IMPLEMENTING A MATHEMATICAL THINKING ASSESSMENT FRAMEWORK: CROSS CULTURAL PERSPECTIVES

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Assessment systems should place more emphasis on the thinking process, not academic achievement alone. This study focuses on comparing Australian and Malaysian teachers' views on the practicality of implementing the Mathematical Thinking Assessment (MaTA) Framework. It involved eight mathematics teachers from Australia and Malaysia. All teachers implemented the MaTA Framework in their schools to assess students' mathematical thinking using a Performance assessment to elicit students' thinking processes during problem-solving. They also used a Metacognition Rating Scale, a Mathematical Dispositions Rating, and a Mathematical Thinking Scoring Rubric. Teachers were interviewed and their views towards implementing the MaTA Framework were reported in this study.

Introduction

The Trends in International Mathematics and Science Study (TIMSS) for Australian and Malaysian Grade 8 students' recorded a gradual decline over the years 2003 and 2007 with average scores of 505, 496, 508, and 474 respectively (Gonzales, Williams, Jocelyn, Roey, Kastberg, & Brenwald, 2008). These results suggest that the Australian and Malaysian Grade 8 students were more inclined to apply basic mathematical concepts than organizing their thinking effectively from the information given. These performances have roused, to a certain degree, national concern about the quality of mathematics education in both countries' education systems.

Therefore, we argue that the Mathematics Curriculum should give priority to fostering students' abilities to think and to organize information, as well as possessing procedural knowledge in solving problems (Ginsburg, Jacobs & Lopez, 1993). With the intention to foster this goal, the Mathematical Thinking Assessment (MaTA) Framework was developed. It aims to assess students' mathematical thinking performance in a holistic domain, which includes mathematical knowledge, mental operations and mathematical disposition. This paper focuses on comparing Australian and Malaysian teachers' perspectives particularly on the practicality of implementation the MaTA Framework in their respective countries.

Mathematical thinking

Mathematical thinking is usually referred to indirectly in the mathematics curricula produced by Australia and Malaysia as an important "process" to foster success in mathematical problem solving. In Australia, for example, "Working mathematically" is one of the important goals in mathematics (Stacey, 2005). Working mathematically is a process strand that comprises investigating, conjecturing, using problem solving strategies, applying and verifying, using mathematical language, and working in context (Australian Education Council, 1994). In Malaysia, the words "think mathematically" are contained in the aims of mathematics curriculum, which is "to develop individuals who are able to think mathematically ..." (Ministry of Education Malaysia, 2005, p. 2). It focuses on cultivating students who are able to possess mathematical content knowledge, and who learn effectively and responsibly in mathematical problem-solving and decision making.

The mathematics curricula in both countries seem to define mathematical thinking somewhat differently. This is to be expected because a well defined meaning or explanation of mathematical thinking has yet to be developed (Lutfiyya, 1998; Cai, 2002). As a result, there is no detailed description of the words "mathematical thinking" in most national mathematics curriculum documents (Isoda, 2006). As such, different perspectives on mathematical thinking are evoked. Mason, Burton and Stacey (1982), for example, defined mathematical thinking as a dynamic process enabling one to increase the complexity of ideas able to handle, and consequently expand understanding. Katagiri (2004) defined mathematical thinking as the ability to think and to make judgments independently while solving mathematics problems. Alternatively, Schoenfeld (1992) proposed five important aspects of cognition involved in mathematical thinking and problem solving: (a) knowledge base; (b) problem solving strategies; (c) monitoring and control; (d) beliefs and affects; and (e) practices (p. 348). His use of mathematical thinking is thus much more grounded in the process of its being used and what the problem solver brings to that process. More recently, Wood, Williams and McNeal (2006) defined mathematical thinking as the mental activity involved in the abstraction and generalization of mathematical ideas, adding further dimensions to the idea.

However, all the above definitions are not totally dissimilar. They seem to highlight three major domains of mathematical thinking: (a) mathematical knowledge; (b) mental operations; and (c) dispositions. This categorization was supported by the model of Component of Thinking proposed by Beyer (1988). Mathematical knowledge refers to mathematical concepts and ideas that one has acquired or learnt, while mental operations can be considered as cognitive activities that need to be performed when thinking (Beyer, 1988). As for thinking dispositions, these refer to a tendency or predilection to think in certain ways under certain circumstances (Siegel, 1999). Examples of relevant dispositions include reasonableness, thinking alertness and open-mindedness, as well as beliefs and affects.

In line with the above, it is proposed that mathematical thinking be characterized as including the following aspects:

1. It is a knowledge-dependent activity;
2. It involves the manipulation of mental skills and strategies;
3. It shows the awareness and control of one's thinking such as metacognition; and

4. It is highly influenced by the dispositions, beliefs, or attitudes of the student.

Based on the foregoing, this study will take *mathematical thinking* to be mental operations that are supported by mathematical knowledge and by certain kinds of dispositions toward the attainment of solutions to mathematics problems.

Mathematical Thinking Assessment framework

The Mathematical Thinking Assessment (MaTA) Framework consists of four components: (a) a Performance assessment; (b) a Metacognition Rating Scale; (c) a Mathematical Dispositions Rating Scale; and (d) a Mathematical Thinking Scoring Rubric. The MaTA Framework is intended to be implemented by teachers with the aim of assessing students' mathematical thinking. The *Performance assessment* component is administered by the classroom teacher to assess students' mathematical knowledge and skills (conceptual, procedural, strategies and skills) while solving particular mathematical problems in one or more content areas that have been the focus of classroom instruction. The *Metacognition rating scale* is used, also by the teacher, to elicit students' cognition awareness, such as monitoring and regulation, during problem solving process. The *Mathematical dispositions rating scale* is used by the teacher to indicate students' predisposition toward learning of mathematics. Finally, the *Mathematical thinking scoring rubric* is used to score and grade students' mathematical thinking according to the domains defined in this study.

Teacher's perceptions

Even though performance assessment promises more fruitful feedback on students' learning progress, the use of this assessment has declined in tests in United States of America (Parke & Lane, 2007). One of the major reasons is because implementing performance assessment is time consuming (Ryan, 2006; Linn & Miller, 2005; McKee & Lucas, 2005) compared to standardized testing. With current teaching workloads, administration duties and class sizes, it is argued that it is not cost-effective for teachers to invest so much time in these aspects of assessment.

Difficulty in implementing performance assessment is another reason why it has proved less popular. According to Baker (1997), performance assessment is difficult and expensive to develop. Mckee and Lucas (2005) also stressed that it is difficult at the beginning. Teachers need new knowledge and skills to implement performance assessment (Stiggins, 1995; Adi Badiozaman Tuah, 2006; Buhagiar & Murphy, 2008). Hence, extensive training is needed for teachers on how to administer performance assessment in the classroom (Aschbacher, 1992).

Methodology

Participants

A total of eight secondary school mathematics teachers, four each from Australia and Malaysia were selected to participate in the study. All the selected teachers have at least five years of teaching experience in Mathematics. Through their teaching experiences, they were believed to be able to implement the MaTA Framework according to its guidelines in their respective schools.

Procedures and data analysis

All the selected secondary school mathematics teachers were briefed and guided by one of the researchers on how to use the MaTA Framework to assess students' mathematical thinking performances. This was conducted in a one-on-one basis where the researcher met the teachers regularly prior to and during the data collection processes. The following summarizes how teachers could be expected to implement the MaTA Framework in their home school context.

Step 1: Designing performance assessment

Based on the procedures or guidelines provided in the MaTA Framework, the teachers designed the performance tasks (i.e. test items or questions) and then administered these to their students. During the assessment, teachers encouraged students to use appropriate approaches to perform the tasks, such as explaining and justifying the answers obtained in their solutions, as required by the MaTA Framework. Usually, this was achieved by including specific prompts in questions, such as asking students to explain their thinking or to justify their solutions.

Step 2: Scoring students' performance

By referring to the scoring criteria and scoring guide for each of the domains in Mathematical Thinking Scoring Rubric, namely conceptual knowledge, procedural knowledge, thinking strategies and thinking skills, teachers were able score their students' levels of performances respectively based on their written solutions. After scoring students' written solutions, the teachers then used the Metacognition Rating Scale to rate students' levels of metacognition based on teachers' classroom observations. Similarly, the levels of performances for students' mathematical dispositions could be determined through a Mathematical Dispositions Rating Scale.

Step 3: Reporting students' mathematics performance

After scoring students' written solutions and rating their metacognition and mathematical dispositions, students' levels of performances for each domain were summarized into a standard report, entitled Teacher's Report on Student's Mathematical Thinking Performance. This report contained band scores and comments from the teacher for each domain of mathematical thinking. This report could then be given to students as feedback on each of the three areas indicating the quality of their performances, based on their written solutions and on their teacher's classroom observations.

Finally, all the teachers involved were interviewed for between 30 minutes to 60 minutes. The interviews allowed the teachers to justify their views concerning the practicality of implementing the MaTA Framework in their respective schools.

Findings and discussions

The findings reveal that teachers from Australia and Malaysia responded positively toward the impact of the MaTA Framework on the teaching and learning in the classroom. One of the Australian teachers commented that the band score provided under the MaTA Framework was consistent with and added value to the score given in the school's current approach to assessment,

> From the teacher perspective, I like the fact that I can compare my mark with another scoring rubric [the MaTA Framework]. I can recognize what I was giving … we were roughly the same. I wasn't being too lenient or too harsh, which is always nice. (Teacher 1/Australia)

On top of this, the MaTA Framework was perceived as able to promote students' thinking through solving and justification of solution, as evidenced by the following teacher,

> Because it encourages students to endeavour the answer…hence this helps the students to answer mathematics problem. For higher level mathematics problems, we are not going to encounter [problem like] one plus zero equal to one, we have to explain a lot. This is what I mean, the impact is great. Because we train the kids to think, endeavour to think! (Teacher 4/Malaysia)

The guideline provided in the MaTA Framework was able to help teachers to grade the students' solution in a systematic and homogeneous way. This helps to ensure consistency of grading and fairness to the students who were being assessed, as illustrated by one of the Australian teacher:

> It's very concrete, very detailed and very specific and therefore it would allow for large amount of consistency across (students and grades)." (Teacher 2/Australia)

Even though teachers responded positively towards the MaTA Framework, there were negative views expressed as well. Eight major aspects concerning the practicality of implementing the MaTA Framework were identified. However, this paper only presents three of them: time limitations, inadequate knowledge, and students' limited English proficiency.

Time limitations

All teachers involved in the study commented that scoring and reporting of students' performances in each domain of mathematical thinking were time consuming. However, teachers from Australia seemed to look at these constraints from wider points of view, such as needed professional development and changing school assessment culture. One teacher argued that "time that is required to implement this versus the amount of benefits that would be achieved, it's not a linear relationship". When he was asked to further elaborated, he said:

> If we could perhaps get really used to it and could become more time efficient, but it requires certain amount of professional learning and change in culture across the whole school, or say among all Maths teachers. It has to be something accepted by all Maths teachers and adopted across the whole country. It would require a fair amount of professional learning to be able to use the method. (Teacher 2/Australia)

By contrast, the Malaysian teachers were inclined to focus on the drawbacks, such as heavy workload, pressure of covering the syllabus and needing to keep the students on track, as illustrated by the following teacher.

> Again…(it) is the time factor. Do we have the time to do it? Now teachers are much overloaded, they have still got to do their report books, and they have still got to do the mark sheets … Even though he is a subject teacher, but the subject teacher could be a form teacher for another class and so on and so forth. So it is extra work for the teacher and then you have to score them individually … question by question. It is time consuming. That is one of…I think the major factor…time which we don't really have.

> Everyone is trying to finish the syllabus, trying to do a lot of revision so that [students] can pass the exam with a 7 or 8 grade. (Teacher 1/Malaysia)

This finding was consistent with what was found by Ryan (2006), Linn and Miller (2005), McKee and Lucas (2005) and Parke and Lane (2007) that longer time was needed to implement performance assessment compared to other types of assessment. Nevertheless, the teachers admitted that this type of scoring and reporting could become easier once they were familiar with the terms or keywords used in the scoring and reporting. As teachers commented,

> It's not complicated, it's quite simple to use. As I said it just takes a while to fill up, once you have marked the actual assessment tasks yourself. (Teacher 1/Australia)

> Once you are familiar, it should be quite easy. (Teacher 3/Malaysia).

This result was again in line with McKee and Lucas (2005) who claimed that performance assessment tends to be more difficult at the early stage of implementation.

Inadequate knowledge and skills

The teachers were familiar with traditional forms of assessment where scoring focuses only on the final answer produced by the students. Therefore, when the teachers were asked to focus more on assessing students' thinking process, they found it more difficult to give fair and appropriate scores to students' performance based on the scoring rubric. The Australian teachers admitted that they had inadequate knowledge and skill to implement the MaTA Framework. They agreed that this inadequacy could be remedied by teacher re-training. As one of the teachers said, "Not many teachers would be confident to be able to handle this type of assessment [the MaTA Framework] accurately. It requires a different teacher training approach for the present school teachers" (Teacher 3/Australia). Malaysian teachers also admitted this inadequacy, but they preferred self-directed learning to introduce themselves to this type assessment. They asked whether there was module provided to guide them through this assessment:

> Normally when we try to create something that is very new, …the Malaysian way is they want something to look at first, to go through first, they want something as examples, as a guideline or reference for them. And from there…I cannot say they want to copy or something, but normally they will follow exactly from there. (Teacher 1/Malaysia)

These responses reflected that the teacher professional development in Australia tends to be more structured, with any implementation of new education policies requiring systematic dissemination and training. However, the situation in Malaysia is quite different where only few experienced teachers get selected for such training; and they are then expected to give "in house" training to other teachers from different schools at a later date. Very often, important information dissipates during the sharing process. Worse still, some teachers are not asked to attend any training due to tight budgets. As a result, many teachers have to learn the new education policies for themselves based on guidelines or modules provided by the Malaysian Ministry of Education.

Limited English proficiency by students

Even though Australia is an English speaking country, the teachers involved gave a surprising remark by saying that limited English proficiency was one of the major drawbacks that caused some students to perform poorly under the MaTA framework. One of the teachers gave the following example which was happening in his class:

> We have a student this year in year 12, he is of Chinese background, his English language is very poor ... he is so frightened of choosing subjects that will affect his marks based on his poor English. He actually chooses to do Further Maths, Maths Methods, and Specialist Maths in Year 12. (Teacher 1/Australia)

Because of students' limited English proficiency, some mathematics teachers focused on teaching mathematical skills, with less emphasis on solving mathematical problems, as commented by one of the teachers:

> Yeah, that's [English proficiency] is critical, one of the reasons why we do mostly skill based teaching in this school is because we've got an extremely high (number of) non-English speaking background students, 80% of them. (Teacher 3/Australia)

Hence, students who were poor in English were not keen on being assessed using the MaTA Framework. This was similar to the Malaysian context where students preferred standardized-testing, such as tests with multiple-choice questions (Hwa, 2010). One of the Malaysian teachers said that,

> They have the idea but they don't know how to explain it, how to write their idea ... because some of our students' English is not good" (Teacher 1/Malaysia).

As a result, students from non-English speaking backgrounds were expected to perform poorly in MaTA Framework. Students who were struggling in mastering English were rarely comfortable with being asked to write justifications of their solutions.

Conclusion

We found that the MaTA Framework provides sufficient information in guiding secondary school mathematics teachers from Australia and Malaysia to assess students' mathematical thinking. The guidelines proposed were effective in enabling the mathematics teachers involved to implement the MaTA Framework in their schools. However, the data also reveal that the MaTA Framework was seen by teachers as lacking in simplicity and ease of use compared to traditional forms of testing. Much more time was needed to prepare performance assessment by using the MaTA Framework. Besides being time consuming, factors such as inadequate knowledge and skills by teachers, and limited English proficiency among students were seen also to affect the practicality of the MaTA Framework in the school context.

These views were expressed in somewhat different ways by Australian and Malaysian teachers towards implementing the MaTA Framework. Hence, in order to give greater attention to the assessment of mathematical thinking, increasing teachers' exposure to the key ideas of a framework such as MaTA, and consequent teacher training through workshops or seminars are necessary. These should increase the quality of the performance assessment, but also foster greater consistency in scoring and reporting of students' mathematical thinking.

Acknowledgement

The authors acknowledge with gratitude the funding of this research study from Endeavour Research Fellowship Award, Australia.

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