TEACHER CAPACITY AS A KEY ELEMENT OF NATIONAL CURRICULUM REFORM IN MATHEMATICS: AN EXPLORATORY COMPARATIVE STUDY BETWEEN AUSTRALIA AND CHINA

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This exploratory study involving Australian and Chinese teachers seeks to characterise teachers' capacity to help students connect arithmetic learning and emerging algebraic thinking. The study is based on a questionnaire given to Australian and Chinese teachers, comprising seven students' solutions of subtraction sentences. Teachers' responses to the questionnaire were analysed in terms of four categories: knowledge of mathematics, interpretation of the intentions of the official curriculum documents, understanding of students' thinking, and capacity to design appropriate instruction in the short and long term. These four categories form the basis of our construct of teacher capacity.

Curriculum focus

In many countries, official curriculum documents now endorse the building of closer relationships between the study of number in the primary school and the development of algebraic thinking. Algebraic thinking is not the same as the use of algebraic symbols. It is about identifying generalisations and structural relations in number sentences and operations. This is very different to what in the past was seen as the study of arithmetic.

Australian Curriculum in Mathematics (ACARA, 2010) presents Number and Algebra as a single content strand for the compulsory years of school. In its overview statement to this strand, ACARA (2010) website states that:

> Number and algebra are developed together since each enriches the study of the other … They (students) understand the connections between operations. They recognise pattern and … build on their understanding of the number system to describe relationships and formulate generalisations. They recognise equivalence and solve equations and inequalities … and communicate their reasoning.

This statement echoes important ideas that have been present for at least five years in related State curriculum documents, for example, in linking Number, Structure and Working Mathematically in the Victorian Essential Learning Standards (VCAA, 2007); and in other official curriculum documents such as the Mathematics Developmental Continuum (DEECD, 2006).

The *Chinese Mathematics Curriculum Standards for Compulsory Education* (Ministry of Education, 2001), also present a single strand entitled Number and Algebra. In Stage 2 which covers Years 4 to 6, two "teaching objectives" refer to the

importance of considering the inverse properties of calculation and to investigating the properties of equivalent sentences. Objective 5 on "operation of numbers" refers "to experience the inverse relation between addition and subtraction, as well as that of multiplication and division in the process of concrete operation and solution on simple practical problems." (p. 21). Objective 3 on "sentences and equations" (p. 21) refers "to understand the property of equal sentences and enable to solve easy equations with the property of equal sentences (e.g. $3x + 2 = 5$, $2x – x = 3$)". Several Chinese researchers, such as Xu (2003), emphasise that closer alignment is needed between the study of number and number relationships in the primary school and the study of algebra in the secondary school in this curriculum reform.

Official documents in both countries clearly endorse a more coherent treatment of number sentences and operations and the development of algebraic thinking in the primary and early years of secondary school; and we argue that teacher capacity is a key dimension in realising that goal. However, the implementation of curriculum change is never simply from the top down. Teachers' interpretations and responses at the level of practice are never simple reflections of what is contained in official curriculum documents. While curriculum documents set out broad directions for change, any successful implementation of these "big ideas" depends on teachers' capacity to apply subtle interpretations and careful local adaptations (Datnow & Castellano, 2000). Teachers' professional insight and agency in translating these ideas into practice must frame any definition of teacher capacity (Smyth, 1995). Moreover, simply focussing on enactment as the defining feature of capacity tends to place any teacher opposition to reform in an entirely negative light.

Research focus

In examining the importance of teacher capacity in building a bridge between number operations and algebraic thinking, our mathematical focus is on students' ability to read and interpret number sentences as expressions of mathematical relationships, rather than seeing them exclusively as calculations to be performed. Specifically, we draw attention to the importance of assisting students to use ideas of *equivalence* and *compensation* to solve number sentences involving subtraction. These methods, Irwin and Britt (2005) have argued, may provide a foundation for algebraic thinking (p. 169). Jacobs, Franke, Carpenter, Levi and Battey (2007) use the term relational thinking to refer to these kinds of strategies. These authors agree that there is still room for debate whether relational thinking in arithmetic represents a way of thinking about arithmetic that provides a foundation for learning algebra, or is itself a form of algebraic reasoning. They argue strongly that "one fundamental goal of integrating relational thinking into the elementary curriculum is to facilitate students' transition to the formal study of algebra in the later grades so that no distinct boundary exists between arithmetic and algebra" (p. 261).

The research instrument and some results

Teachers in both countries were invited to complete a written questionnaire based on a "scenario" where some researchers had visited their school and gave students (either in Year 6 or Year 7) the following number sentences, asking them to write a number in the box to make a true statement, and in each case to explain their working briefly.

These two questions, according to the scenario, had been accompanied by similar questions dealing with addition, and were intended to see how students interpret and solve number sentences involving different operations:

> For each of the following number sentences, write a number in the box to make a true statement. Explain your working briefly.
>
> 39 – 15 = 41 – ☐
>
> 104 – 45 = ☐ 46

The Australian and Chinese teachers were then presented with seven responses selected from actual responses by Australian and Chinese students in a study reported by Stephens (2008). In the Australian sample (see Appendix A), two Australian students, A and B, correctly found the missing number by calculating the result of the subtractions 39 –15, and 104 – 45, and then used these results to calculate the value of the missing numbers on the right hand side. Student C refrained from calculating, attempting to use equivalence, but compensated in the wrong direction to get answers of 13 and 103 respectively (or mistook the operation for + instead of -). Two students, D and E, successfully argued that since 41 is two more than 39 the missing number has to be two more than 15 to keep both sides equivalent. They applied similar reasoning to the second problem. Student F used arrows connecting the two related numbers (e.g. 39 and 41), and also connecting the other number (15) to the unknown number. Above the arrows Student F wrote +2 for the first problem and +1 for the second problem, obtaining correct answers. Finally, student G placed the letters A and A1 beneath 39 and 41, and B and B1 beneath 15 and the unknown number, and found correct values for the unknown numbers using an explanation based on equivalence and compensation. While the answer to the first problem is correct, Student G's written explanation contained a small error.

The Chinese sample contained parallel examples as far as possible. Students A and B gave calculated solutions that were almost identical to their Australian counterparts. Students C and D, in the Chinese sample, gave correct and clearly articulated relational explanations. Student E used a diagrammatic representation almost identical to Student F in the Australian sample. Student F in the Chinese sample used compensation correctly in the first problem, but in the wrong direction for the second problem (like Student C in the Australian sample) giving an answer of 103. Student G in the Chinese sample also used compensation in the wrong direction in the first problem, obtaining an answer of 13. However, in the second problem Student G gave the missing number as 59 which is the result of calculating 104 – 45.

Teachers were then asked three key questions, with each question on a separate A4 page. Firstly, teachers were asked to comment briefly on each of the seven samples. Secondly, Australian teachers were asked how they would respond specifically to Students A, B and C if they were students in their class. They could respond to other students if they wished. Chinese teachers were asked to respond specifically to Students A, B and F. Thirdly, all teachers were asked: "In planning your teaching program, how do you want to move students' thinking forward in regard to these and related questions? How will you develop the kind of mathematical thinking that

students need to solve these kinds of number sentences? You can talk about a short design of one or several lessons, or a longer plan over the year."

The sample

Both samples used in this exploratory study were convenience samples. The Australian sample consisted of 20 Numeracy coaches working in Victorian government schools who were participating in an extended professional development program. All 20 were school-based with time release to support mathematics teaching in their home school or in other local schools. 17 were based in primary schools. Two of the three coaches who were based in secondary schools were not mathematics specialists, although all were teaching mathematics. The Chinese sample of 20 teachers was randomly selected from a larger group of more than 100 specialist mathematics teachers who had agreed to complete the questionnaire (Chinese version) during several teacher professional development programs in Nanjing, Whenzhou and Chongqing. All Chinese teachers were teaching Mathematics across several grades; and 18 were teaching in elementary schools.

The analytical framework: Four criteria

Teacher capacity to build effective bridges between the teaching of number and thinking algebraically about number sentences using equivalence and compensation is defined in this study in terms of four criteria: *Criterion A:* Knowledge of mathematics; *Criterion B:* Interpretation of the intentions of official curriculum documents; *Criterion C:* Understanding of students' thinking; and *Criterion D:* Design of teaching (See Table 1, over). This construct of teacher capacity is similar to the construct of mathematical knowledge for teaching elaborated in two important papers by Ball, Thames and Phelps (2008) and by Hill, Ball and Schilling (2008). Our Criterion A is intended to capture their category of *Specialized Content Knowledge*; our Criteria B and C are derived from their category of *Knowledge of Content and Students*, that is, knowledge that combines knowing about students and knowing about mathematics; and our Criterion D gives emphasis to their category of *Knowledge of Content and Teaching*, which combines knowing about teaching and knowing about mathematics. Our construct of teacher capacity differs from the construct of mathematical knowledge for teaching in giving a greater emphasis to knowledge of official curriculum documents.

Qualitative analysis

Each criterion of our analytical framework was expressed in terms of four specific indicators (see Table 1). In the case of the first two criteria, these indicators expressed how well teachers' responses indicated a clear understanding of the mathematical thinking that the two problems were intended to examine; and in the second criterion how this thinking reflected key ideas of current official curriculum documents in the respective countries. Indicators of capacity associated with the third criterion looked specifically at how well teachers could describe and interpret key features of performance expressed by the seven students, and how well they could respond to what the students had done in terms of immediate classroom teaching. Finally, those for the fourth criterion looked at how well teachers could plan for teaching that

fostered a deeper appreciation of the mathematical thinking embodied in these and related tasks, especially in fostering ideas of equivalence and compensation.

Table 1. Four criteria and associated indicators.

Criterion A – Knowledge of relevant Mathematics: (1) Does the teacher recognise that there are two mathematical approaches to solving these kinds of problems – using calculation; or using equivalence and compensation for the operations of subtraction or difference? (2) Does the teacher recognise that students need to attend specifically to subtraction or "difference" when using equivalence? (3) Does the teacher refer specifically to mathematical terms such as "equivalent difference" or "difference unchanged"? (4) Does the teacher understand that equivalence using subtraction is compensated differently from addition, and/or that the key idea of equivalence also applies to the other operations?	**Criterion C – Understanding of students' thinking:** (1) Does the teacher recognise that Australian students D, E, F & G (or Chinese students C, D & E) were correctly using relational thinking although expressed in different ways? (2) Does the teacher identify the typical error (compensating in the wrong direction) shown in solution C of Australia sample (or solutions F(2) and G(1) of China sample)? (3) Does the teacher recognise the importance of identifying those students who can *only* use calculation? (4) Do Chinese teachers see that solution G(2) suggests a deeper misunderstanding; or do Australian teachers recognise that student G has a clear understanding of equivalence although makes a small error in the explanation for question 1?
Criterion B – Interpretation of the intentions of official curriculum documents: (1) Does the teacher realise that "Mathematical Thinking" should be treated as an important consideration whilst calculation remains valued? (2) Does the teacher understand and support the intention of the curriculum to link number learning and algebraic thinking? (3) Does the teacher show in his/her descriptions of children's responses, an awareness of the key curriculum goal of moving students from reliance on calculation to using equivalence in number sentences, here with respect to "difference" or subtraction? (4) Does the teacher's response use terms, words or expressions that are found in official curriculum documents?	**Criterion D – Design of teaching:** (1) In designing teaching, does the teacher focus on the important aspects of mathematics to be taught and fostering mathematical thinking, not on general strategies? (2) Does the teacher have a short-term teaching plan to respond to selected students in the next lesson? Does the teacher recognise that it is *more* important to let students who can think relationally explain their thinking to the whole class, but not so important for those who used calculations? (3) Does the teacher have a longer-term teaching plan to move students' relational thinking forward? How well does this plan reflect knowledge of students' thinking (Criterion C)? (4) Does the teacher give teaching examples or use teaching with variation to help students' learning and thinking?

Qualitative evidence of demonstrated teacher capacity

Criterion A: Knowledge of Mathematics

Chinese teacher 1 commented: "If the same number is added to both minuend and subtrahend, the difference represented by the number sentence will be unchanged … this is also called the *law of difference unchanged*." Similarly, Australian teacher 11 said: "Although the process is the same with both + and - , the students often misunderstand whether they have to add or subtract to get the equivalent value on both sides of the equals sign."

Criterion B: Interpretation of the intentions of official curriculum documents

Australian teachers 3 and 4 referred to *Key Characteristics of Effective Numeracy Teaching P-6* (DEECD, 2009). Teacher 4 pointed to the need to:

> engage students in identifying and using arithmetic relationships within number sentences to solve problems without calculating and teach a repertoire of strategies – (using) guess-guess-check (systematic trial and error), logical arithmetic reasoning and inverse operations to solve a wider range of number sentences.

Chinese teacher 15 said: "In the elementary teaching of number and algebra, integrity and coherence need to be embodied".

Criterion C: Understanding of students' thinking

> Despite their different responses, students C, D, E, F & G were all using the relations to solve the questions which is different from students A & B. This is a better method and to be encouraged because it is closer to the structural thinking that students need when learning algebra. These number sentences have been carefully chosen to make this method better. Student C spotted the relationships between the numbers being used in both algorithms (addition and subtraction) …. s/he has added to one of the numbers, (whereas) s/he needed to subtract from the other. (Australian teacher 5)

Chinese teacher 13 said, "It is not easy to judge whether A and B solve it through calculation, or through the reverse principle between addition and subtraction," noting the importance of distinguishing between those students who can only use calculation. Chinese teacher 8 says that "After students' agreement on Type 2 (relational thinking), further explain the rationale of type 2 to help students understand it."

Criterion D: Design of teaching

One Australian teacher 16 gave a well-designed five-stage plan to move students thinking forward with each stage reflecting a different level of mathematical thinking.

Chinese teacher 7 suggested: "Explore variations, change the '–' in both sides into '+' or have the change in one side and leave the other unchanged." Teaching with variation is used effectively in the following teaching examples:

1. Fill in ">" , "<" or "=" in ○.	2.Fill in "+" or "–" in ○, and numbers in □.
45 – 36 ○ (45 + 3) – (36 + 3)	87 – 45 = (87 ○ □) – (45 - 3)
45 – 36 ○ (45 – 3) – (36 – 3)	98 – 36 = (98 – 5) - (36 ○ □)
198 – 42 ○ (198 + 2) –(42 + 2)	184 – 56 = (184 ○ □) – (56 + 8)

A weak or inappropriate response to Criteria B and D

Australian teacher 7 said: "I am not familiar with working in this area of the school I would need to consult the Maths (Developmental) Continuum ... I need further help as I was probably looking in the wrong progression point."

A dissenting response to Criteria A and B but strong on D

Chinese teacher 7 showed a clear understanding of the mathematical elements of the questions and designed very clear teaching examples to help students develop relational thinking. However, teacher 7 had very strong resistance to fostering mathematical thinking other than ensuring students' correct calculations:

> The solutions of Students A & B need to be energetically popularized (to the whole class), because most students can master them ... The deep thinking of Student C

deserves praise, *but* it shouldn't be introduced, because it is not very good and some students may be confused by it and cause mistakes like that of Student F.

An exploratory quantitative analysis

By assigning a score of 1 if one of the four indicators was evident in a teacher's response, and 0 if it was omitted from their response or answered inappropriately, it was possible to construct a score of 0 to 4 for each criterion, and hence a maximum score of 16 across the four criteria. While the four listed indicators for each criterion are, in our opinion, the most relevant in terms of reflecting teacher capacity, they are not the only possible indicators. We allowed for the possibility that teachers' might provide convincing alternatives to the four indicators that we had listed.

For the Chinese sample, the highest score was 15 and the lowest score was 5, with a median score of 9. For the 20 Australian teachers, the highest score was 14 and the lowest was 2, with a median score of 10. The respective mean scores were 9.05 (Chinese) and 8.9 (Australian) with standard deviations 2.31 and 3.54 respectively. In the Australian sample, four teachers scored less than 5, whereas a score of 5 was the lowest for the Chinese teachers. Table 2 shows means that were calculated for each of the Criteria, and a global mean score calculated across all four Criteria.

Table 2. Means for each criterion and global means.

Sample	Criterion A	Criterion B	Criterion C	Criterion D	Total
Chinese	3.0	2.2	1.75	2.1	9.05
Australian	2.3	2.45	2.1	2.05	8.9

An initial classification of Teacher Capacity

Those teachers who scored between 11 and 16 were classified as demonstrating High Capacity; those scoring between 6 and 10 were classified as having Medium Capacity; and those scoring less than 6 as having Low Capacity. An initial classification of the two samples is shown Table 3.

Table 3. Classifications of teacher capacity.

Capacity	Chinese	Australian
High	4	6
Medium	15	10
Low	1	4

Discussion and conclusion

Among Chinese and Australian teachers, High Capacity to make an effective bridge between the teaching of number and fostering of algebraic thinking was demonstrated by teachers' clear understanding of the mathematical nature of the tasks students had been engaged in; their capacity to relate these tasks to relevant curriculum documents; their high interpretative skills when applied to each of the seven samples of students' work; and their extensive range of ideas for designing and implementing a teaching program to support the development of students' mathematical thinking. Medium

Capacity was shown by other teachers who, while possessing knowledge and skills supportive of these directions, clearly need to increase their current levels of professional knowledge and skills. In both samples, Low Capacity was evident in a minority of teachers who appeared unable to express a clear articulation of the mathematical nature of the tasks, or what differentiated the seven responses used in the questionnaire. These teachers were unable to point to how the tasks related to what is contained in official curriculum documents, or to describe how they would plan a program of teaching to foster these and related mathematical ideas.

Chinese and Australian teachers in the sample appeared to perform similarly on Criteria B (Interpretation of the intentions of official curriculum documents) and D (Design of teaching). However, Chinese teachers appeared to perform better than their Australian counterparts in elaborating the mathematical thinking embedded in the tasks that the students were asked to work on. On the other hand, Australian teachers appeared slightly better than the Chinese sample in responding to Criterion C (Understanding of students' thinking). These apparent differences call for further investigation. An initial pair-wise comparison of the four criteria shows a significant correlation at 0.05 level between Criteria A & B, and A & C; and at 0.01 level between Criteria A & D, B & C, and B & D. Similar analysis at the level of indicators should also be explored, and a factor analysis could also be used.

As a basis for a study involving a larger sample of teachers in both countries, with a more carefully stratified sample with respect to specific mathematical training, years of experience and location, this exploratory study has been successful in several respects. The questionnaire using the seven samples of students' work, and its three key questions, was effective in eliciting teachers' responses. In turn, teachers' responses were able to be used as a basis for examining teacher capacity in terms of teachers' mathematical knowledge, knowledge of official curriculum documents, understanding of students' thinking – that is, ability to analyse and interpret students responses and to frame appropriate responses to individual students – and to design credible sequences of teaching to foster the underlying mathematical ideas. These interpretations and professional dispositions go well beyond the "big ideas" or "general statements of intent" that are typically expressed in official curriculum documents. These subtle interpretations and the ability to frame immediate and longer term instructional responses are pre-requisites of any successful implementation of the "big ideas". In this paper we have elaborated a definition of teacher capacity firmly based on these characteristics. We feel confident that the conceptual and analytical framework of this exploratory study is robust enough to guide a larger study examining teacher capacity and curriculum reform in China and Australia.

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Appendix A