YOUNG CHILDREN'S UNDERSTANDINGS ABOUT "SQUARE" IN 3D VIRTUAL REALITY MICROWORLDS

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This paper reports an investigation of primary school children's understandings about "square". 12 students participated in a small group teaching experiment session, where they were interviewed and guided to construct a square in a 3D virtual reality learning environment (VRLE). Main findings include mixed levels of "quasi" geometrical understandings, misconceptions about length and angles, and ambiguous uses of geometrical language for location, direction, and movement. These have implications for future teaching and learning about 2D shapes with particular reference to VRLE.

Introduction

When asked "What is a square?" or "What do you know about squares?", children would give a variety of responses such as:

It has four equal sides.
They're a quadrilateral, a regular quadrilateral. They have four corners, four right angled corners.
Squares can go onto 3D shapes, like on the bottom of a pyramid.
Square based pyramid.
And a cube and rectangular based prism. (Year 4s)

They're 2D shapes.
They have four lines of symmetry.
It's an enclosed shape.
Every side is the same.
There's four angles.
Four right angles. (Year 5s)

Is a 2D shape. It has four edges, four vertexes.
Four sides are the same.
Four corners of the square are 90 degrees. (Year 6s)

It times a number by itself.
It's a regular quadrilateral means it has 4 equal sides and 4 equal angles.
The four angles are all 90 degrees, if not 90 degrees it can't be a square.
All sides are even.
It has 2 sets of parallels. (Year 7s)

Upon reviewing their responses, it seems that the critical properties of a square (i.e., four equal sides and four equal angles) are all understood by year 4-7 children. However, examining their understanding simply by spoken language is not necessarily sufficient. In order to probe more deeply into children's thinking and understanding, the researchers have developed a 3D virtual reality learning environment (VRLE), allowing young children to express their thinking and construct their understanding about shapes and geometry via a variety of semiotic resources (Yeh & Nason, 2004a).

This study originated from the *Spatial Thinking And Reasoning* [STAR] project, in which the VRLE named VRMath 2.0 is the vehicle for investigating and developing young children's spatial abilities. VRMath 2.0, puts simply, is a combination of 3D LOGO and Web 2.0 environments. Traditionally, LOGO turtle graphics has been a powerful tool for learning geometry. However, its significance has been limited by its 2D graphics. A 2D square drawn in a traditional LOGO environment is as a square drawn on paper—a legitimate bird's-eye view of a square. What we taught young children traditionally was also based on this mindset and communication in either symbolic language or visual concrete materials has focused on the top view of the 2D square. The geometrical understanding based on this is what we would call "quasi" understanding that needs to be challenged and further qualified. This study is informed by "new paradigms for computing, new paradigms for thinking" (Resnick, 1996, p. 255) and "empowering kids to create and share programmable media" (Monroy-Hernandez & Resnick, 2008, p. 50), and has introduced new practices to traditional turtle graphics. The 3D LOGO graphics (and Web 2.0[1]) are providing new opportunities for young children to develop a more holistic learning and geometrical understanding, and new opportunities for researchers to reveal how this new practice enables young children to think and do things differently. This paper reports the first trial of the STAR project about how young children develop their ideas of squares in VRLE.

Literature review

Semiotics as the epistemological stance

This research has taken a semiotic view about meaning-making as its epistemological stance. Semiotics is the study of signs, where a sign (representamen), is something that stands to somebody (interpretant) for something (object) in some respect or capacity (system of signs) (Peirce & Buchler, 1955). Human cognition or "meaning-making" is irreducible to any one element of this triadic relation among the sign, object, and interpretant. Signs are incomplete representations of the objects and thus meaning making must be an on-going process, and meaning must be constantly qualified and challenged.

The multiple semiotic resources for learning mathematics proposed by Lemke (2001) are of particular relevance to this research project, is. Lemke outlined three semiotic resources—typological, topological, and social-actional—which have informed the design of VRMath 2.0 for mathematical meaning-making. According to Lemke, typological semiotics represents meanings by types or categories such as spoken words, written words, mathematical symbols, and chemical species. They are discrete, point-

[1] Due to the complexity and scope of this paper, the sharing aspect of Web 2.0 is not included in this report.

like, and distinctive signs. In contrast to this, topological semiotics makes meaning by continuous variations in such as size, shape, position, colour spectrum, visual intensity, pitch, loudness, and quantitative representation in mathematics. Social-actional semiotics provides a context such as building a bridge that constantly reinforces the meaningfulness of mathematics in the real world situation. The epistemological assumption for this study thus is that better learning occurs when multiple semiotic resources (i.e., typological, topological, and social-actional) are provided for meaning making.

Learning and understanding about 2D shapes

Recognition of basic 2D shapes including circles, triangles, rectangles, and squares is usually developed quite early in lower primary years, or even before school. Children at this stage are able to name the above basic shapes when see them but are not noticing properties of those shapes such as the number of sides and angles. In terms of Van Hiele's (1986) level of geometric understanding, most children in years 4 and 5 have understanding of Level 1 (visualisation, recognise figures by appearance) and Level 2 (analysis, recognise and name properties of geometrical figures), but not Level 3 (abstraction, perceive relationships between properties and between figures).

Concrete materials and computers have been the main resources for teaching and learning about 2D shapes and regular polygons. In particular, the LOGO programming language and its 2D turtle graphics has been widely used for learning and creating 2D shapes during the 1980s and 1990s (e.g., Clements, 1999; Noss, 1987). In the LOGO environment for example, a square has been transformed or represented typologically as:

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FD 50 RT 90 FD 50 RT 90 FD 50 RT 90 FD 50 RT 90
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or

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REPEAT 4 [FD 50 RT 90]
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This typological transformation of a square represented a new paradigm of thinking and doing, as well as a different level of geometric understanding.

However, as stated earlier, the traditional LOGO environment was limited by its 2D graphics. Furthermore, the traditional LOGO environment lacked the topological representations of a square. The 3D LOGO environment of VRMath 2.0 provides continuous viewpoints of a square in 3D virtual space, where learners' fixed mindsets about a bird's-eye view of a square can be challenged.

Revisiting Microworlds

The term "microworld" arose in the context of introducing the LOGO programming language (Papert, 1980). However, the idea of a microworld is not necessarily limited to LOGO programming. Edwards (1995) found that microworlds were analogously used in a variety of environments such as simulations, intrinsic models, interactive illustrations, and discovery-based learning environments. He then concluded that microworlds should be seen as being the embodiments of mathematics. He argued that the value of microworlds went beyond their reifying link between the representation and the mathematical entity to providing the opportunity for learners to kinaesthetically and intellectually interact with a system of mathematical entities, as mediated through the symbol system of a computer program.

Hoyles, Noss, and Adamson (2002) noted that the microworlds environments led directly to the idea of constructionism, arguing that effective learning will not come from finding better ways for the teacher to instruct but from giving the learners better opportunities to construct. This idea has been central to the development of VRMath 2.0, aiming to provide better opportunities for learners to construct and engage learners in interlinked mathematical entities and representations (semiotic resources).

The instrument: VRMath 2.0

VRMath 2.0 is a new implementation of its predecessor VRMath (Yeh & Nason, 2004b) enhanced by technological changes. Figure 1 is a snapshot of a square drawn in the prototype used in this study.

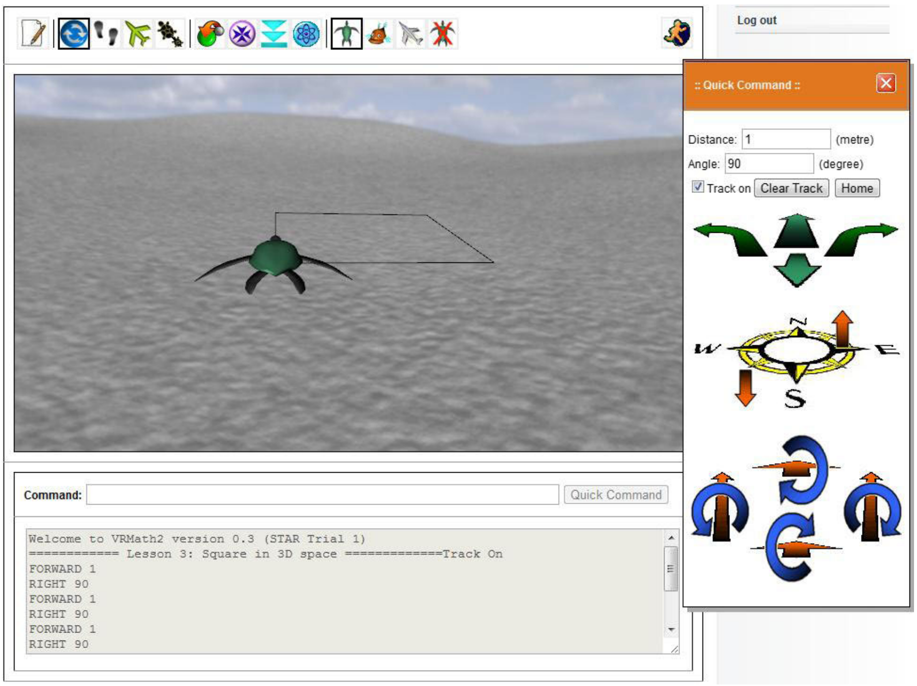

Figure 1. VRMath 2.0.

The VRMath 2.0 environment is rich in typological and topological resources. Typological resources include the Tool bar with icons, the Quick Command window, the Command field and the Message box. These resources have certain discrete meanings imposed on them. For example, the Quick Command has sets of icons that produce moving (change location) and turning (change direction) commands such as *forward 1* and *right 90*. These language commands, when clicked, will produce a topological change of the turtle in the 3D virtual space. Another main topological resource is the 3D navigation. When learners navigate in the 3D virtual space, for example, they perceive continuous views of the square in 3D space.

Because it employs virtual reality (VR) technology, the 3D space is measured in metres instead of pixels. The 3D space also enables full 3D rotations on three axes with six turns, and six fixed movements (i.e., up, down, east, west, north and south). These moving and turning commands can be classified into two groups as egocentric (i.e., forward, back, and the six turns) and fixed frame of references.

Method

There were 12 participants, three from each of grades 4 to 7. Each of the three students from the same grade were administered a lesson of 45 minutes as a small group teaching experiment (Steffe & Thompson, 2000). The lesson administered for this study was the third lesson titled "Square in 3D space", which involved (1) discussion about squares; (2) drawing a square on paper; (3) interpreting square procedures, and (4) drawing a square in VRMath 2.0. Prior to this lesson, all 12 participants had been introduced to the interface and environment of VRMath2 in lesson 1 and 2.

During the lesson, the discussions were audio-recorded and later transcribed. Participants' drawings were collected and their interactions (e.g., using Quick Command and navigation) with VRMath 2.0 were automatically logged into an online database. Field notes were also taken if the researchers observed any developments.

Results

Discussions about squares

Participants' responses to "What is a square?" or "What do you know about squares?" were presented in the introductory session. The critical properties of a square were all mentioned, with some additional information. For example: "2D squares can be found on 3D shapes" and "a regular quadrilateral" by year 4; "four lines of symmetry" by year 5; "four edges and four vertexes" by year 6; and "times a number by itself", "two sets of parallel lines" by year 7. This initial assessment showed that from year 4 to 7, students were able to articulate some properties of a square.

Another question that the researchers asked was "Is a square a rectangle?" All participants answered "No", except one year 7 student who had previously said that a square has two sets of parallel sides. The prominent reason for arguing that a square is not a rectangle was that a rectangle must have two long sides and two short sides. The year 4s' discussions were typical:

Researcher: Is a square a rectangle?
J: It can be. Stretched out a bit.
N: If the sides aren't equal.
J: My brother told me this and our teacher was telling us about this too. She used to be a year 6 teacher. They had a argue about this [sic]. She said a square can be a rectangle, but a rectangle can't be a squ ... I mean, a rectangle can be a square, but a square can't be a rectangle. But now I don't believe that anymore because the squares have four equal sides and four corners and the rectangle has two long sides and two short sides. So, I believe that it's partially a square but it can never be a full square.
Researcher: So a rectangle can't be a square…?
J: Fully.
Researcher: But can the square be called a rectangle?
R: Not really.
J: No, they're partially, they're bits and pieces that are the same.
R: A rectangle is basically a square stretched out.
J: Yes, and a square has four equal sides.
Researcher: So a rectangle can't have four equal sides?

J: No, because then it would be a square.

Draw a square on paper

When asked to draw a square on paper, all participants again drew quite consistent views of squares, as shown in Figure 2.

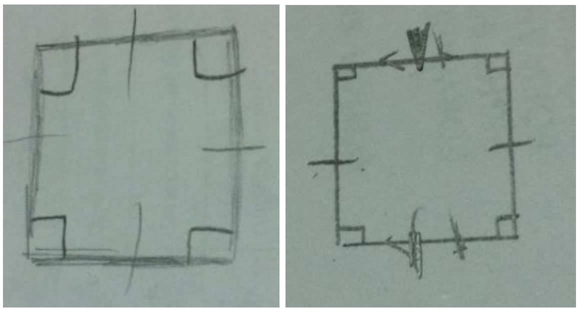

Figure 2. Drawings of squares.

Participants from year 4 to 7 all used the same notation. They used right-angle brackets to denote right angles, and a small dash on every side to denote the same length. One error found on the right hand side square in Figure 2 was the year 7 child who used one dash and two dashes to express his idea again about "two sets of parallel lines". However, this could actually denote that the four sides are not equal.

Interpreting square procedures

After drawing squares on papers, the participants were given two sets of procedures and asked to interpret them to see if they created a square:

Which sequence will make a square?

FORWARD 1	NORTH 1
RIGHT 90	EAST 1
FORWARD 1	SOUTH 1
RIGHT 90	WEST 1
FORWARD 1	
RIGHT 90	
FORWARD 1	
RIGHT 90	

Because these commands had been introduced in a previous lesson, the researchers were testing the participants' understanding about these commands. To our surprise, participants had quite different interpretations, particularly to the first procedure. A year 4 girl drew on paper as she interpreted the first procedure (see Figure 3).

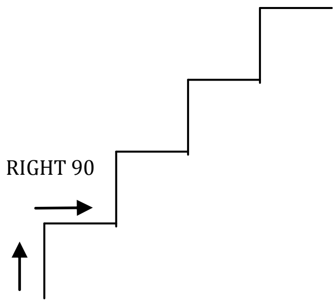

Figure 3. Year 4's interpretation (zig-zag).

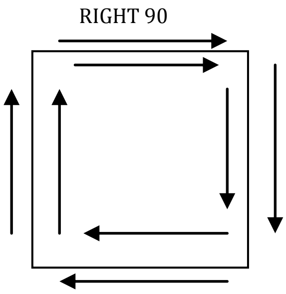

Figure 4. Year 6's interpretation (2 squares).

A year 6 boy also misinterpreted RIGHT 90 and thus predicted the first procedure to be two squares (see Figure 4). Year 5 children were also puzzled by RIGHT 90 but soon realised it was a turn when they acted it out physically. Year 7 participants did not make any mistake on the first procedure. All participants were able to recognise that the second procedure produces a square. And from that, all agreed that using compass movements is easier to create a square.

Draw a square in VRMath 2.0

Participants first tested the two square procedures in VRMath 2.0 to see if they really produced squares. As they followed the procedures, they created a square in virtual space. But due to the perspective in the 3D environment, the square does not look like a square. The researchers then asked, “Is that a square?” and “How do you know that that’s a square?” Despite having just interpreted the procedures as squares, and being aware of the distance set to 1 metre and degrees set to 90, the participants started to give surprising responses.

Year 4s seemed to be quite confident. They thought it should be a square although it did not look like one. A year 4 girl said “It looks like one. If you go up like …”, then navigated above the square to get a top view. Two year 5s also navigated to confirm that it was a square. Another year 5 boy was not so sure. He did not navigate (claiming that he “didn’t know how to move it up”): instead he said “I don’t know because I can’t see because … I don’t think it is”. Later, this boy actually commented that “That’s not a square, it’s a trapezium.” One year 6 was very much in doubt about the square. He did navigate to try to make it a square but could not get a perfect viewpoint. When the researcher questioned how he could be certain whether or not it was a square, this year 6 participant tried to use a ruler and a protractor to measure the square on the computer screen. The year 7s were very sure that the two procedures would produce squares. One year 7 did not even navigate to see but simply recognised the square. A noticeable behaviour for another year 7 was that he navigated to a good viewpoint (top view) whenever he drew a line.

The participants were then encouraged to create different procedures for a square. The results were very creative. They created squares on different planes in the 3D virtual space. One year 7 created a very simple procedure for a square using both egocentric and fixed frame of reference (FOR) commands.

FORWARD 1 EAST 1 BACK 1 WEST 1

While the above procedure did create a square, the researchers challenged “what if the turtle starts at RIGHT 45 direction? Would your procedure still produce a square?” He then predicted yes but found it to be a diamond (see Figure 5).

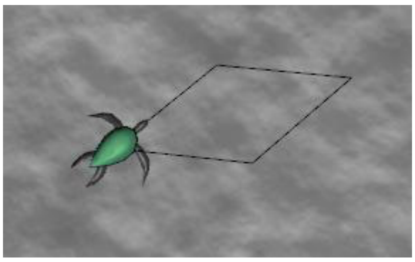

Figure 5. Diamond shape produced by mixing two FORs.

He quickly realised and explained to the researcher that because "east is always that way" and "west is always that way" (pointing to west)—a step closer to the big idea that the fixed FOR does not change the turtle's direction.

Discussion and conclusion

In this paper, we have presented our new practice that builds on the traditional LOGO. The traditional LOGO microworld is still a powerful field. It links mathematical concepts/entities with symbolic and visual representations. However, when examined under a semiotic framework, this tradition lacks the continuous representations of topological resources. In light of this, our new practice employs the 3D LOGO microworld with Web 2.0 technology to form VRMath 2.0. VRMath 2.0 is a vehicle for investigating and developing young children's spatial abilities. After its first trial, we have identified the following points of discussion.

1. Children may only develop "quasi" geometric understandings within traditional teaching and learning. For this paper, we probed 12 year 4–7 children's understandings about "square". Their seeming understanding about 2D squares became fragile when challenged in a 3D environment. We found that the traditional and legitimate bird's-eye view to be rigid. When viewing a 2D square from a 3D perspective, or when creating a 2D square in 3D space using LOGO programming language, young children's understandings about squares could be changed easily, even when they were fully aware of a square's critical properties.
2. It was also noticed that in 3D movements, some children were confused about the moving (change location) and turning (change direction). In real world situations, we are often changing the location and direction together. However, in mathematics, moving and turning have to be separated. If the turtle is changing direction, then it will not change its location and vice versa. We would like to term this as "component movement", which is essential for mathematical reasoning.
3. In terms of language use, this study found that the word RIGHT (as a turning command), can be interpreted as a moving command. When this happens, the children have ignored the number of degrees following the RIGHT command. No doubt this could be a diagnostic indicator of children's understanding, but it also presents a semantic, rhetorical, and communication problem of the learning environment.
4. Children seemed to have developed a new definition of rectangle, which unfortunately is not in line with that of the wider mathematics community. The source of this misconception is unknown. Educators should be informed about this misconception so they can communicate with children using the same mathematical language and discuss alternate definitions with them.

To conclude, we would like to return to our epistemological stance on semiotics. VRMath 2.0 can be seen as a complex sign system. It provides multiple semiotic resources including typological, topological, and social-actional representations for mathematical meaning-making. Due to the scope of this paper, we have not yet reported the sharing and social aspect of VRMath 2.0. We believe that VRMath 2.0 is a pertinent vehicle for investigating and developing spatial abilities and geometric understanding, which, including VRMath 2.0 itself, will need to be constantly challenged and qualified.

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