CABRI GEOMETRY: A CATALYST FOR GROWTH IN GEOMETRIC UNDERSTANDING

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In a recent study (Vincent, 1998), van Hiele levels were used to monitor students' progress in geometric understanding when learning with Cabri geometry. The following report focuses on the experiences of two of the twelve participants in the study: Student D who was initially at van Hiele Level 0/1 and Student L who was at Level 2 for most concepts.

INTRODUCTION

Cabri geometry was developed in 1986 by Jean-Marie Laborde, Yves Baulac and Franck Bellemain, at the Université Joseph Fourier in Grenoble, France. Cabri primitives may be classified as pure drawing primitives, where the user can draw a point, line or circle as in any computer drawing tool, or geometric primitives such as perpendicular bisectors or midpoints.

Visual Versus Geometrical Strategies with Cabri

While it might be expected that dynamic geometry software such as Cabri would enhance students' learning of geometry, research has shown that students experience considerable difficulty constructing particular drag-resistant shapes such as rectangles or equilateral triangles since they base their constructions on visual appearance. They expect to be able to apply the same by-eye methods as with pencil and paper.

Laborde (1993) suggests that the van Hiele levels can be recognised in students' use of the drag mode:

> One can recognize the van Hiele levels in the use of the variations of the drawing. At a low level the figure is viewed as an entity but not analysed into parts or elements: all parts of the drawing must move together under the drag mode. At an intermediate level the figure is viewed as a shape which can be distinguished from other shapes, the drawings are instances of the shape but not yet analysed. At a higher level the figure is made of elements linked by relations which remain invariant when dragging the drawing. (p. 66)

In her experimental observations, Laborde (1995) notes the occurrence of three strategies: purely visual strategies, combinatorial use of geometrical primitives without definite intention and geometrical strategies aimed at a definite result. Students did not necessarily follow these pathways hierarchically - they would sometimes revert to "messing-up" constructions even after successfully completing quite difficult drag-resistant constructions. When faced with a new problem, students were observed to revert initially to by-eye strategies, even though they had previously found these visual strategies to be unsuccessful. Healy, Hoelzl, Hoyles and Noss (1994) made similar observations when working with 14-year old pupils in a London comprehensive school. Despite the pupils' previous experience of "messing-up" they usually started with a by-eye drawing and "were not always convinced that a shape which looked correct might still be wrong" (p. 16). Noss and Hoyles noted that the pupils' by-eye approach provided them with a pathway which showed them that their initial solution was inadequate as well as leading them to a resolution. Cabri "provided a route which could be progressively abandoned as an alternative strategy presented itself" (p. 128).

THE STUDY

The study investigated students' progress in geometric understanding when using the dynamic geometry software, Cabri geometry™, MS-DOS version 1.7. The participants in the study were twelve 11-12 year old girls in a private girls' school in Melbourne. The girls had their own notebook computers, but had not used Cabri prior to this study.

Methodology

The study was essentially a case study: both quantitative, involving measurement of changes in van Hiele levels associated with the use of Cabri, and qualitative, documenting and analysing students' geometric language and methods of construction of geometric figures with Cabri. Data collection, in the form of students' test responses, saved Cabri files and taped conversations, took place over a period of four weeks. The methodology involved the following stages:

1. PowerPoint presentation of triangles and quadrilaterals by each student.
2. Pre-tests:
   Geometric Terms test
   Shapes Recognition test
   CDASSG Project multiple choice van Hiele test
   Mayberry/Lawrie van Hiele test

At this stage, on the basis of the van Hiele pre-tests, the twelve students were divided into two groups: Group I comprising the students who were at Level 0 or 1 on most concepts and Group II comprising those who were at Level 2 or 3 on most concepts.

3. Cabri worksheet lessons (each lesson was of 45 minutes duration):
   Lesson 1: Cabri introduction where students played with the software.
   Lesson 2: Exploring the difference between the various Cabri points.
   Lessons 3-8: Learning with Cabri using structured worksheets.
4. Cabri constructions for Group II students.
5. Shapes Recognition post-test
6. Van Hiele post-test (Mayberry/Lawrie van Hiele test).
7. Further Cabri constructions for selected students (letter A and House shapes)

The Cabri Lessons

During lessons 3-8 the students recorded information from their Cabri activities and answered questions on their worksheets. It was anticipated that these activities would develop Level 2 thinking by encouraging the students to think about properties of the figures, but no explicit reference was made to relationships between properties.

The Cabri Constructions

Students were required to produce drag-resistant geometric figures: parallelogram, rectangle, right-angled triangle and isosceles triangle. The *History* option of Cabri enabled the students' construction steps to be retraced in the saved files and assisted in analysing the levels of thinking displayed by the students in their constructions,

RESULTS AND DISCUSSION

PowerPoint Presentation

The purpose of this activity was to ascertain the students' prior understanding of triangles and quadrilaterals using the drawing tool shapes which could be freely rotated. In her slide for Squares Student D placed her three squares in standard positions while Student L rotated two of her squares and classified squares as rectangles. In her Rectangles slide Student D described rectangles as having "two equal sides which are parallel", indicating that she was unable to correctly verbalise her understanding.

Geometric Terms Test

The Geometric Terms test involved matching a geometric term with a diagram. The correct (✓) and incorrect (✗) responses of Students D and L are shown in Table 1.

Table 1.
Responses in the Geometric terms test

Student	Intersection	Parallel	Scalene triangle	Rectangle	Perpendicular	Isosceles triangle	Midpoint	Perpendicular bisector	Quadrilateral	Parallel-ogram
D	✓	✓	✗	✓	✗	✓	✓	✗	✓	✓
L	✓	✓	✓	✓	✗	✓	✓	✗	✓	✓

Neither student, in common with most of the other students in the study, knew the meaning of *perpendicular* or *perpendicular bisector*. Student D confused isosceles and scalene triangles, although there were inconsistencies in her selection of isosceles triangles in the Shapes Recognition test, as shown in Table 2.

Shapes Recognition Test

The Shapes Recognition test involved selecting rectangles and parallelograms from a page of quadrilaterals and selecting isosceles triangles from a page of triangles.

Table 2
Numbers of Shapes Correctly Identified in the Shapes Recognition Test

	Isosceles triangles	Rectangles	Parallelograms
Possible number of correct responses	10	6 (including 3 squares)	10 (including 2 rhombuses and 6 rectangles)
Student D	5 + 1 incorrect	3 rectangles (no squares)	2 (included 1 rotated rectangle + 3 incorrect trapezia)
Student L	8	6	4 (included rhombuses but not rectangles)

Student D made several incorrect selections and did not recognise class inclusion. By contrast, Student L classified rhombuses as parallelograms and squares as rectangles, although she did not include rectangles as parallelograms.

CDASSG van Hiele Test

This test consisted of the first 15 items, representing van Hiele Levels 1-3, of the 25 item test developed in the CDASSG Project (Usiskin, 1982). Table 3 shows the numbers of correct responses and the assigned van Hiele levels. Student D was classified as Level 1 using the CDASSG 3 of 5 criterion but as Level 0 using the CDASSG 4 of 5 criterion. Student L could not be assigned a level as she scored 5/5, 2/5 and 4/5 respectively for the Level 1, 2 and 3 questions, perhaps indicating that she was in transition from Level 2 to Level 3.

Table 3
Assigning van Hiele Levels from the CDASSG Test (Items 1-15)

	Number of correct responses			Level	
Student	Questions 1-5	Questions 6-10	Questions 11-15	3 of 5 criterion	4 of 5 criterion
D	3	0	0	1	0
L	5	2	4	NC	NC

NC Not classified

Mayberry/Lawrie van Hiele Test

This written test used the 25 items concerning the concepts squares, right-angled triangles, parallel lines and isosceles triangles from the 58-item Mayberry/Lawrie test (Lawrie, 1993). Table 4 shows the levels assigned by the test for the four concepts for Students D and L. Student D, who performed poorly for isosceles triangles on the Shapes Recognition test and Geometric Terms test, was at Level 0 for isosceles triangles. These observations suggest that she was probably in transition from Level 0 to Level 1. Although Student L could not be assigned a level on the CDASSG test, she was clearly at Level 2 for three of the four concepts on the Mayberry/Lawrie test.

Table 4
Assigning van Hiele Levels from the Mayberry/Lawrie Test

Student	Squares	Right-angled triangles	Isosceles triangles	Parallel lines
D	1	1	0	1
L	2	1	2	2

Following completion of the Cabri worksheet lessons, the students were re-tested with the Mayberry/Lawrie van Hiele test and the Shapes Recognition test. Table 5 compares the pre-test and post-test responses for the Shapes Recognition test. Student D showed very little change except that she made fewer incorrect selections on the post-test. Student L, on the other hand, now clearly identified rectangles as parallelograms, although she incorrectly classified two parallelograms as rectangles.

Table 5
Comparison of Responses in the Shapes Recognition Pre-test and Post-test

Student	Isosceles triangles (10)		Rectangles (6)		Parallelograms (10)	
	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test
D	5 + 1 incorrect	7	3	3	2 (included 1 rotated rectangle + 3 incorrect trapezia)	2
L	8	9	6	6 + 2 incorrect parallelograms	4 (included rhombuses but not rectangles)	9 (missed 1 rhombus)

Table 6 compares the pre-test and post-test van Hiele levels assigned from the Lawrie/Mayberry van Hiele test. Both students progressed in three of the four concepts, with Student L increasing from Level 1 to Level 3 for right-angled triangles. Shaded cells indicate where an increase in van Hiele level occurred.

Table 6
Pre-test and Post-test van Hiele Levels for Four Concepts.

Student	Squares		Right-angled triangles		Isosceles triangles		Parallel lines	
	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test
D	1	2	1	2	0	1	1	1
L	2	3	1	3	2	2	2	3

Based on the van Hiele levels assigned by the tests, Student D was placed in Group I and Student L in Group II.

Cabri Constructions

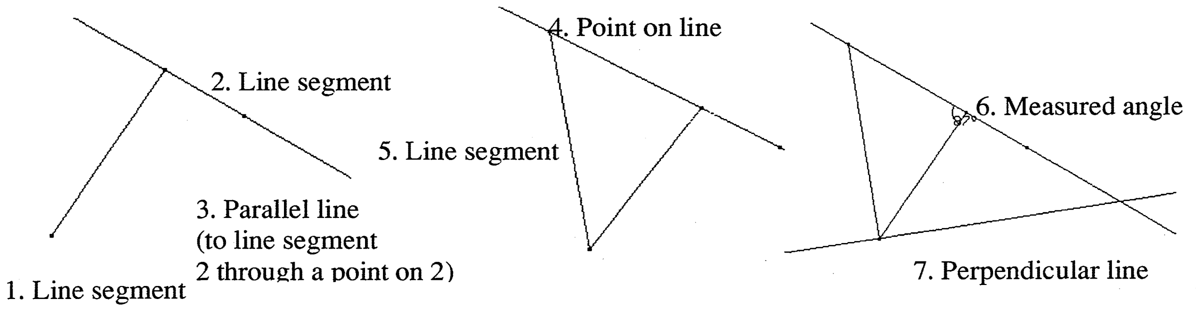
Student D

Since Group I students were still consolidating their Level 2 thinking with relationships between properties not yet being recognised, the construction tasks were generally not given to these students. However, Student D, when asked to draw an isosceles triangle, used the triangle tool, dragging it until it appeared to be isosceles. When she realised that it did not remain isosceles when dragged, she had no idea how to proceed and became distressed: she could not see beyond a by-eye construction. Student D differed in this respect from the Group II students, who were not worried by the "messing up" of their by-eye constructions, but used it to help them with a geometric construction.

Student L: Right-angled Triangle

Student L at first used a by-eye method (Figure 1), starting with two *line segments* (1 and 2). She confused *parallel* and *perpendicular* lines and, instead of constructing a line *perpendicular* to line segment 2, she constructed a *parallel line* (3) to this line segment. She then constructed a *point on line* (4) and a *line segment* (5) to form a triangle. After measuring the angle (6), Student L abandoned this angle as her right angle and constructed a *perpendicular line* (7) but then decided not to complete this triangle.

Figure 1
Student L: Right-angled Triangle 1



In her second attempt (Figure 2), Student L again initially confused parallel and perpendicular and, although she did construct a right-angled triangle, it could not be rotated since the triangle depended on a basic line which cannot be rotated once placed. Student L's third attempt (Figure 3) satisfied the requirements of the task.

Figure 2
Student L: Triangle 2

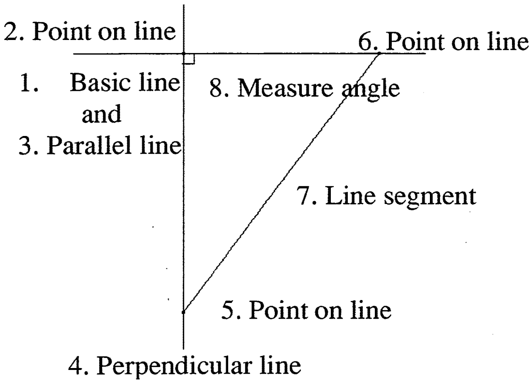
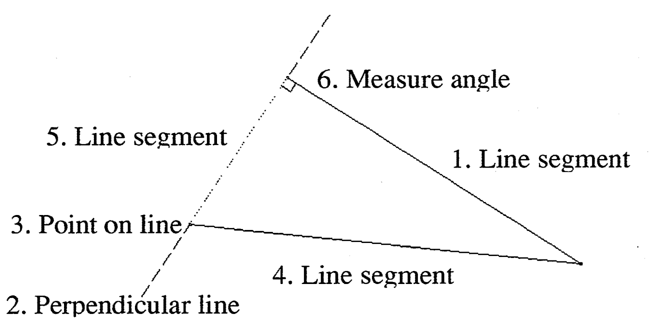


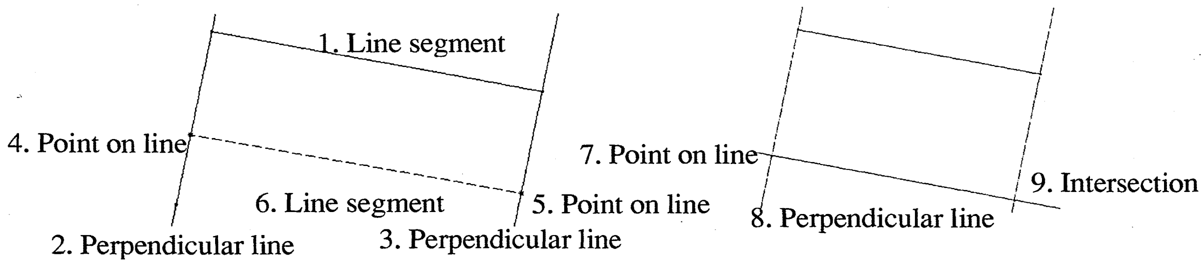
Figure 3
Student L: Triangle 3



Student L: Rectangle

Student L completed her rectangle (Figure 4) with little difficulty, having already discovered the problems of by-eye methods with her right-angled triangle and now understanding the meaning of *perpendicular*. She quickly realised that *line segment* (6), which was based on by-eye placing of *points* (4) and (5), would not stay parallel to *line segment* (1), so she deleted *points* (4) and (5), constructing another *point* (7), a *perpendicular line* (8) and an *intersection* (9).

Figure 4
Student L: Rectangle



Student L: Letter *A*

Despite her previous experiences, Student L's first attempt at constructing the letter *A* was a by-eye placing of three *lines by 2 points* (1, 2 and 3) - see Figure 5. In her second attempt (Figure 6), she constructed a pair of perpendicular lines using *line by 2 points (1)* and *perpendicular line (2)* to form the "legs" of her Letter *A* but then placed the cross-line by eye: *points on lines* (3 and 4) were connected with a *line by 2 points* (5). Student L then asked "This is not one of those impossible puzzles is it? "

Figure 5
Student L: Letter A (1)

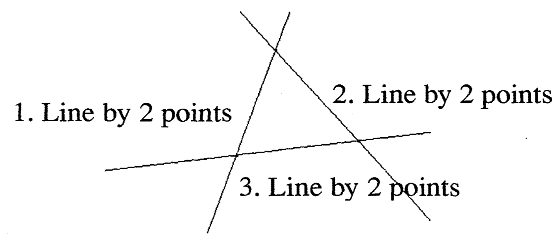
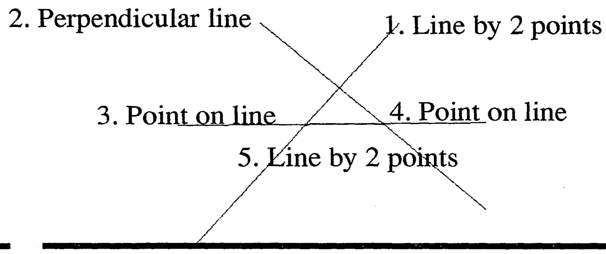


Figure 6
Student L: Letter A: (2)



After being assured that the construction was possible, Student L abandoned the idea of perpendicular lines and tried using a *triangle* (Figure 7), dragging it until it appeared to be an equilateral triangle. This triangle became the framework for constructing three *lines by 2 points* to make the Letter *A* shape. Finding that this distorted when dragged, Student L placed a *point* (2) visually above the centre of a *line by 2 points* (1) (Figure 8) but seemed unsure how to continue. She had realised that visual constructions would not work but her progress was perhaps hindered by her lack of understanding of *perpendicular bisector*. Although she had explored the Cabri menu options, unlike other Group II students, she made no attempt to use this construction tool.

Figure 7
Student L: Letter A (3)

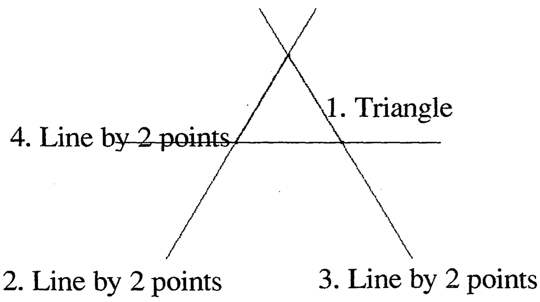
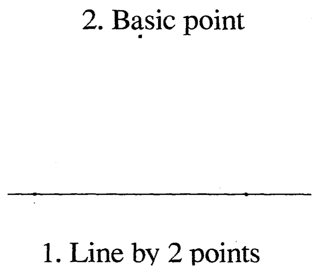
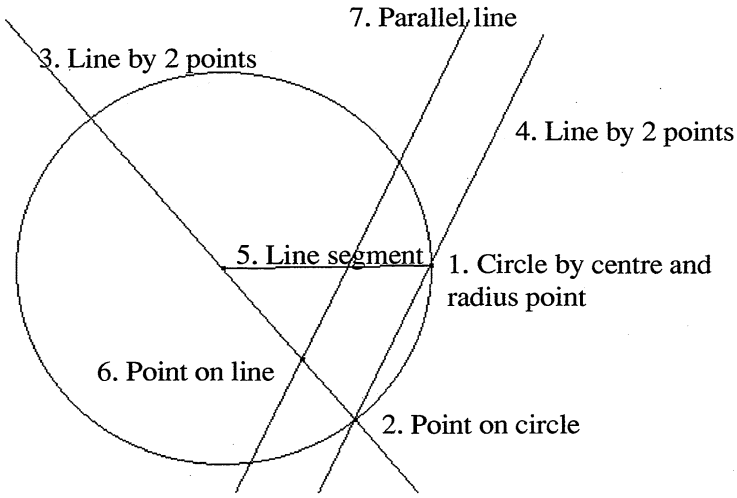


Figure 8
Student L: Letter A (4)



Having apparently been trying to utilise the symmetry property of the letter *A* in her first four attempts, Student L instead focused on the equal sides of an isosceles triangle in her next attempt, using two radii of a circle to construct two equal sides of an isosceles triangle (Figure 9). By constructing a line (7) parallel to her secant line (4) she was able to successfully complete the letter *A*.

Figure 9
Student L: Letter A (5)



CONCLUSIONS

Student D, who initially had very limited understanding of geometric figures or properties, was now able to correctly identify the relevant geometric shapes and was aware of at least one property of each. Unlike Student L, however, she was not yet aware of relationships between properties and was not ready for using Cabri to construct drag-resistant geometric figures. After her initial by-eye drawings, Student L realised that appropriate geometric constructions were required and was able to analyse a geometric figure and make links between its properties and the properties of another figure. Healy, Hoelzl, Hoyles and Noss (1994) suggest that "finding a 'solution' by-eye so that they know where they are going, and thereby have something on the screen on which to reflect, seems to be crucial scaffolding for many children" (p. 16).

Student L's ultimate successful construction of the letter *A* involved the linking of several properties: radii of a circle are the same length, the equal sides of an isosceles triangle can be constructed using the radii of a circle and the required "horizontal line" of the Letter *A* can be achieved by constructing a line parallel to the base of the triangle. In her search for a method by which Cabri would allow her to construct equal sides for her isosceles triangle, Student L used a property independent of the isosceles triangle, thereby linking properties of different geometric figures. Student L's constructions support observations by Laborde (1993), who noted that the use of visualisation by students, both in exploratory tasks and in validating constructions, is related to their conceptual understanding of the figure they have constructed.

Students L's preliminary visual attempts seemed to focus her thinking on which aspects of the letter *A* must remain invariant and, because she was unfamiliar with the term *perpendicular bisector,* she did not recognise the role this Cabri tool could play in the construction. Van Hiele-Geldof in Fuys, Geddes and Tischler (1984) notes that "concept and language can be distinguished, but cannot be separated. The thinking operation itself first has to be made conscious through language symbols and the language symbols are a consequence of the thinking operation" (p. 232). Observations of another pair of students in the study, who worked together on the letter *A* and House constructions, suggest that Cabri has greater potential to enhance the understanding and use of appropriate geometric language when students are actively discussing their constructions.

The study suggests that Cabri can result in significant progress in understanding of geometric properties and relationships even after relatively few lessons. Further studies are essential to explore the potential of dynamic geometry software to develop higher levels of geometric understanding.

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