TEACHERS' INTERACTIONS WITH STUDENTS LEARNING THE "EQUAL ADDITIONS" STRATEGY: DISCOURSE PATTERNS

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The study investigated interactions between nine teachers and their Year 5–6 students during a lesson on the "equal additions" strategy for subtraction problems involving difference. Two quantities were compared (e.g., $445 vs. $398), a quantity was added to both (rounding up the subtrahend), and students asked about the two differences ($447–$400 and $445–$398). Teachers' use of so-called "indicator words" was analysed. Those using words such as "difference" and "how much more" frequently had more students who chose the equal additions method to solve post-test problems. The findings reflect the challenges of bringing about deep and lasting change in teaching (and learning) mathematics.

Introduction

Mathematics education reform in western countries has resulted in a shift in emphasis away from training students in the use of rote-learned skills and procedures, towards helping students to develop deep conceptual understanding (Fraivillig, Murphy, & Fuson, 1999; Goya, 2006; Skemp, 2006). Problem solving processes, including thinking, reasoning, and communicating mathematically, have received far greater attention than in the past (see Ministry of Education, 1992, 2007). Sfard (2008) links these together, defining thinking as self-communication.

Researchers have become increasingly interested in the nature of the learning that takes place during classroom mathematics lessons, and there has been a sharpened focus on the interactions between teachers and their students (Rye, 2011). Discourse analysis, or conversation analysis, has become a popular means of gaining insights into the teaching and learning processes within classrooms (Cohen, Manion, & Morrison, 2007; Mercer, 1995; Mercer & Littleton, 2007; Perakyla, 2005). Such an analysis looks at the "organisation of ordinary talk and everyday explanations and the social action performed in them" (Cohen et al., 2007, p. 389). It has been characterised as a kind of psychological 'natural history' of the phenomena that has interested researchers. Cohen et al. suggest that researchers need to be highly sensitive to the "nuances of language". According to Hodgkinson and Mercer (2008) classroom talk, the means by which children make sense of the ideas of their teachers and peers, "is the most important education tool for guiding the development of understanding and for jointly constructing knowledge" (p. xi). Consequently more attention needs to be given to improving the quality of classroom talk.

Several writers have noted that teachers exert a high degree of control over the ways children engage in conversation in the context of classroom learning, and sometimes children are prevented from engaging productively by the actions of their teachers (Hodgkinson & Mercer, 2008), and in particular teachers who put a high priority on the management of behaviour, and who control who gets to talk, when they talk, and about what.

Classroom talk and thinking has been categorised in many different ways. For example, Mercer (1995) has distinguished "Exploratory talk" (where speakers engage critically but constructively with each others' ideas) from "Cumulative talk" (where speakers build positively but uncritically on what others have said), and "Disputational talk" (which is characterised by disagreement and individualised decision-making). Talk can also be examined using a *linguistic* lens (talk as spoken text) or a *psychological* lens (talk as thought and action) (Mercer, 1995). Barnes (2008) contrasts "Exploratory talk" (new ideas being tried out that are often hesitant and incomplete) with "Presentational talk" (well-shaped talk, adjusted to the needs of the audience).

Several researchers have examined the nature of classroom talk in the context of mathematics lessons (e.g., Mercer & Dawes, 2008; Mercer & Littleton, 2007; Mercer & Sams, 2006). Solomon and Black (2008) noted that children's opportunities to contribute, and the type of talk directed towards them by the teacher, varies. Their work focuses on the way that some children readily develop an identity of engagement with mathematics, while others adopt an identity of exclusion from mathematics—a process that may begin from quite early in a child's school career. Further, teacher questioning can narrow the range of possible responses when teachers continue to ask questions in order to get a pre-determined answer (i.e., "cued elicitation"). Mercer and Littleton examined the incidence of "indicator words" assumed to reflect the thinking that occurred during the exploratory talk of students engaged in joint problem solving.

The aims and focus of the research

The present study set out to explore the use of language by teachers while teaching the "equal additions" strategy for solving subtraction problems with a Compare structure. In contrast to the more common Separate structure that involve taking away an amount from a *single* quantity, Compare problems involve comparing *two* different quantities to find the difference (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Fuson, 1992) —see Table 1. Teachers' language is examined, alongside the use of the equal additions strategy by their students in solving post-test problems.

Table 1. Problem structures for "Separate" and "Compare" problems from Carpenter et al. (1999).

Separate (Change: Take from)	<u>Result Unknown</u> Ana had 13 plums. She gave 5 to Sam. How many plums did Ana have left? 13 – 5 = □	<u>Change Unknown</u> Ana had 13 plums. She gave some to Sam. Now she has 8 plums left. How many plums did Ana give Sam? 13 – □ = 8	<u>Start Unknown</u> Ana had some plums. She gave 5 to Sam. Now she has 8 plums left? How many plums did Ana start with? □ – 5 = 8

Compare (Difference)	*Difference Unknown* Ana has 13 plums. Sam has 5 plums. How many more plums does Ana have than Sam? 13 – 5 = □	*Compare Quantity Unknown* Sam has 5 plums. Ana has 8 more plums than Sam. How many plums does Ana have? 5 + 8 = □	*Referent Unknown* Ana has 13 plums. She has 5 more plums than Sam. How many plums does Sam have? 13 – 5 = □

Method

Nine teachers (7 female and 2 male) of Year 5-6 (nine- to eleven-year-old) students from four schools (serving communities ranging from low to high socioeconomic status), each with one instructional group (a total of 64 students) participated in the study (see Table 2). One teacher from each school had previously worked with the researchers, and that teacher agreed to ask the other teacher/s working at the same level also to be involved in the study. Teachers' classroom experience ranged from two to approximately 25 years. Experience in working with the Numeracy Project approach ranged from two to about eight years.

Students were given some written assessment tasks prior to the first lesson, then a similar assessment after the third lesson. This study focuses on the second lesson, which was designed to teach the equal additions strategy for subtraction (Ministry of Education, 2008, pp. 38-39). During the lesson, the teacher wore a portable digital audio-recorder attached to a flexible belt, with a lapel microphone to pick up his/her language to the children (and some responses from children who were close to the teacher). The researchers observed the lesson and noted non-verbal (contextual) information that could assist with the interpretation of the transcripts of audio-recordings. Actions with materials, written recording in the group workbook, and in students' individual mathematics books were photographed to capture some of this nonverbal information.

In the Equal Additions lesson, the first scenario used in Book 5 is as follows:

> Problem: "Debbie has $445 in her bank account, and her younger sister Christine has $398. How much more money does Debbie have?"
> Make piles of $445 and $398. "Now suppose that Grandma gives Christine $2 to give her a 'tidy' amount of money. To be fair, Grandma gives Debbie $2 also." Discuss why 445 – 398 has the same answer as 447 – 400 and then record 445 – 398 = 47 on the board or modelling book.

The book then provides other examples of equations that can be turned into word problems and solved using materials (e.g., paper money).

Results

Data from the transcripts of the teachers' language while teaching the Equal Addition lesson were analysed to check the use of particular terminology during the lesson. Students' responses on the written assessment tasks given after the third lesson was analysed to see which students chose to use equal additions to solve the Compare problem and other subtraction problems. Table 2 shows the frequencies for teachers' use of particular terminology and the identities of particular students in their groups who used equal additions for the compare problem (those who used it for another subtraction problem are shown in brackets). Gail referred to "difference" far more often than the other teachers (n = 30). She was also the second highest user of "how much more” (*n* = 6). She chose to illustrate the idea of difference using small numbers (4 vs. 2), showing what happens when one is added to both numbers (5 vs. 3), that the difference remained the same. Several times she referred to the way “the distance between [the two numbers] stays the same”. At the very beginning of the lesson, she referred to a number line activity the students had done prior to the lesson, then part way through the lesson she asked students to:

> Think of a number line… and you’re looking for the difference between six and two, the difference there is a space of one, two, three, four, right, now if you add two to both of those, one, two. Has the difference between both of them changed? [A student says “No”] It hasn’t, has it, but if we went like this and you added two to one and not the other, okay, it’s bigger isn’t it. The difference between it has changed, so it becomes bigger.

It was interesting to note that three of the five children in Gail’s group used equal additions on a post-test problem, the greatest proportion of any group. Cara referred to “difference” 12 times and was the most frequent user of “how much more” (*n* = 7). She also referred to the “distance between the two [numbers].” Four of her ten children used equal additions on the post-test. Ben started the lesson by using the Separate (“take away”) structure rather than Compare, but later referred to “difference” nine times. He only used “how much more” three times. Two of his students used equal addition on a post-test problem, and one student (B6) used equal subtraction for one problem. Three teachers (Ann, Dot, and Iris) did not refer to “difference” at all.

Table 2. Number of times particular words or expressions were used by teachers and children who chose to use Equal Addition to solve a Compare problem (or another subtraction problem) on the post-test.

Teacher	“Difference”	“How much more”	“Why”	“How”	Group size	Children using Equal Addition
Ann	0	1	6	20	8	
Ben	9	3	8	39	8	B4 (B5)
Cara	12	7	7	22	10	C3, C4, C7 (C9)
Dot	0	0	32	29	8	D8
Ed	4	4	7	31	5	E4
Fay	1	3	3	47	6	F3
Gail	32	6	23	31	5	G1, G2 (G5)
Hana	2	4	14	48	7	H1, H4
Iris	0	2	6	15	7	I5

Several other key issues that emerged were the importance of teachers using consistent language and their awareness of problem structure. Although it was not clear whether or not any of the teachers understood about different problem structures, Cara and Gail were very careful in their use of mathematical language and special terminology with the children.

Several teachers, including Ann, Dot, and Iris, took the first example from the book (see description above), which was structured as a *Compare* (difference) problem and turned it into a *Separate* problem. Dot said:

> Right, I had $445 right, K had, K asked me if she could have a loan of $398 and being the giving, caring person that I am, I said sure. How much money did I have left over? Right, I want you to think about the tidy numbers, using tidy numbers.

One student (D1) was concerned that if two was added to one number, it needed to be taken off later. Dot explained to D1:

> [D1], what I think you've been confused with is if we did it to one of these numbers, if we added two the one number, then yes, we do have to take it away but we did it to both numbers. If we just added two to 398 and 445 the same, then yes, we would have to take that two away, but because we do the same treatment to both numbers the gap remains the same.

Several students commented at this point that they were lost, so Dot then decided to bring the lesson to a close as they had run out of time for further explanations. When Dot was asked in the post-lesson interview if she planned to follow up anything particular from the lesson in the future, she did not have a plan to address the confusion described above. It was interesting to observe later that on the post-test, D1 continued to subtract from the difference the amount she had added to the subtrahend initially, making her answers consistently incorrect.

Ben introduced his lesson by sharing with the group how, in preparing for this lesson, his own mathematics had been extended.

> This is one of these really cool exercises, now I mentioned before that since doing this, my understanding of maths has really improved. What this next lesson is, is actually a really cool lesson for an area that I think we've got a bit of a weakness in as a class, looking at one particular type of operation. Now, so what we are going to do is, we're going to look at, looking at [Reading the learning intention for the lesson] how to solve subtraction problems by Equal Addition that turns one of the numbers into a tidy number.

Ben then asked the students "What sort of problem are we looking at?" One student (B1) suggested a missing addend structure: "398 plus what equals 445?" Ben would not accept this missing addend structure because the learning intention in the resource book focused on subtraction. He said:

> Oh okay, so [B1], you've gone for that first one, reversing strategy, so you've gone for 398 plus what equals 445, yeah. If we just look at the learning intention, which is to solve subtraction problems by using Equal Addition, are we using a subtraction problem here?

One child answered "No." Ben continued:

> Is this still a good strategy? Yep, but we're going to look at just using subtraction, so what problem am I going to write down here to show subtraction?

Another child suggested "445 take away 398". Ben affirmed that response:

> Nice, so we are going to use 445 take away 398 equals, we think it might be 47 [suggested earlier by one of the other students].

Ben then asked whether adding two to both numbers would change the answer. Some students thought it would increase the answer by four, but others believed that the answer was still the same. Ben tried to get those students to explain why:

> What are we actually looking at, we're looking at, what? ... So when I give the answer, what's the answer? Okay, if we take the answer we say let's say that's 47, we're happy that it's 47. What does the 47 actually mean?

One student suggested "Numbers?" to which Ben responded "Excellent, nice, okay."

There was further discussion but it did not appear to produce what Ben was wanting so he explained:

> Okay with subtraction, we're really looking at the difference between these two numbers, so the difference between 445 and 398 is 47, so we're just looking at difference, so the numbers here, you can change the numbers either way and it's not going to affect the outcome. Does that make sense?

At least one child agreed, but another was concerned about what happens if different amounts are added to different numbers. Ben responded:

> Ah now, good question. Will that affect the answer, if you're not adding the same amount to each side—because you're looking at difference? But that's a good question, that's a very good question.

He then gave them another problem, but did not stick consistently to either a Compare or a Separate structure.

> Okay, let's have a look. I'm going to give you another problem. Here you go. This time [B1]'s got 367 apples and [B5] would like to have some—he thought he could probably eat 299 apples 'cause he's sort of feeling a little bit hungry, he hasn't eaten for a while. So [B1] started off with 367 and he is going to give [B5] 299 of those 'cause he's quite generous. Now can you predict, now thinking about using that Equal Addition, will that help us solve the problem?

At least one child responded "Yes."

> Keeping in mind that we're looking at the difference between these two numbers, not necessarily the numbers themselves.

One student (B4) suggested that the answer was 68. When asked by Ben, how he did it, he responded:

> I gave each of them one more.

Ben then pressed for understanding.

> So just while we are doing this, but with [B4] adding on one more, have we changed the difference between the numbers?

The students responded with both "Yes" and "No."

> We've changed the numbers, but have we changed the difference between the two numbers?

This time the students knew they were expected to answer "No." However, it was not clear whether or not they really understood why they had answered "No."

Discussion

The analysis of indicator words showed a consistent pattern in terms of the relationship between the frequency of teachers using the term "difference" and the number of students from their instructional group who chose to use equal additions to solve a problem on the post-test at least one week later. Mercer and Littleton (2007) used the relative incidence of indicator words to examine improvements in children's talk from *before* to *after* a programme designed to increase the quality of their talk during group-based learning. However, this tool has also proved useful in the present study for analysing differences among the teachers in their awareness of the Compare structure for subtraction. Finding that the highest incidence of using equal additions on post-test problems (60% of students in instructional group G) was associated with the teacher who had the highest incidence of referring to "difference" (Gail) suggests that the content of teachers' language may be important in revealing critical differences in the effectiveness of their teaching of mathematics. Teachers who did not refer to "difference" had no more than one student who chose to use the equal additions strategy on the post-test.

The findings of this study suggest that teachers' understanding of problem structure may be an important component of their content and pedagogical content knowledge (PCK) in mathematics. This is consistent with the work of several writers who stress the importance for primary teachers of having a deep and connected understanding of mathematics in order to teach it effectively (Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008). However, bringing about reform in mathematics education is challenging and time consuming (Anthony & Hunter, 2005).

Analysis of the lesson transcripts showed that teachers stuck very closely to the lesson description in the resource book (Ministry of Education, 2008, p. 38), mostly using the IRE (Initiation, Response, Evaluation) pattern in their interactions with the students. Although the teachers appeared committed to teaching for understanding, many of the lessons were taught in a fairly procedural manner. An alternative to following the instructions in the resource book for the scenario described in the procedure could have been to begin the lesson by letting the students solve the problem in their own preferred ways. If no student spontaneously used equal additions, then the teacher could suggest trying out this strategy to check its effectiveness. When this approach was used with Bachelor of Teaching (Honours) students, they seemed to be particularly impressed with the elegance and efficiency of the equal additions strategy after having initially tried a less efficient strategy of their own choosing. Alternatively, multiple ten-frames with beans, including some grouped in canisters of ten, seem to show far more clearly than paper money the number of beans that need to be added to the subtrahend to make it into a tidy number. It would have been good to see the Yr 5–6 teachers encouraging the students in their groups to discuss their ideas with peers, justify their solution strategies, and resolve differences in viewpoints.

Conclusions

Observing the equal additions lesson highlighted for us just how complex a process like subtraction can be. Teachers need to have a deep and connected understanding of mathematics, including knowledge of problem structure and number properties. Although resource books such as the one used for this lesson include some useful activities, it is vital that the underlying purpose and structures are clearly articulated, and teachers realise that they need to study the lesson until they fully understand it. Otherwise teachers may pick up such a book and follow a lesson prescriptively, and because they missed the point of the lesson, cause further confusion for their students. The findings of this study highlight the fact that mathematics education reform is difficult, and it takes considerable time to shift classroom discourse patterns.

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