An arithmetic game with strategic components - playing procedures of primary school children

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The project *children playing arithmetic games with strategic components* concerns with the existence and with types of strategies used during playing. In a qualitative study with an interpretive background we have observed primary school children of different grades playing arithmetic games with strategic components. In this paper we shall present in detail the procedure of children of grade 3 playing an addition-game with strategic character.

Introduction

The value of using learning games in the primary mathematics classroom is widely acknowledged. Nevertheless, until now few empirical studies have investigated their effectiveness. Our project deals with research on existence and development of strategic facilities with younger children. For this purpose we use arithmetic games with underlying strategic components. The games are designed to promote an active and discovery-oriented arithmetical learning process. They allow children to discover relationships between numbers and patterns such as the commutative law of addition and multiplication. They also embody frequently neglected strategic components such as recognizing multiple ways of achieving a score. Optimal strategic behaviour requires far-sighted and deductive thinking and reasoning.

The structure of the games is always nearly the same and resembles those games which children know from playing outside school. The different games vary in the arithmetic operations - each game has only one operation - and in the tasks which have to be solved.

Normally the games are planned for two players. Concerning our interest on discussions about playing procedures we changed this situation. In our studies there are two teams playing against each other, each team consisting of two children and the teams sitting in different rooms. The communication about the proceedings of the game takes place by telephone.

Paradigmatically we watched one team very carefully by video-camera. We transcribed the video-tape and will present the transcript in full length in this paper.

The following questions can be formulated:

- do the children anticipate that there are several tasks with the same result,
- do they use mathematical rules when choosing a task,
- what are they mainly motivated by.

Presentation and Analysis of the game

45	+	2	5	4	3	65
46	53					64
47	55					63
48	51					62
49	54					61
50						60

51 52 53 54 55 56 57 58 59

The game consists of a matrix of tasks called *task field*. The tasks are determined by the matrix numbers and the given operation. The results of the tasks are arranged around the matrix. We call the numbers around the matrix *result field*. There are more numbers in the result field than can be achieved by calculating the tasks of the task field. To mark the elements in the task field the playing teams use tokens in different colours.

Playing rules:

1. Playing teams in turn place their tokens on free elements of the matrix, calculate the sum and place another token of the same colour on the correct result of the result field.
2. In case an element in the result field is already occupied, the playing team is allowed to replace the token of the opponent - or even its own token - by one of its own.
3. The game is over when all elements of the matrix are occupied. The winner is the playing team with the most occupied elements in the result field.

The tasks of the game are of such kind that they can easily be solved by children of third grade so that there would be enough capacity to discover mathematical relationships and rules and to plan strategic proceedings.

There are sixteen different tasks which yield only eight different results. At the end of the game there will be thirteen elements of the result field unoccupied.

Frequency of occupation in the result field

There are

- three elements which can be occupied only once - the numbers 53, 54, 60 - called *one-result elements*,
- two elements which can be occupied twice - the numbers 55, 59 - called *two-result elements*,
- three elements which can be occupied threetimes - the numbers 56, 57, 58 - *called three-result elements*.

Optimal strategic proceeding

The playing teams should first occupy the one-result elements in the result field. The three-result elements should follow in due course and they should avoid to be the first team to occupy a two-result element. When using an optimal strategy the team which starts in second will be the winner. To play in an optimal way they must not be the first team to occupy a two-result element. This refers also to three-result elements which are already once occupied.

Possibilities of variation

The game can be used in all classrooms of the primary school. There can be variations of

- size of the matrix
- operation
- tasks for calculation - less or more difficult-
- number of elements in the result field with multiple occupations
- frequency of results

Mathematical background of the tasks

The grade of difficulty of the tasks is rather low. There is only one task with result 60, all other tasks have results between 53 and 59, so that there is only once a change of the tens. These facts enable the children to develop strategies by using mathematical relationships and rules (see Table 1).

Result field	Tasks	Relationships and Rules	Frequency
53	51+2		One-result elements
54	51+3		
60	55+5		
55	51+4 53+2	 53+2 = (51+2) + (4-2)	Two-result elements
59	54+5 55+4	Commutative law at the ones	
56	51+5 53+3 54+2	 53+3 = (51+2) + (5-2) 54+2 = (51+3) + (5-3)	Three-result elements
57	53+4 54+3	Commutative law at the ones	
	55+2	Neighbour numbers compared to 54+3	
58	53+5 55+3	Commutative law at the ones	
	54+4	Neighbour numbers compared to 55+3	

Table 1

Transcript

The two boys - Sieg and Stev - whose playing behaviour we describe had as opponents another team of two boys. All four boys were third-graders. To give each team the possibilitiy of discussing strategies the two teams sat in different rooms and communicated about their moves by telephone. The opponents of Sieg and Stev started with the game. The tokens of Sieg and Stev are symbolized by ⌘, the tokens of their opponents by ■. The leader of the inverstigation is called L.

Sieg and Stev knew the rules of the game since they had already played and won a similar game with subtraction as operation. At that occasion they had also been second to start.

Sieg: Who starts?
L: The other team.
Sieg: Ah, very good. Now we will beat them again. Is it important who is the better calculator?

1st move: 55+5 = 60 (opponents)

45	+	2	5	4	3	65
46	53					64
47	55		■			63
48	51					62
49	54					61
50						■

51 52 53 54 55 56 57 58 59

Sieg: 60. We'll beat this.
Stev: 53, 54.
Sieg: No, 53+2. No.
Stev: Minus 5.
Sieg: Too bad. You can't beat them.
Stev: 55+5.
Sieg: 58, 60. Oh that's unkind of them. You can't beat this at this place. So we'll do also something unkind, so that they can't do anything. 54 + 5, you agree?
Stev: No. 53+2.
Sieg: Then they have 55. They can do it also.
Stev: How?
Sieg: Look, if they here (points to 55) plus 3. I tell you what they will do: they do simply 51+4. Then we do better 54+5 equals?
Stev: 59.
2nd move: 54+5 = 59.
Sieg: They won't throw us off any more. But they have an advantage.
Sieg: Now we must hope that they don't throw us off.
Stev: I don't think so.
Sieg: Oh dear they can do it yet. But they are not as clever as we are.

3rd move: 55+4 = 59 (opponents)

45	+	2	5	4	3	65
46	53					64
47	55		■	■		63
48	51					62
49	54		⌘			61
50						■

51 52 53 54 55 56 57 58 ■

Sieg: Bad boy. Wait, we also throw them off. 53+...No. 55 plus 2, 57, plus 3, 58.
Stev: How shall we do it?
Sieg: I don't know. They are well in advance. Now let's take the biggest task. 60, ok?
Stev: Is already.
Sieg: And then we throw them off. Not possible. Then we can't throw them off. Then let's take 55+2.
4th move: 55+2 = 57.
Sieg: Hopefully they can ...They can beat us again. Or? Yes they can (points to54+3). Then they catch us.
Stev. Anyhow we loose.

Sieg: That's not yet clear. There are still many fields to do. When they make a mistake and don't beat us then we'll win.
Sieg: We can't beat them. Not on 60 and not on 59.

5[th] move: 54+3 = 57 (opponents)

45	+	2	5	4	3	65
46	53					64
47	55	⌘	■	■		63
48	51					62
49	54		⌘		■	61
50						■

51 52 53 54 55 56 ■ 58 ■

Sieg: Oh, look. We'll beat them. Plus 5 (points to 53). No. Plus 2.Plus 4. Yes. Now away.

6[th] move: 53+4 = 57.

Sieg: They will be annoyed now. We can still do something. Now they can't throw us off that field. They make the first and we can do the second. If they do anything with 58 we can throw them away by 55+3. I hope they won't make 57.

7[th] move: 51+2 = 53 (opponents)

45	+	2	5	4	3	65
46	53			⌘		64
47	55	⌘	■	■		63
48	51	■				62
49	54		⌘		■	61
50						■

51 52 ■ 54 55 56 ⌘ 58 ■

Sieg: So, can we beat them? 53? Let's take 51+...No. We don't have 50. 54. No. 54+2 = 56. Can they do anything with it? Plus 5 works as well. If we do 55+3 they can do 53+5. They can beat us nearly everywhere. No. 54 they can't (points to 51+3, then to 51+4). 55. Can they do anything with 55? No. They can still beat us. 56 also. No. 8...Let's do 58.

8[th] move: 54+4 = 58.

Sieg: Let's look what to do. Oh dear that also works (points to 55+3). I hope they won't do that.

9[th] move: 51+3 = 54 (opponents)

45	+	2	5	4	3	65
46	53			⌘		64
47	55	⌘	■	■		63
48	51	■			■	62
49	54		⌘	⌘	■	61
50						■

51 52 ■ ■ 55 56 ⌘ ⌘ ■

Sieg: Shall we throw them off? Let's take 51. No. 53. No. 54 plus...No. 51+3, ok? No, plus 4. 55. No, then they will throw us off. 58 is good, this is occupied by us.

55 or 56, ok?

10th move: 53+3 = 56.

Sieg: They beat us. Perhaps they do even 51+4.

11th move: 54+2 = 56 (opponents)

45	+	2	5	4	3	65
46	53			⌘	⌘	64
47	55	⌘	■	■		63
48	51	■			■	62
49	54	■	⌘	⌘	■	61
50						■

51 52 ■ ■ 55 ■ ⌘ ⌘ ■

Stev: Oh, no, no.

Sieg: We can take this away. Shall I say what we will do?

Stev: 51+ ...

Sieg: Exactly. 51+5. Away with you. Now they can't do anything more.

12th move: 51+5 = 56.

Sieg: Now they can't beat us. Or? Plus 2 doesn't work. Plus 4 doesn't work. Plus 3 doesn't work. Yes, they can't beat. But perhaps they are in advance with one more.

Stev: No, we have all the same. They have six and we have six.

13th move: 53+5 = 58 (opponents)

45	+	2	5	4	3	65
46	53		■	⌘	⌘	64
47	55	⌘	■	■		63
48	51	■	⌘		■	62
49	54	■	⌘	⌘	■	61
50						■

51 52 ■ ■ 55 ⌘ ⌘ ■ ■

Sieg: Can we throw it off? Let's think. Let's just do 55+3 = 58.

14th move: 55+3 = 58

Sieg: Now they can't do anything more. And you said we would loose (to Stev). If they do 53+2 = 55 then we will win, no deuce. The result will only be 55 (points to 51+4). Then we will beat them.

15th move: 51+4 = 55 (opponents)

45	+	2	5	4	3	65
46	53		■	⌘	⌘	64
47	55	⌘	■	■	⌘	63
48	51	■	⌘	■	■	62
49	54	■	⌘	⌘	■	61
50						■

51 52 ■ ■ ■ ⌘ ⌘ ⌘ ■

Sieg: Perhaps we will still win. So, and now we put it there (points to 53+2).

16th move: 53+2 = 55.

16th move: 53+2 = 55 (Sieg and Stev)

45	+	2	5	4	3	65
46	53	⌘	■	⌘	⌘	64
47	55	⌘	■	■	⌘	63
48	51	■	⌘	■	■	62
49	54	■	⌘	⌘	■	61
50						■

51 52 ■ ■ ⌘ ⌘ ⌘ ⌘ ■

Sieg: We won.
Stev: One, two, three, four, five, six, seven, eight.
Sieg: Did we win?
Stev: No. Deuce.
Sieg: One, two, three, four. Deuce. Good as well.

Results and Conclusion

The two children are mainly motivated by removing the opponent's token from the result field (e.g. before the first move of the opponents "...we will beat them again."). Immediately after each move of the opponents they make remarks like "we'll beat this" or „we can throw this off".

There is no evidence that the two children had planned their moves at the very beginning of the play. There are no indicators that they have recognized the optimal strategy, they never occupy a one-result element Their main strategy is the immediate removing of an opponent's token from the result field whenever this is possible. This strategy is very predominant. In the moment of the move it prevents them from thinking of their own token being eventually removed afterwards by the opponents. Nevertheless they show far-sighted behaviour in an initial stage by thinking about consequences when their move is done. This happens after each of their moves. But they also recognize a two-result element and its disadvantage in advance (discussion after the first move (opponents) about the two-result element 55). We could not find that they also recognized a three-result element in advance.

The use of mathematical rules is shown in the discussion after the seventh move (opponents) where they refer to the commutative law "5+3 = 3+5".

The game gives a lot of opportunities to practice calculating. If not in the first run the children could develop and improve far-sighted thinking by playing the game several times.

Refences

Flitner, A.: Spielen - Lernen. Praxis und Deutung des Kinderspiels. Piper und Co., 3. Auflage, München 1974.

Krummheuer, G./ Voigt, J.: Interaktionsanalyse von Mathematikunterricht. Ein Überblick über einige Bielefelder Arbeiten. In: Maier, H./ Voigt, J. (Hrsg.): Interpretative Unterrichtsforschung. Aulis, Köln 1991, S. 13 - 32.

Mosel-Göbel, D./ Peter, A./ Sorger, P.: Arithmetic games with strategic components in the early primary mathematics classroom. In: Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education. Valencia 1996, vol. 1, 225.