not distorted by the research praxeology created on top of them. Thanks to this construction, each cross-experimentation became both a research praxeology for the two teams involved, rooted in their specific cultures, and a research praxeology which took these as object of study for answering questions related to networking.

This original construction was essential to the success of ReMath whose outcomes cannot be summarized in a few sentences. It helped us understand better our respective research cultures and the ways these impacted our vision of technological affordances in terms of representation, to build a consensus around these affordances structured around 10 assertions, each of these supported by experimental evidence and expressed in a language accessible to a wider audience, to enrich with trans-cultural connections the initial landscape, to identify productive complementarities but also to point out some limitations to networking efforts and understand the reasons for these.

## Final Comments

In this text, starting from the fact that the field of mathematics education is a multicultural field, and relying on my personal experience of this multiculturalism, I have tried to show that accepting the full consequences of such an intellectual position is not so easy as it could appear. I did it first at an institutional level, taking as an illustrative example the history of ICMI, the International Commission for Mathematical Instruction, and showing its slow and still unachieved progression towards true internationalization, despite its foundational aim of international communication and exchange. From this institutional and international level, I skipped then to an individual level, reflecting on my personal trajectory and on the difficulties I had to face. I also tried to show how the global evolution of the field of mathematics education makes that fortunately the situation is quite different for the new generations. This is certainly due to the intensification of international exchanges and to the fact that the dominant educational cultures of the past are today challenged by other educational cultures, for instance those of East Asia. But this is also more fundamentally due to the fact that the field of mathematics education as a field of research has drastically evolved, moving from the cognitive study of the single learner to the analysis of the complex joint action of teachers and students (Sensevy, 2011), seen as a socially and culturally situated process, and to the analysis of the condition and constraints which up to the most global levels shape the functioning of didactical systems and of their actors. This is also because we have accepted to acknowledge that the field of mathematics education is a field where knowledge cannot be separated from values. Acknowledging the multiculturalism of mathematics education is in fact a political position.

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