EXPLORING DIVERSITY: YEAR 2 STUDENTS' RESPONSES TO QUESTIONS CONCERNING SIMPLE 2D SHAPES

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Recent developments in the Level theory of van Hiele have provided a framework in which the thinking of young students can be explored. This study involved 12 Year 2 students and explores in detail the diversity in responses of six students as they explained their understandings of simple 2D shapes. The focus of the research was on interpreting and exploring the implications of students' language and it highlighted important issues relating to the relevance of the van Hiele model to primary school education.

BACKGROUND

The van Hiele theory (van Hiele, 1986) has been subjected to extensive research in English-speaking countries for nearly 20 years. As a result, there is a large body of empirical support for the hierarchical levels (particularly Levels 1 to 4) identified with the theory and the testing of senior primary, secondary and tertiary students.

Despite this overall positive support for the level aspect, the theory has not been useful in explaining the thinking of younger students at primary school (Clements and Battista, 1992; Lehrer, Jenkins & Osana, 1998). This has been due in part to the fact that the van Hieles were secondary mathematics teachers and hence the intent of their theory was directed towards the group that was most relevant to them. However, recent work on the theory (Pegg, 1997) has offered the potential to address this issue. Of value, has been the splitting of van Hiele's Level 2 (where the student identifies a figure in terms of its properties but these are seen as being independent) into two categories referred to as Level 2A where the focus is on a single property and Level 2B where the focus is on more than one property. This development offers the opportunity for a more focused exploration of the movement from Level 1 (where shapes are known by their global appearance) to Level 2A. While a developmental pattern has been hypothesised (Pegg & Davey, 1998) these writers have also called for the need to explore student diversity during this period of transition between levels. This paper takes up this challenge by considering the detailed responses of six Year 2 students who were interviewed concerning their understanding of simple 2D shapes following the completion of a written questionnaire.

RESEARCH DESIGN

The research question which guided the study was: What diversity can be identified in young students' descriptions of, and language concerning, basic 2D shapes?

In order to examine responses between Level 1 and Level 2A, and thus explore the development of property concepts, a study was carried out with 12 Year 2 students aged between 7.5 and 8.5 years. This age cohort was chosen as this appears to be the time at which responses in geometry are more likely to move from Level 1 towards Level 2.

Sample

The sample for the study was taken from a co-educational primary school in an inner-city suburb of Sydney. The teacher of a Year 2 class nominated twelve students to participate in the study. The teacher selected four students in who were considered to be above average, four who were considered to be average and four who were considered to be below average in their spatial ability.

The sample chosen by the teacher consisted of five girls and seven boys from a variety of socioeconomic and cultural backgrounds. All students had completed units of work on 2D shapes prior to the investigation, with a particular focus on investigating triangles.

Data Collection

The first phase of the study centered on the 12 students' responses to a written questionnaire containing five items. Following this, continuing access to the sample meant that six students from the original sample participated in an individual interview session. These sessions took an average of 30 minutes and were audio taped, and later transcribed. Both the questionnaire and the interview were designed to explore the students' understandings of squares and oblongs.

In this paper, only data pertaining to those children who participated in both the questionnaire and interview phases of the study are discussed.

Questionnaire Items

Items 1- 4: Required students to name the figure drawn and to explain their reasoning in naming it. The figures presented in each item appear below. The same questions accompanied each figure, these were:

This is a _____.

Why?

Item 1 Item 2 Item 3 Item 4

Item 5: This question was included to give students the opportunity to both add to their descriptions of the individual shapes in Items 1-4 and/or to express similarities and/or differences which they saw between figures. The students were asked:

Can you tell me anything else about these shapes?

Comprehension Procedures: It was important to ensure that the students were not being assessed on reading comprehension ability and that a consistent structure was used with all students. As such, if a student was unable to name a figure presented, s/he was encouraged to "think about what (they) would call it". If a student was unable to explain their naming of a figure, the student's name for the figure was used in asking the question "how did you know this was a _____?".

Interviews

The interview consisted of two parts. In the first part students were given the opportunity to discuss and/or explain their responses to the written questionnaire they had completed. This consisted of clarifying students' written responses by asking questions of the type "what did you mean by _____ ?".

In the second part of the interview, students were asked a series of eight questions (with some probes and prompts) using concrete materials in order to explore further their understandings of squares and oblongs. The interview schedule followed is provided below.

Question Number	*Question (notes in italics)*	*Prompt/Probe (if necessary)*
q1	Can you draw me a square?	
q2	What sort of things did you have to think about when you were drawing your square?	
q3	How did you know that you had a square when you were finished drawing?	
q4	*(Place example drawing of a square next to student's square.)* Is your drawing the same as mine? Why/why not?	Is it the same shape? What is the same/
q5	What else can you tell me about your drawing?	
q6	If I do this to your drawing *(add length measurement in centimetres to one side of student's square),* what else can you tell me about your square?	
q7	*(Make square out of paddlepop sticks.)* What shape is this? Why? *(Move paddlepop sticks, to change orientation of square.)* What is it now? Why?	Is it still a square? Why/why not?
q8	*(Make square out of paddlepop sticks.)* What shape is this? Why? *(Move paddlepop sticks and add one to make a trapezium.)* What is it now? Why? 	Is it still a square? Why/why not?

RESULTS

Below are the case studies which summarise the responses of the six students who completed questionnaires and participated in interviews. The case studies are listed in order of increasing sophistication of the responses.

In order to distinguish between written responses and conversation, conversation is reported in italics. Questionnaire responses are reported as written by the students, as such, the meanings of any words that may not be obvious have been included in brackets.

Case Study 1: 'Liam'

Liam used the words "sqwer" (square), "dimnd" (diamond), "reptangle" (rectangle) and "oblonge" (oblong) to name the figures presented in items 1-4, respectively. His reason for each of the names was exactly the same – "because it look like one". He did not write any further information at item 5.

In the first part of the interview, Liam was unable to explain his written answers any further. He named the shapes in the same way as he had done on the written test and simply reiterated *"because it looks like one"*.

In the second part of the interview, Liam drew a square (q1) and, when prompted, recognised that the sample square was the *"same shape"* as his drawing (q4). He was unable to explain any features of squares (q2 - q6). Liam also identified a square made out of paddlepop sticks. When the orientation of the square was changed, he called it a *"diamond"* (q7). Liam was unable to give a name for the trapezium presented at question 8.

Case Study 2: 'Rhimi'

Initially, Rhimi did not understand the written task, due to the demands it placed on his reading skills. Therefore, the questions were read to him.

Rhimi used the words "scwer" (square), "dimen" (diamond), "reek tangoll" (rectangle) and "reek tangoll" (rectangle) to name the figures presented in items 1-4, respectively.

Item 1 was a square because it had "for sids" (four sides); item 2 was a diamond because "it has the same corns (corners)"; item 3 was a rectangle because "it is Loug (long)" and item 4 was a rectangle because "it has for sids" (four sides). At item 5, Rhimi compared item 1 to a box, item 2 to a necklace, item 3 to a present and a mirror, and item 4 to a mirror. He explained that the figures in items 1, 2 and 3 each had four corners: "the dimen has for coones and the scwer to ~~ad~~ and the reektangoll".

In the first part of the interview, Rhimi repeated his written answers in the same form as he had in his writing as well as clarified that when, at item 2, he had written "it has the same corns" he had meant *"the diamond has the same corners as the square"*.

In the second part of the interview, Rhimi drew a square (q1). Rhimi also recognised and named the sample square and the squares in standard orientations presented at questions 4, 7 and 8. He restated that a square had four sides (q2-3, q7-8), and was able to add that a square had *"four corners"* (q5). When the orientation of the square was changed (q7), Rhimi called it a *"diamond"* (q7). He named the trapezium a *rocket* and did so consistently regardless of orientation because *"the rocket's just flying the other way"*.

Case Study 3: 'Angela'

Angela used the words "squr", "diemend", "rectangel" and "dieagenul rectangel" to name the figures presented in items 1-4, respectively. Item 1 was a square "because it has four sides and it has four coners (corners)"; item 2 was a diamond "because it has four coners (corners) and it looks like a triangel"; item 3 was a rectangle "because it has four sides like a squer"; and item 4 was a diagonal rectangle "because it has four sides like a rectangel and it is strayt (straight)". At item 5, Angela commented that the diamond and the diagonal rectangle also both "look like" the square, that the square "looks like" the diamond, and that the "rectangel has four coners (corners)".

In the first part of the interview, Angela clarified that the diamond *"looks like a triangle because there is one at the top and one at the bottom"* and that a rectangle *"is like a square because it has four sides and four corners"*. Angela explained that she named the figure in item 4 a diagonal rectangle because *"it looks like that rectangle"* (item 3) *"but on a diagonal"* and that when she wrote that the diagonal rectangle "is strayt (straight)" she was referring to the fact that the sides were straight.

In the second part of the interview, Angela drew a square (q1). She also recognised and named the sample square and the squares in standard orientations (q 4, 7 & 8). Angela restated that a square had four sides and four corners (q2-3, q7-8), and added that a square had *"straight sides"* (q5). When the orientation of the square was changed (q7), Angela called it a *"diamond"* (q7) because *"it's on a diagonal"*. Angela was unable to give a name for the trapezium presented at question 8 but described it as *"a bit like a square"*.

Case Study 4: 'Nicole'

Nicole used the words "square", "dianglar", "rectangular" and "oblong" to name the figures presented in items 1-4, respectively. Item 1 was a square "because it has four sides and it has Pointy edges on each side"; item 2 was a "dianglar" "because it looks Like a hexagon"; item 3 was a "rectangular" "because it's big and long"; and item 4 was an oblong "because it looks like a rectangle and it looks like a stick if it was skinny". At item 5, Nicole drew comparisons between the square and the "dianglar", and between the "rectangular" and the oblong:

> The square and the dianglar ~~are~~ look like the same and I think they are because the diangler is the same but the diangler is been tipped over.
>
> The rectangular and the oblong are the same and they are the same ~~shape But~~ the rectangular and the oblong ~~aren't and~~ are the same Height.

In the first part of the interview, Nicole explained that her written comment about the "Pointy edges on each side" referred to the place *"at the end of the sides where they join"*, and that when she described the "rectangular" as "big and long" in the written questionnaire she had meant that *"it's like a big long square"*. She also explained that the oblong in item 4 was "like a stick if it was skinny" because skinny sticks go *"up and down"*, while *"fat sticks go across"*.

In the second part of the interview, Nicole drew a square (q1) and was able to recognise and name the sample square and the squares in standard orientations (q 4, 7 & 8). She reiterated that a square had four sides (q3), and went on to explain that a square *"has four sides with pointy edges"* (q7-8) and had to have four *"straight"* sides (q2). When the orientation of the square was changed (q7), Nicole again called it a *"dianglar"* because *"it's like a diagonal rectangular"* which has four sides but is *"crooked"*. Nicole was unable to give a name for the trapezium presented at question 8 but noted that it was *"pointy"*.

Case Study 5: 'Sam'

Sam used the words "sqaure", "diamond", "rectangle" and "dieagnal rectangle" to name the figures presented in items 1-4, respectively. Item 1 was a square "because it has four sides and four corners"; item 2 was a diamond "because it is a turn up side down square"; item 3 was a rectangle "because it is a wider sqaure"; and item 4 was a "dieagnal rectangle" because "it is ~~short~~ diffent to the other ~~Oo~~ One because it is tiped over" At item 5, Sam noted:

I realised they all have four sides and four corners And One of them is not strate (straight). Two of them ar both sqaure shaped. Except One of them are diamand shaped. Not all of them are eaqual achaul (actually).

In the first part of the interview, Sam explained that the figure described as "not strate" was that presented in item four. After much discussion it became clear that Sam was trying to describe that the oblong in non-standard orientation was the only shape on the questionnaire which did not have a vertical axis of symmetry, though these terms were not used by the student. The squares in items 1 and 2 were both seen to be *"square shaped"* but the square in a non-standard orientation (item 2) was still *"diamond shaped"*. The reference to not all of the shapes being equal was explained by Sam to be a comparison between squares and rectangles - both have four sides but the sides are *"not all the same"*.

Sam also explained his descriptions of the diamond and the rectangle (items 2 & 3). He explained his description of the diamond as a "turn up side down sqaure" to mean that *"it's got four sides and corners like a square but it's been turned upside down"*. Similarly, the description of the rectangle as a "wider sqaure" meant that *"it's got four sides and corners but it's wider than the square"*.

In the second part of the interview, Sam drew a square (q1) and was able to recognise and name the sample square and the standardly oriented squares (q4, 7 & 8). He continued to describe a square as having *"four sides and four corners"* (q2-4, 7-8) and noted that the sides also had to be *"straight"* (q5). When the orientation of the square was changed in question 7, Sam continued to call it a *"diamond"*. He was unable to give a name for the trapezium (q8) but observed that *"it still has four sides"*.

Case Study 6: 'Sean'

Sean used the words "square", "diamond", "rectangle" and "oblong" to name the figures presented in items 1-4, respectively. Item 1 was a square "because it a (has) four sides and angles and it has all sides the same"; item 2 was a diamond "because if you know what a square is this is a diagonal square"; item 3 was a rectangle "because it has two sides

shorter then top and bottom"; and item 4 was an oblong because "it is the same as the rectangle except the shorter parts are at the top". Item 5 allowed Sean to express that if the diamond was "cut in half you would have an isosceles triangle", that all four figures presented could be used together to make a picture of a man, and that if you took a square and "turned it diagnaly (diagonally) you would have a diamond".

In the first part of the interview, Sean explained that his description of the diamond (item 2) as "a diagnal square" included the previous description he had written of a square in that the diamond has *"four sides the same and four angles but it's on a diagonal"*. Similarly, his description of the oblong (item 4) as "the same as the rectangle except the shorter parts are at the top" meant that *"it still has four sides but the top and bottom are shorter than the sides"*.

In the second part of the interview Sean drew a square (q1). He also recognised and named the sample square and the squares in standard orientations (q4, 7 & 8). His description of a square as having *"four sides the same and four angles"* remained constant (q2-5, 7 & 8) but at question 5, Sean added that the *"sides are straight"* and the *"angles all look the same"*. When the orientation of the square was changed at question 7, Sean named it a *"diamond"* but continued to move around the shape on the floor. When he sat down, he said *"you would think it's a square"* (and from my perspective the square was standardly oriented), though to Sean it was still *"really a diamond"*.

Sean named the trapezium (q8) a *"squashed square"* because it *"looks like someone has pushed it down"*. Although Sean was able to explain that the sides of a square had to be equal or the *"same"*, the addition of a side-length measurement at question 6 prompted no further explanation of this equality.

DISCUSSION

The case studies above illustrate effectively the enormous range of responses which Year 2 children provide when answering questions about and discussing simple 2D shapes. Of the six cases reported, only one student was unable to explain or describe any features of the figures presented. All of the five remaining students recognised that a square has four sides. From this point, however, explanations and understandings diverge.

While Rhimi was able to explain that a square has *"four sides"*, Angela, Nicole, Sam and Sean all made some reference to the fact that those sides also had to be *"straight"*. Only Sean, however, was able to articulate that the sides of a square are *"the same"*. Even Sean, however, was unable to interpret the implications on the length of the sides of a square given that one was 3cm long (q5). Thus, while these students all understand that a feature of a square is that it has four sides, the development of the individual aspects of 'straightness' and 'equality' which is characteristic of those sides provides us with insight into the complexity of their understandings.

The word *"straight"* was used by four students to describe sides. One of these students, Sam, also used the term "not strate (straight)" to convey his idea that the oblong presented in item 4 differed from the other three items on the questionnaire in that it did not have a vertical axis of symmetry. This highlights the necessity of interviews in the correct interpretation of students' written responses.

Another area in which considerable diversity was observed involved the concept of angles. Four students were able to combine the number of sides and the existence of angles in their descriptions of a square. The idea of angles first emerged in Nicole's description of the "pointy edges" which are *"at the end of the sides where they join"*. Sam and Angela both spoke of *"corners"*, but only Sean used the term *"angle"*. Sean was also the only student to acknowledge that the angles in a square also *"all look the same"*.

Students' descriptions of *"diamonds"*, *"rectangles"* and *"oblongs"* during interviews highlighted the supplementary nature of their written responses. If the student had written a description of a shape and then referred to another shape as being "like" it, the student often did not see the need to repeat themselves. For example, Sam wrote that a square "has four sides and four corners" and that a rectangle is a "wider sqaure". When interviewed, Sam explained that a rectangle is a "wider sqaure" because it has *"got four sides and corners but it's wider than the square"*. Similarly Sean's description of a diamond as "a diagnal square" included his description of a square and therefore meant that the diamond has *"four sides the same and four angles but it's on a diagonal"*. While these two examples come from students who were able to provide comparatively more sophisticated responses than their peers, it appears that it is not only these high ability students who may see their descriptions as cumulative. For example, Nicole described her "rectangular" as "big and long" in the written questionnaire but clarified this by saying *"it's like a big long square"* in her interview. While this does not add any new features to the description, it does clarify it. This implies that Nicole sees links between squares and *"rectangulars"*. This issue of a number of written descriptions having a cumulative nature is not unequivocal, however, and may be traced to the working memory demands, which a particular task places on students. For example, "a rectangle is a long square" may not imply that the student is aware of any features of either shape.

Interestingly, no students gave the same name to the two squares presented in different orientations (items 1 & 2), and only one student, Rhimi, gave the name *"rectangle"* to both oblongs presented (items 3 & 4). Four students used the name *"diamond"* to describe the square in non-standard orientation even after they had saw the entire square moved in front of them, of these, three students made some reference to the diamond being *"like"* a square or *"square shaped"* but *"on a diagonal"* or *"like a square but it's been turned upside down"*. Some students used non-conventional names of their own composition that indicated the similarities between figures presented and their conviction that, due to their orientation, they were still *"not the same"*. These non-conventional names include Nicole's use of the word *"dianglar"* for the square in a non-standard orientation (item 2), and the use of *"diagonal rectangle"* by both Angela and Sam for the non-standardly oriented oblong (item 4). Three students consistently used the word *"rectangle"* for the oblong in standard orientation, and the word *"oblong"* for the oblong in a non-standard orientation. Again, these students saw similarities between the two figures but were not convinced that they were the same shape and, as such, they could not be given the same name. This difficulty, with orientation being seen as a feature of the shape itself, was clearly illustrated by Sean in his interview. At question 5, Sean named the square in standard orientation as a *"square"* but when the orientation of the figure was changed he named it a *"diamond"*. After viewing the *"diamond"* from various locations, Sean remarked that *"you would think it's a square"* (due to my different perspective) even though it was still *"really a diamond"*.

CONCLUSION AND IMPLICATIONS

This study has illuminated the diverse range of responses to questions on 2D shapes provided by the sample of Year 2 students. The discussion above has emphasised key aspects of this diversity. However, this study has also brought to the fore two issues, which require a deeper analysis. These issues appear central to our knowledge of how young children acquire geometric competencies.

First, the descriptions "four sides and four corners" and *"four sides the same and four corners"* while not contradicting the developmental descriptors hypothesised by Pegg and Davey (1998), are not easily accommodated into their model.

The second issue concerns the possible differentiation between global attributes which may build towards a Level 1 understanding and features which are building blocks towards the notion of a property at Level 2A. The clarification of this issue is of fundamental importance to the appropriateness of the van Hiele model if it is to be relevant in describing the development of young children's geometric understandings.

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