META-RULES OF DISCURSIVE PRACTICE IN MATHEMATICS CLASSROOMS FROM SEOUL, SHANGHAI AND TOKYO

LIHUA XU
University of Melbourne
xulh@unimelb.edu.au

This study extends a previous study on spoken mathematics (Clarke & Xu, 2008) and seeks to compare the discursive practices in classrooms from Seoul, Shanghai, and Tokyo, with a particular focus on meta-discursive rules (Sfard, 2001) that regulate exchanges between the teacher and students. The analysis centres on the events when the topic of linear equations was introduced. The similarities and differences of the three classrooms suggest that while the shared macrocultural values and beliefs frame the social activity of the classrooms in similar ways, the meta-discursive rules of classroom microculture determine the opportunities for student learning in mathematics.

Introduction

The benefits of engaging students in mathematics classroom dialogues have been highlighted in a number of recent publications (e.g. Alexander, 2008; Walshaw & Anthony, 2008) and the intensity and quality of classroom discourse have been the focus of many studies (e.g., Mercer, 1996). While there seems to be an universal assumption about the significance of student mathematical talk in learning mathematics, our studies on spoken mathematics in 22 well-taught classrooms internationally (Clarke & Xu, 2008; Clarke, Xu, & Wan, 2010) revealed significant differences among those classrooms characterised as "Asian" in the opportunities that each classroom afforded for the students to employ relatively sophisticated mathematical terms in both public discussion and private student interactions.

Extending the previous study, the study reported in this paper attempts to compare the discursive practice in classrooms from Shanghai, Seoul, and Tokyo and to examine the role played by culture in the constitution of that practice. In this study, I want to go beyond simply considering culture as a set of values and beliefs that are brought in by the participants or as external influences that are imposed on them, but to see culture as an integral part of how the work in the classroom was carried out and sustained. For the clarity of the paper, I define "culture" to be "any aspect of the ideas, communications, or behaviours of a group of people which give them a distinctive identity and which is used to organize their internal sense of cohesion and membership" (Scollon & Scollon, 1995, p. 127). In this paper, I distinguish microculture from macroculture. I use the word macroculture to refer to a set of ideas, communications, or behaviours embraced by the majority of people in a particular society (e.g. Chinese culture), whereas microculture defines regularities and patterns of interactions specific to mathematics classrooms, usually from the perspective of the researcher. The main purpose of this paper is to examine the microculture of mathematics classrooms with a particular focus on the meta-discursive rules that regulate patterns of classroom exchanges between the teacher and the students.

Meta-discursive rules in mathematics classrooms

Studies of mathematics classroom microculture have been focused on the "normative" aspects of teaching and learning. These include studies of patterns of social interactions and those rules and norms specific to a content area. For example, the work by Cobb, Yackel, and others studied classroom social norms, sociomathematical norms, and classroom mathematical practice (Yackel & Cobb, 1996). Examples of socio-mathematical norms include what counts as a mathematically different, efficient, sophisticated, or acceptable solution. Sfard (2001) addressed more general aspects of the normative aspects of classrooms, and coined the phrase "meta-discursive rules" to encompass those rules that regulate or govern discourse rather than those object-level rules concerning the relationships between mathematical objects. According to Sfard (2001), the meta-rules in mathematical discourse include those that underlie the uniquely mathematical ways of defining and proving; rules that regulate and guide interpersonal exchange and self-communication; the way symbolic tools should be used in the given type of communication, and so on. These meta-rules are the observer's construct and mostly act "from behind the scenes".

The importance of meta-rules of classroom discourse has been acknowledged by several studies. For example, van Oers (2001) argued that "participation in a mathematical discourse presupposes the observation of a set of meta-rules that regulate the discourse and the practice in general" and these rules are "culturally bound, intersubjective entities" (p. 79) that are developed as a result of participating with others in a community of practice. In addition, in a study of classrooms in Korea and the US, Pang (2000) provided evidence to show that sociomathematical norms rather than those general social norms determine the opportunities for student learning of mathematics.

Methodology

Based on the work by Yackel and Cobb (1996) and Sfard (2001), a particular focus of this paper is on the meta-rules underlying the discursive practice in classrooms. The analysis of the lessons centres on the events in which a new mathematical topic was introduced. I selected three classrooms located in Shanghai, Seoul, and Tokyo respectively, from the dataset of the *Learner's Perspective Study* (LPS) because of a shared focus of content on "linear equations" or "linear function". The LPS research design was detailed elsewhere (Clarke, 2006). In brief, three teachers who were considered as competent by local standards, from three different schools, were selected in each city. A sequence of lessons was videotaped for each teacher using three cameras (teacher camera, whole class camera and focus student camera) and video-stimulated post-lesson interviews were conducted with both the teacher and the students. Other materials collected include student written work, instructional materials, and so on.

This paper reports the analyses of the first three lessons from each of the classrooms studied and the teacher interviews. The guiding question of the analysis is "What are the similarities and differences of meta-rules that regulate the discursive practice in the three classrooms?" To address this question, the data analysis was conducted in two phases. In the first phase, three lessons from each of the three classrooms were analysed to reveal the forms and functions of activities involved in introducing the new content. In the second phase, classroom dialogues and interview accounts were examined in detail to uncover the meta-discursive rules governing those exchanges. The paper discusses the meta-discursive rules related to:

- *The nature of mathematics:* What is mathematics and who defines the rules and principles?
- *Ways of learning mathematics:* How is mathematics learned in the classroom?
- *Mathematical language:* What is considered to be the appropriate use of mathematical language?
- *Mathematical explanation:* What counts as a valid and acceptable explanation?
- *Mathematical solution method:* What is regarded as an acceptable solution method?

I will discuss these meta-rules in relation to the beliefs, values and expectations from a broader macroculture and the traditions of a particular education system. Based on the comparison of the meta-discursive rules in the three classrooms, I conclude the paper by examining the affordances of these rules on student mathematics learning and drawing some implications for studies of mathematics classrooms.

It should be emphasized that the selection of these three classrooms is not intended to signify any form of national typification. Instead, I want to illustrate the distinctive pedagogy that each classroom employs and to show how the meta-discursive rules shaped the forms of knowledge allowable in each classroom.

Introducing linear equations in the three classrooms

Despite a common focus on linear equations, observation of the lessons showed different tasks and activities employed in each classroom. In the Shanghai classroom (SH1), the topic of the first lesson was on linear equations in two unknowns and solutions. Particular attention was paid to clarifying the meaning of linear equations in two unknowns and the concepts of a solution and a solution set. The second and the third lessons introduced the rectangular coordinate axes and coordinates as "a graphical method" for solving linear equations in two unknowns.

In the Seoul classroom (KR1), the emphasis of the first lesson was on the difference in the graphs of a linear equation in two unknowns, when the condition for variable X is a natural number as compared to the graph when X is a real number. Lesson 2 focused on the notion of the intersection of the two straight lines as the solution of the simultaneous equations, and Lesson 3 continued this focus and introduced the method of elimination by addition and subtraction.

The three lessons in the Tokyo classroom (JP1) were conducted around the same task: a staircase problem, which served as a context to introduce general forms of linear function. In the first lesson, the teacher invited the students to brainstorm about the variables that can be examined in the stair problem, and the class explored the relationship between the number of steps and the perimeter of the stairs in three forms of representation: a table, a formula, and a series of figures. In the second lesson, the class was asked to relate the mathematical relationship between the number of steps and the perimeter to the changes displayed in the figures. The students were also asked to formulate relationships between two variables of their choice. The definition of a linear function was introduced in Lesson 3.

Meta-discursive rules in the three classrooms

Doing mathematics as a collective activity

The early analysis of spoken mathematics in LPS classrooms revealed both similarities and differences in the way classroom dialogue was orchestrated in each classroom. Figure 1 shows the number of teacher utterances, student utterances, and choral utterances in each lesson analysed in this paper. The figure demonstrates that while teacher talk was the most dominant form of talk in all three classrooms, there are significant differences in the way in which choral utterances and individual student utterances were valued. While very few choral utterances were found in the Tokyo classroom, this form of utterance was the most important means through which the students were given voice in the classroom in the Seoul and Shanghai classrooms.

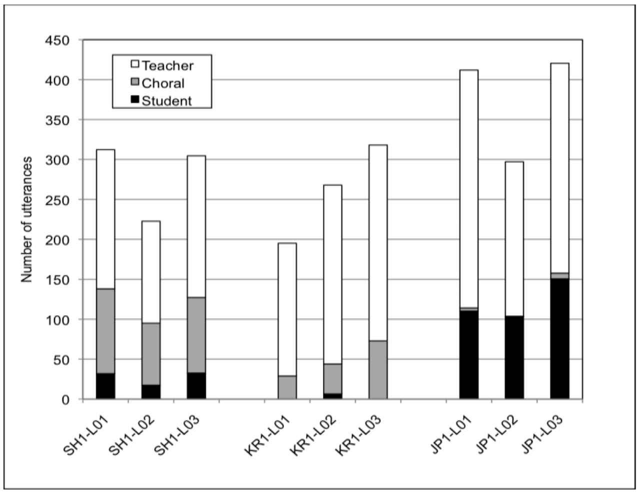

Figure 1. Number of public utterances in each lesson.

Further analyses of the classroom data revealed differences in the role of the students and the value attached to student contribution in public classroom discourse. In the Shanghai classroom, despite the classroom discussion being regarded as heavily guided by the teacher, the students were given many opportunities to contribute to the public classroom discourse, usually through teacher invitation. The activities, such as drawing a coordinate plane or defining the quadrants, were conducted in a way that the conclusion could be seen as the result of the collective contribution of the whole class. And this was crystallized in the form of board notes. Such an approach of building on student contribution was expressed in this Shanghai teacher's interview:

> One characteristic (of a typical lesson) is that the teacher is the facilitator of learning. This lesson shows that students are the active agent in learning, from the beginning till the end. That is…(I raised) questions that let them to answer, and towards the end, students generate their conclusions. Even when we talk about the sample problems, the teacher does not tell them the conclusion directly. <u>It is the students who have to think and talk about the problems by themselves. The role of the teacher is only to guide them. In other words, students are the active agent.</u> (SH1-IntT2)

While the Shanghai teacher weaved student input into a coherent ongoing classroom discourse, student contribution to the public discussion in the Seoul classroom was minimal, with most student responses consisting of a simple mental calculation or agreement with a statement made by the teacher. The teacher's reluctance to the "new" way of teaching was clearly expressed in his interview:

> These days there are many open classes in which students actively discuss in the class, I think the way of teaching is changing. But I think the teacher should teach. I think it is better. In the beginning, I teach and in the last part of the class I make students discuss what they learned. It is a good way to teach math. I don't oppose to the open class. But I think teacher's explanation is more important in teaching math. (KR1-IntT2)

Compared with the emphasis on collective action in both the Shanghai and the Seoul classrooms, the students in the Tokyo classroom were given autonomy to generate their own formulation and come up with their own method of solving the problem. In the interviews, the teacher stated the importance of students having their own opinions and raising these opinions in the public discussion. For example, in one interview, she said:

> Um, it went totally different from what I have planned, so I wouldn't be able to evaluate this class. But I had another thing I wanted to do in class if it had gone as I planned. That plan was to begin talking about a graph of a linear equation in general. So I had two plans for this lesson. But it was not important to do as planned. Students discuss with each other, and have their own opinions—that is the most important. And I think it is what was good about this lesson. (JP1-IntT2)

This Japanese teacher valued the opportunity for the students to share their opinions with their peers, which was considered more important than teaching the lesson as planned. The observation of the Japanese lessons also showed that student expression of lack of understanding was acceptable in the classroom and adequately resolved by the teacher. Arguably, this classroom is a different place from the one in which students are rarely given the chance to voice their own opinions.

While all the three classrooms can be regarded as belonging to a collectivist culture associated with Confucius Heritage, the form of collectivism was differently performed in each classroom. While in the Shanghai and Seoul classrooms, the students were given opportunities to verbally participate in the classroom discourse as a collective, the teacher in the Tokyo classroom respected the different opinions of individual students, and orchestrated the classroom discussion so that these student opinions were voiced and shared within the classroom as a community.

The use of mathematical language in the classroom

The significance attached to the use of standard mathematical language also differs. In comparison with the other two classrooms, the Shanghai classroom showed a distinctive emphasis on the accuracy of mathematical language (see Figure 2). Through the classroom discursive interactions, the students were assimilated and institutionalised into a discourse of school mathematics that encourages the accurate use of standard mathematical terms. The modelling of mathematical language by the teacher was a deliberate strategy, and the students were expected to follow such a model.

The value attached to the use of accurate mathematical language and the completeness of student response was clearly conveyed in the teacher interviews.

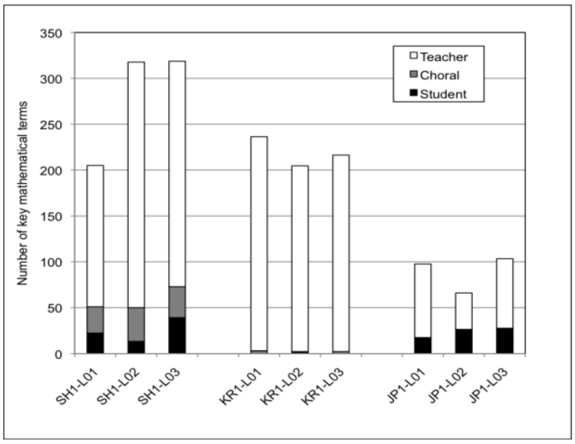

Figure 2. Frequency of key mathematical terms employed in each lesson.

For example, in the second interview, the teacher said:

> I asked one student to answer me. He could tell me what was the first step, what was the second step. <u>The answer was quite complete</u>, especially he said the first step is to transform an equation to an algebraic expression with unknown to represent another unknown. What he said is very good. He said the second step was…put this algebraic expression into another equation to substitute the unknown in that equation. That is to make the system of linear equations in two unknowns into an equation in one unknown. Then … after that … what to do after finding out this unknown. Find another unknown by substituting the value of the other unknown got. <u>This language, that is this mathematics language, is good.</u> (SH1-IntT2)

The use of standard mathematical language can be regarded as a normative aspect of this particular classroom. This finding is consistent with Leung's (1995) study of Beijing classrooms, in which he reported that 15 out of the 36 lessons observed demonstrate the stress placed on the use of accurate and rigorous mathematical language. Compared with the Shanghai classroom, the accurate rehearsal of mathematical language was much less prominent in the other two classrooms. This suggests that the emphasis on the verbalization of mathematics language may represent a distinct feature of Chinese classrooms.

Mathematical explanations

In many mathematics classrooms, it is not sufficient for students to simply provide an answer to a problem. Providing explanations is considered to be an essential component of mathematics discourse (Lampert, 1990). In the Shanghai classroom, the students were frequently asked by the teacher to provide explanations for their answer to a particular mathematical problem. Many of these explanations required the students to employ mathematical concepts or rules to justify their responses. This systematic way of defining and applying mathematical concepts (mediated by specifically designed tasks) could be seen as a reflection of beliefs about the nature of mathematics and beliefs about what students should be able to do in mathematics. This is well grounded in a tradition of school mathematics in China that emphasizes basic knowledge and basic skills. As Li (2006) observes, under this tradition, the teaching process is usually deliberately organized to ensure that teachers and students concentrate on concepts, theories, rules, skills and techniques.

Compared with the Shanghai classroom, in the Seoul classroom the rules or principles of solving a linear equation or simultaneous equations were given with little explanation from the teacher nor requested from the students in terms of the underlying meaning of the mathematical operation. The focus of the lessons was to help the students understand the procedures of solving particular groups of equations rather than an explicit focus on the meanings of concepts or the relationship between different representations. The emphasis on procedures in Korean classrooms was also reported in the study by Park and Leung (2006). Such an approach can be regarded as reflecting a view that mathematics is composed of a given body of knowledge and truth, and the task of teaching is to impart this body of knowledge to the students. In addition, such a "transmissive" way of teaching might be influenced by the male dominant culture in Korea in that this is a class in a girls' school with a male teacher.

In the Tokyo classroom, students' contributions were accepted and acknowledged no matter whether or not they were "mathematical" in a strict sense. In this classroom, mathematics was about formulating relationships and expressing them in different representations such as a table, a formula or figures. The students were interrogated to explain their understanding of the underlying relationships between variables and between representations of different form. For example, in the second lesson, the students were probed about their understanding of the proportional relationship between the number of steps and the perimeter of the stairs displayed in different representations.

It can be argued that the rules governing the legitimacy of mathematical explanations in classrooms reflect the different priorities that each teacher had in developing their students' mathematical understanding. In the Shanghai classroom, the intention is to get the students to understand the meaning of mathematical concepts, such as a solution. The Japanese teacher, on the other hand, tried to get the students to understand the mathematical relationship between two variables by using a range of representations. In comparison, the Korean teacher tried to get the students to understand the procedures for solving different types of equations. While one may argue that these differences are constrained by the different mathematical tasks presented in each of the classrooms, the meta-rules for the acceptance of certain student explanations and the rejection of others reveal more fundamental differences in the teachers' pedagogy and their beliefs about the nature of mathematics and mathematics learning.

Diversity and simplicity of solution methods

Rather than restricting the class to a particular way of solving mathematical problems as demonstrated by the teacher in the Seoul classroom, different methods or solutions were encouraged by the Tokyo teacher. The encouragement of diverse ideas was demonstrated in two interrelated aspects: firstly, the students in this Tokyo classroom were given autonomy to generate their own formula about the variables of their choice; secondly, the students were encouraged to consider the relationships displayed in different representational forms from various perspectives. The teacher's respect for diversity of solution methods was conveyed in her interview:

> <u>I think it is important to make them raise their hands when we had some opinions opposing to each other.</u> It is not for deciding by majority. I do this to see what each student has in their mind. (JP1-IntT2)

The Shanghai classroom also provided the students with opportunities to display various solution methods, but the purpose of displaying different solution methods was to examine which method was better and simpler in solving particular types of problems.

> In this way, we list students' different ways of solution, and compare them. We can analyze which method is better and students can get the correct way in the process of solving the problems ... This problem, students can do it themselves. But after solving the problem, most of them do not think whether there is a simpler method. … Some students do the problem correctly, but in a very complicated way. But a few of students do it correctly, and use a simpler method. We encourage students to make it simpler when solving a problem. (SH1-IntT3)

Arguably, the emphasis on diversity and on simplicity represent two different meta-discursive rules, each having consequences for student learning. The respect for diversity of solution methods without evaluation of their superiority in the Tokyo classroom could foster student creativity, but it might overlook the consideration of the relative validity of those methods. On the other hand, the public evaluation of different solution methods may help students to see the merits of certain methods in terms of their simplicity and efficiency, but it might encourage rigid approaches to problem solving by fostering a belief in one single "best method". Indeed, as Sekiguchi (2006) argued, maintaining the productivity of mathematical activity requires a delicate balance between the three components of a value system: validity, efficiency, and creativity.

Conclusion

From the outset, there are similarities among the three classrooms studied, such as teacher-dominated whole-class teaching as the predominant mode of instruction in all three classrooms. However, this superficial similarity masks the different functions of whole class discussion and the distinctive characteristics of such discussion displayed in each setting. As I have demonstrated in the above comparisons, the balance between uniformity and individualization was differently maintained in each classroom. While the Shanghai teacher expected the conclusions to be built upon student inputs, the Seoul teacher conceived that the role of the students was to follow the examples set by the teacher. Moreover, both the Shanghai and the Seoul classrooms encouraged uniform and collective action by the students. In comparison, the students in the Tokyo classroom had opportunities to raise their individual opinions.

The comparison of meta-discursive rules also reveals some fundamental differences in the criteria that each teacher used to make judgement about what is "mathematical" and what constitutes "student capability in mathematics". In the Shanghai classroom, the students were required to use standard mathematical language as modelled by the teacher. In addition, to be considered as mathematically capable, the students should not only be able to articulate their understanding of the mathematical concepts or principles in standard mathematical language, but also be able to apply their understanding in solving mathematical problems. In the Seoul classroom, to be regarded as mathematically capable, the students were required to understand the conditions of X and the consequence of these conditions on the solutions and the graphs of an equation. In this classroom, understanding means to know and to be able to apply those "established" mathematical routines and principles in solving various. In contrast, the students in the Tokyo classroom were interrogated by the teacher regarding their understanding of the relationships between different representations. Understanding, here, meant being able to see the underlying relationships between the variables as expressed in different representational forms and the connections between these. Since each meta-discursive rule affords different opportunities for student learning in mathematics, it can be argued that students in the three classrooms were in fact learning different "mathematics" in spite of a common focus on the topic of linear equations.

The similarities and differences between the three classrooms have implications for cross-culture comparative studies and studies of teacher competence. The findings suggests that while an examination of shared macrocultural values and beliefs (e.g. respect for authority) may help us to understand the similarities in the social organization (teacher-dominated whole class instruction), we need to look for meta-discursive rules of the classroom microculture in order to understand what determines opportunities for student learning in mathematics. More importantly, the diversity of discursive practices demonstrated in the three classrooms that are usually characterised as "East Asian" in several major international studies (e.g. TIMSS) suggests that teacher competence should indeed be conceived as a cultural construct reflective of local cultural norms, national aspirations, and traditions of particular educational systems.

Reference

Alexander, R. (2008). *Towards dialogic teaching: Rethinking classroom talk*. Cambridge: Dialogos.

Clarke, D. (2006). The LPS Research Design. In D. Clarke, C. Keitel & Y. Shimizu (Eds.), *Mathematics classrooms in twelve countries: The insider's perspective* (pp. 15–37). Rotterdam: Sense Publishers.

Clarke, D., & Xu, L. (2008). Distinguishing between mathematics classrooms in Australia, China, Japan, Korea and the USA through the lens of the distribution of responsibility for knowledge generation. *ZDM Mathematics Education, 40*(6), 963–981.

Clarke, D., Xu, L., & Wan, V. M. E. (2010). Spoken mathematics as a distinguishing characteristics of mathematics classrooms in different countries. *Mathematics Bulletin – A journal for educators* (China), *49*, 1–12.

Lampert, M. (1990). When the Problem is not the question and the solution is not the answer: mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29–63.

Leung, F. K. S. (1995). The mathematics classroom in Beijing, Hong Kong, and London. *Educational Studies in Mathematics, 29*(4), 297–325.

Li, S. (2006). Practice Makes Perfect: A key belief in China. In F. K. S. Leung, K.-D. Graf & F. J. Lopez-Real (Eds.), *Mathematics Education in Different Cultural Traditions - A Comparative Study of East Asia and the West* (Vol. 9, pp. 129-138). New York: Springer.

Mercer, N. (1996). The quality of talk in children's collaborative activity in the classroom. *Learning and Instruction, 6*(4), 359–377.

Pang, J. (2000, 24–28 April). *Implementing student-centered instruction in Korean and the U.S. elementary mathematics classrooms.* Paper presented at the American Educational Research Association, New Orleans, LA.

Park, K., & Leung, F. K. S. (2006). Mathematics lessons in Korea: Teaching with systematic variation. In D. Clarke, C. Keitel & Y. Shimizu (Eds.), *Mathematics classrooms in twelve countries: The insider's perspective* (pp. 247–261). Rotterdam: Sense Publishers.

Scollon, R., & Scollon, S. W. (1995). *Intercultural communication*. Oxford: Blackwell Publishing.

Sekiguchi, Y. (2006). Mathematical Norms in Japanese Mathematics Lessons. In D. J. Clarke, C. Keitel & Y. Shimizu (Eds.), *Mathematics Classrooms in Twelve Countries: The insider's perspective* (Vol. 1, pp. 289-306). Rotterdam: Sense Publishers

Sfard, A. (2001). There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. *Educational Studies in Mathematics, 46*, 13–57.

van Oers, B. (2001). Educational forms of initiation in mathematical culture. *Educational Studies in Mathematics, 46*, 59–85.

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research, 78*(3), 516–551.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458–477.

.