EFFECTS OF USING HISTORY OF MATHEMATICS ON JUNIOR COLLEGE STUDENTS' ATTITUDES AND ACHIEVEMENT

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Many researchers advocate the use of history of mathematics in education. However, empirical research in this area is scarce. To address this issue, a quasi-experiment is used to investigate the relationship between the use of history of mathematics, and the attitudes and achievement of junior college students in Singapore. Multivariate analysis of covariance and analysis of covariance, with pre-test scores as covariates, and post-test scores as dependent variables, suggest that history of mathematics can improve students' attitudes and achievement in mathematics.

Introduction

The benefits of using history of mathematics in education have been widely discussed by researchers globally. A list of these benefits are summarised in Fauvel and Van Maanen (2000). Many researchers have suggested that the use of history of mathematics in education are related to positive students' learning outcomes (Calinger, 1996; Fauvel, 1991) and supported its inclusion in national curricula (Fauvel & Van Maanen, 2000). However in practice, history of mathematics is rarely used in schools (Fried, 2001), as teachers cannot commit the time to prepare the relevant teaching materials (Fauvel, 1991; Fowler, 1991; Gulikers & Blom, 2001). To reap the benefits of using history in mathematics lessons, researchers must first convince education planners and teachers that the time spent in preparing these teaching materials can translate to better students' learning outcomes. One possible way is to provide empirical evidences that demonstrates positive relationships between the history of mathematics and desirable students' learning outcomes. However, such studies are either lacking or dated. Hence this study uses a quasi-experiment to investigate the relationship between the use of history of mathematics in classrooms, and students' attitudes and achievement in mathematics.

Literature Review

Lim and Chapman (in preparation, a) review definitions of history of mathematics used in education by various researchers and suggest that it should include (1) the use of anecdotes and biographies of mathematicians (Bidwell, 1993; Wilson & Chauvot, 2000); (2) the discussion of historical motivations for the development of content (Katz, 1993); and (3) the use of original materials from historical sources (Arcavi & Bruckheimer, 2000; Jahnke et al., 2000). This study adopted these definitions.

Theoretical literature review

Many researchers have argued that the use of history in mathematics lessons can lead to better attitudes and achievement in mathematics (e.g., Fauvel, 1991; Gulikers & Blom, 2001). Firstly, the use of anecdotes and biographies of mathematicians make lessons more interesting and dynamic (Perkins, 1991; Siu, 1997). Hence students should find their mathematics lessons more enjoyable when history is used.

In addition, students are more motivated to learn about mathematics if they are able to identify the important role that mathematics play in human culture through history (Tymoczko, 1994). In addition, students are able to appreciate the usefulness of mathematics in real life through history, as mathematical concepts are often developed to solve real-life problems in the past (Burton, 1998). Consequently, the use of history in lessons can improve students' perception about the value of mathematics to mankind and motivate them to learn mathematics.

Moreover, learning takes place more effectively when a learner retraces the key steps in the historical development of the subject (Gulikers & Blom, 2001). As mathematical concepts are often oversimplified in textbooks and by teachers (Freudenthal, 1991; Siegel & Borasi, 1994), students may not be able to understand these concepts which are often broken up into smaller parts and presented to them from an expert's viewpoint (Tall & Vinner, 1981). Showing students the historical development of mathematical concepts can help them to see the links between the broken parts and improve their understanding of these concepts (Furinghetti, 2000), which may translate to better achievement test scores.

Finally, teachers can better understand the common difficulties faced by current students by examining the errors and misconceptions of past mathematicians (Moru, Persens, Breiteig, & Ndalichako, 2008; Sierpinska, 1992). They can then take pre-emptive measures to ensure more effective learning (Sfard, 1994). At the same time, when students realise that it is common for mathematicians to commit errors and learn from their own or others' mistakes, they appreciate that collaboration and perseverance are necessary to derive mathematical concepts which they often feel are beyond their ability to derive or understand initially. This gives them the confidence to explore and participate in mathematical activities (Siu & Siu, 1979). Hence, the use of history in mathematics lessons is a feasible technique to improve students' attitudes and achievement.

Empirical Literature Review

Although many articles discuss about the benefits of using history of mathematics in education, few empirical studies on history of mathematics have been conducted (Gulikers & Blom, 2001). Amongst these studies, most suggest that history of mathematics leads to better student learning outcomes. For instance, a qualitative study by Ponza (1998) reported that Grade 7 students displayed better attitudes toward mathematics after they worked on a project about the life of a French mathematician. In another qualitative study (Dittrich, 1973), Grade 11 and 12 participants showed greater interest in mathematics after the researcher used biographies of mathematicians and historical sources in their lessons. No significant improvement in the mathematics achievement of these participants was observed as improving problem solving skills was not one of the foci of the course. These qualitative studies suggest a positive relationship between the use of history and students' attitudes toward mathematics. However, quantitative studies are necessary to further convince educationists of this relationship.

One of the earliest quantitative studies was conducted by McBride and Rollins (1977) who used original proofs by famous mathematicians in a college algebra course in the United States. ANCOVA, with initial attitudes scores as covariates, showed a significant improvement in the experimental group's attitudes toward mathematics.

More recently, a quasi-experiment was conducted by Lit, Siu and Wong (2001) to examine the effect of using the historical development of the Pythagorean Theorem on the attitudes of Secondary 2 (Grade 8) students in Hong Kong. *T*-test results indicated a significant improvement in attitudes toward mathematics. However, the experimental group did not display a significant improvement in mathematics achievement. As the participants were assessed on different topics for pre-test and post-test, the achievement tests used in this study produced questionable results. In addition, *t*-test does not account for initial differences between the experimental and control groups. ANCOVA, with pre-test scores as covariates, may produce more meaningful results.

Ng (2006) investigated the effects of an Ancient Chinese Mathematics Enrichment Program (ACMEP) on the mathematics achievement of 414 secondary two (Grade 8) students in Singapore. ANCOVA, with achievement pre-test scores as covariates, showed that the experimental group performed significantly better than the control group. However, the results need to be interpreted with care as participants could choose whether or not to participate in the ACMEP. Students who opted to participate in the program might have better attitudes toward mathematics in the first place. Future experiments should remove this assignment bias to obtain more conclusive results.

Conscious of the dearth of studies on the use of history of mathematics, especially at the junior college and polytechnic level in Singapore, Ho (2008) conducted action research on the effects of using history of algebra on the attitudes of 102 polytechnic students. Participants in the experimental group performed better in two domains of attitudes, namely belief and perseverance, than their counterparts in the control group. As part of the study, Ho also developed a test to measure attitudes toward mathematics. However, the test had not been assessed for reliability and validity. Hence, the results of this study may be disputable.

In view of the weaknesses in the above-mentioned studies, this study aims to use a quasi-experiment to investigate the relationship between history of mathematics and students' attitudes and achievement in mathematics. This study hypothesizes that students who are introduced to history of mathematics in their lessons have better attitudes and achievement post-test scores than students who do not, after pre-test score differences are statistically controlled using multivariate analysis of covariance (MANCOVA) and analysis of covariance (ANCOVA).

Methodology

A quasi-experiment in which participants were selected from existing classes in a junior college was used in this study.

Participants

Four college year one (Grade 11) classes taught by the same teacher participated in this quasi-experiment. Two classes, one with class size 26 and the other 25, were assigned to the experimental (history of mathematics) group (total n = 51) and the other two classes, each with class size 26, were assigned to the control (no history of mathematics) group (total n = 52). The assignment of the classes to the two groups was done randomly. The participants had an average age of 17 years.

Research Design

Table 1 illustrates the design of this research. The use of history of mathematics is denoted by X. The same teacher went through a standard set of tutorial questions with both the experimental and control groups. To ensure that the same amount of time was spent with each group, the teacher went through the tutorial questions at a slower pace with the control group. All participants took three sets of achievement pre-tests and post-tests on three calculus topics: (1) techniques of differentiation, (2) applications of differentiation, and (3) integration. To minimize carry-over effects, the same set of attitudes tests was administered only twice to both groups of participants, once before the treatment and once after the last treatment session.

Table 1. Research design of the study.

Achievement	Control (n = 52)	O_1		P_1	O_2		P_2	O_3		P_3
	Experimental (n = 51)	O_1	X	P_1	O_2	X	P_2	O_3	X	P_3
Attitudes	Control (n = 52)	A_1								A_2
	Experimental (n = 51)	A_1	X			X			X	A_2

Legend:
X: Treatment (history of mathematics)
O_r: Achievement pre-test r, for r = 1, 2 and 3.
P_r: Achievement post-test on calculus topic r, for r = 1, 2 and 3.
A_1: Attitudes pre-test
A_2: Attitudes post-test

Variables

The independent variable in this study is the use of history of mathematics in lessons and the dependent variables are students' attitudes toward mathematics and achievement in mathematics. The confounding variables are pre-test differences between the experimental and control groups. Table 2 shows the variables that are held constant across both the experimental and control groups.

Treatment

The teaching package for the experimental group was aligned with the definitions of history of mathematics used in this study.[1] For instance, the dispute between Sir Isaac Newton and Gottfried Wilhelm Leibniz on who has precedence over the development of the fundamental theorem of calculus (Hall, 1980) was used as an anecdote to arouse students' curiosity and interest. The experimental group was also introduced to the

[1] Readers may contact the author for the complete teaching package on history of mathematics that was used with the experimental group.

motivation behind the development of mathematical concepts in the past, such as the development of differential calculus by Newton to solve physics problems on motion (Grabiner, 1983). Furthermore, original materials from historical sources such as the proof for Snell's law using differentiation was used to illustrate to students the use of calculus to prove a formula that they were asked to memorise without understanding when they were in lower grades.

Table 2. List of variables held constant.

Variables Held Constant	**Remarks**
Age and year of study	Same age group of 16 and 17 year old college year 1 students taking the General Certificate of Education Advanced Level (GCE 'A' level) 9740 H2 mathematics examination administered by the University of Cambridge-London Examination Syndicate (UCLES).
Participating school	All participants were from the same junior college.
Number and duration of lessons	All participants attended 16 lectures and 22 tutorial sessions, conducted over a period of four months. Each lecture session and each tutorial session lasted one hour. History of mathematics was used in at least 40% of the tutorial time for the experimental group.
Teacher and teaching approach	All participants were taught by the same teacher using a teacher-centred approach.
Students' notes and tutorial questions	The same set of lecture notes and tutorial questions without any mention of history of mathematics was used for both groups and for all topics.

Instruments

Mathematics attitudes

The Attitudes Toward Mathematics Inventory (ATMI) (Tapia & Marsh, 2004) and the modified Academic Motivation Scale (AMS) (Lim & Chapman, in preparation-a) were used for both pre-test and post-test. The psychometric properties and factor structures of both instruments had been established using participants with similar profiles to the participants of this study by Lim and Chapman (in preparation-a, in preparation-b).

The ATMI consists of 40 items that measure general attitudes toward mathematics using four factors, namely enjoyment, general motivation, self-confidence and value, on a five-point Likert Scale that ranges from strongly disagree to strongly agree. On the other hand, the modified AMS is made up of 20 items that address the question "Why do you spend time to study mathematics?" It measures participants' motivation toward mathematics using five factors that exist on a self-determination continuum (Deci & Ryan, 1985). These five factors can be categorised from lower to higher level of self-determination, into amotivation, external regulation, introjection, identification and intrinsic motivation. Each factor consists of four items and is measured on a 5-point Likert scale that ranges from "does not correspond at all" to "corresponds exactly".

Mathematics achievement

Three sets of achievement tests which were equivalent in terms of difficulty level and content were constructed to measure mathematics achievement. The questions for the tests were modified from past years GCE 'A' level questions. To assess the content validity of the achievement tests, a setter of the GCE 'A' level 9740 H2 mathematics paper from UCLES, Dr. Lionel Elliot, was invited to review the test items and mark schemes, and to provide feedback on their validity. He also verified that the content and difficulty level of the three sets of tests were similar. All the participants' scripts were evaluated by the same teacher based on the mark schemes. To assess the alternative form reliability (Clark-Carter, 2004) of the tests, a pilot study was conducted on 73 participants who were similar in profiles to the participants of this study. High reliability over a one-month duration for all pre-tests and post-tests was reported (average r = 0.87, p = 0.01).

Results and Discussion

All results of this study were analysed using SPSS 19 (SPSS Inc., 2010). Table 2 presents the mean scores and standard deviations of the post-tests scores of the ATMI, the modified AMS and the three achievement post-tests. The achievement scores are given in percentages. Pre-test scores are used as covariates which are statistically controlled using MANCOVA and ANCOVA when comparing the post-test scores of the experimental and control groups. All the underlying assumptions of MANCOVA and ANCOVA are supported by the data.

Table 2. Descriptive statistics of ATMI, the modified AMS and achievement test scores.

		Control (n = 52)		**Experimental (n = 51)**	
	Factors	**Mean**	**SD**	**Mean**	**SD**
Attitudes Tests					
ATMI	Enjoyment	3.31	0.80	3.40	0.75
	Motivation	3.10	0.86	3.13	0.75
	Self-confidence	3.54	0.71	3.71	0.84
	Value	3.65	0.59	3.88	0.45
Modified AMS	Amotivation	1.82	0.89	1.65	0.84
	Intrinsic motivation	3.00	0.85	3.28	0.75
	Identification	3.20	0.72	3.42	0.73
	Introjection	2.72	0.85	3.14	0.99
	External regulation	2.81	0.67	2.90	0.99
Achievement Tests					
Test 1 (Techniques of Differentiation)		68.46	18.93	77.16	12.18
Test 2 (Applications of Differentiation)		53.01	17.90	62.55	14.69
Test 3 (Integration)		40.77	19.57	56.21	19.70

Results and discussion on mathematics attitudes

Except for amotivation which contains negatively-worded statements, the experimental group reported higher scores for all domains of attitudes in the ATMI and the modified AMS.

Results of ATMI

There is a significant effect of the treatment variable (history of mathematics versus no history of mathematics) on the combined dependent variables of the ATMI, $F(4, 94) = 2.70$, p = 0.035, partial η^2 = 0.103. Analysis of each dependent variable shows that the experimental group performs significantly better in terms of value, at a Bonferroni adjusted α level of 0.0125, $F(1, 97) = 6.75$, $p = 0.011$, partial $\eta^2 = 0.065$.

Results of the modified AMS

After reversing the scores for the amotivation items, the results of the modified AMS show a marginally significant effect of the treatment variable on the combined dependent variables, $F(5, 90) = 2.31$, $p = 0.051$, partial $\eta^2 = 0.031$. Analysis of each dependent variable shows that the experimental group performeds significantly better in terms of intrinsic motivation ($F(1, 94) = 4.94$, $p = 0.029$, partial $\eta^2 = 0.050$) and introjection ($F(1, 94) = 7.07$, $p = 0.009$, partial $\eta^2 = 0.070$), at a Bonferroni adjusted α level of 0.01.

Discussion

Burton (1998) proposes that history of mathematics allows students to see the usefulness of mathematics in real-life. This is supported by the results of this study as the experimental group performed significantly better than the control group in terms of value. The experimental group is also significantly more intrinsically motivated and introjected than the control group after the use of history. This finding is in line with the arguments of D'Ambrosio (1995) who suggests that students are more motivated to learn mathematics if they explore mathematicians' cognitive activities in the past, as this allows them to appreciate the role of human minds in constructing mathematical knowledge.

Although the experimental group performed better in other domains of attitudes such as enjoyment and self-confidence as proposed by Perkins (1991) and Siu and Siu (1979), results are not significant. Future research with a bigger sample size may produce more conclusive results. In general, the positive results on attitudes after the use of history of mathematics are similar to those of previous empirical studies such as McBride and Rollins (1977) and Lit, Siu and Wong (2001). Although results are significant only for a few domains of attitudes, namely value, intrinsic motivation and introjection, these domains of attitudes are positively related to desirable student learning outcomes such as high academic achievement, low anxiety and low dropout rate from school (Gottfried, 1982; Vallerand et al., 1993). This implies that history of mathematics is beneficial to students and should be used by teachers in schools.

Results and discussion on mathematics achievement

The experimental group performed better in all three achievement post-tests. ANCOVA was conducted on each of the three achievement tests, with pre-test score on the same topic as covariate. There is a statistically significant effect of the treatment variable on Test 1 ($F(1, 100) = 9.72$, $p = 0.002$, partial $\eta^2 = 0.089$) and Test 3 ($F(1, 100) = 15.78$, $p = 0.001$, partial $\eta^2 = 0.136$).

This result affirms Furinghetti's (2000) suggestion that learning can be made more effective through the use of history. Interestingly, the achievement results by Lit, Siu and Wong (2001) are in contrast to the results of this study and Ng (2006)'s study. As the study by Lit, Siu and Wong took place over only three weeks, while this study and Ng's study took place over four months and seven months respectively, time may be a necessary factor to observe better achievement test results after the use of history.

However, the improvement in achievement may be a result of better attitudes, rather than a direct consequence from the use of history of mathematics. Further research is required to investigate the effects of history of mathematics on academic achievement.

Conclusion

This study aims to convince educators about the benefits of using history of mathematics in schools through a quasi-experiment. Results show that there is a significant and positive relationship between history of mathematics and students' achievement and certain domains of attitudes that are in turn positively related to other desirable students' learning outcomes. Hence the use of history of mathematics in classrooms is highly recommended.

However, this study is not without limitations. Firstly, the experiment involved the teaching of calculus topics only. Secondly, the participants of this study came from only one junior college in Singapore. Further studies that involve other mathematical topics, and more participants from different schools and countries are necessary to generalise results. Qualitative data can also be collected through the administration of journals, interviews, and recordings of classroom activities to better understand how students' attitudes and achievement are affected by the use of history in their mathematics lessons.

References

Arcavi, A., & Bruckheimer, M. (2000). Didactical uses of primary sources from the history of mathematics. *Themes in Education, 1*, 55–74.

Bidwell, J. K. (1993). Humanize your classroom with the history of mathematics. *Mathematics Teacher, 86*(6), 461–464.

Burton, L. (1998). The practices of mathematicians: What do they tell us about coming to know mathematics? *Educational Studies in Mathematics, 37*(2), 121–143.

Calinger, R. (Ed.). (1996). *Vita mathematica: Historical research and integration with teaching*. Washington, DC: Mathematical Association of America.

Clark-Carter, D. (2004). *Quantitative Psychological Research*. New York, NY: Psychology Press.

D'Ambrosio, B. S. (1995). Implementing the professional standards for teaching mathematics: Highlighting the humanistic dimensions of mathematics activity through classroom discourse. *Mathematics Teacher, 88*(9), 770–772.

Deci, E. L., & Ryan, R. M. (1985). *Intrinsic motivation and self-determination in human behavior*. New York, NY: Plenum Press.

Dittrich, A. B. (1973). An experiment in teaching the history of mathematics. *Mathematics Teacher, 66*(1), 35–37.

Fauvel, J. (1991). Using history in mathematics education. *For the Learning of Mathematics, 11*(2), 3–6.

Fauvel, J., & Van Maanen, J. (2000). *History in mathematics education: The ICMI study*. Dordrecht: Kluwer.

Fowler, D. (1991). Perils and pitfalls of history. *For the Learning of Mathematics*, *11*(2), 15–16.

Freudenthal, H. (1991). *Revisiting mathematics education: The China lectures*. Dordrecht: Kluwer.

Fried, M. N. (2001). Can mathematics education and history of mathematics coexist? *Science and Education*, *10*(4), 391–408.

Furinghetti, F. (2000). The history of mathematics as a coupling link between secondary and university teaching. *International Journal of Mathematical Education in Science and Technology, 31*(1), 43–51.

Gottfried, A. E. (1982). Relationships between academic intrinsic motivation and anxiety in children and young adolescents. *Journal of School Psychology, 20*(3), 205–215.

Grabiner, J. V. (1983). The changing concept of change: The derivative from Fermat to Weierstrass. *Mathematics Magazine, 56*, 195–206.

Gulikers, I., & Blom, K. (2001). 'A historical angle', a survey of recent literature on the use and value of history in geometrical education. *Educational Studies in Mathematics, 47*(2), 223–258.

Hall, A. R. (1980). *Philosophers at war: The quarrel between Newton and Leibniz*. New York, NY: Cambridge University.

Ho, W. K. (2008, 10–11 March). *Using history of mathematics in the teaching and learning of mathematics in Singapore.* Paper presented at the 1st RICE, Singapore: Raffles Junior College.

Jahnke, H. N., Arcavi, A., Barbin, E., Bekken, O., Dynnikov, C., Furinghetti, F., & Weeks, C. (2000). The use of original sources in the mathematics classroom. In J. Fauvel & J. Van Maanen (Eds.), *History in mathematics education – The ICMI Study* (pp. 291–328). Boston, M.A.: Kluwer.

Katz, V. J. (1993). Using the history of calculus to teach calculus. *Science & Education, 2,* 243–249.

Lim, S. Y., & Chapman, E. (in preparation-a). *A psychometric investigation of the Academic Motivation Scale*.

Lim, S. Y., & Chapman, E. (in preparation-b). *A psychometric investigation of the Attitudes Toward Mathematics Inventory*.

Lit, C. K., Siu, M. K., & Wong, N. Y. (2001). The use of history in the teaching of mathematics: Theory, practice, and evaluation of effectiveness. *Educational Journal, 29*(1), 17–31.

McBride, J. C., & Rollins, J. H. (1977). The effects of history of mathematics on attitudes towards mathematics of college algebra students. *Journal for Research in Mathematics Education, 8*, 57–61.

Moru, E. K., Persens, J., Breiteig, T., & Ndalichako, J. (2008). Epistemological obstacles in understanding the limit of a sequence: A case of undergraduate students at the National University of Lesotho. In L. Holtman, C. Julie, Mikalsen. Ø., D. Mtetwa & M. Ogunniyi (Eds.), *Some developments in research in science and mathematics in Sub-Saharan Africa* (pp. 47–42). Somerset West, South Africa: Africa Minds.

Ng, W. L. (2006). Effects of an ancient Chinese mathematics enrichment programme on secondary school students achievements in mathematics. *International Journal of Science and Mathematical Education, 4*, 485–511.

Perkins, P. (1991). Using history to enrich mathematics lessons in a girls' school. *For the Learning of Mathematics, 11*(2), 9–10.

Ponza, M. V. (1998). A role for the history of mathematics in the teaching and learning of mathematics: An Argentinean experience. *Mathematics in School, 27*(4), 10–13.

Sfard, A. (1994). What history of mathematics has to offer to psychology of mathematical thinking. In J. P.da Ponte, & J. F. Matos (Eds.), *Proceedings of the 18th international conference for the psychology of mathematics education,* Vol. 1 (pp. 129–132). Lisbon, Portugal.

Siegel, M., & Borasi, R. (1994). Demystifying mathematics education through inquiry. In P. Ernest (Ed.), *Constructing mathematical knowledge: Epistemology and mathematics education* (pp. 201–214). Washington, DC: Falmer.

Sierpinska, A. (1992). On understanding the notion of function. In E. Dubinsky & G. Harel (Eds.), *The concept of function: Elements of pedagogy and epistemology, MAA Notes, 25*, 25–58.

Siu, M. K. (1997). The ABCD of using history of mathematics in the (undergraduate) classroom. *Bulletin of the Hong Kong Mathematical Society, 1*(1), 143–154.

Siu, M. K., & Siu, F. K. (1979). History of mathematics and its relation to mathematical education. *International Journal of Mathematics Education for Science and Technology, 10*(4), 561–567.

SPSS Inc. (2010). *IBM SPSS Statistics.* Retrieved 21 January 2010, from http://www.spss.com/statistics/

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Journal Educational Studies in Mathematics, 12*(2), 151–169.

Tapia, M., & Marsh, G. E., II. (2004). An instrument to measure mathematics attitudes. *Academic Exchange Quarterly, 8*(2), 16–21.

Tymoczko, T. (1994). Humanistic and utilitarian aspects of mathematics. In D. F. Robitaille, D. H. Wheeler & C. Kieran (Eds.), *Selected lectures from the 7th International Congress on Mathematical Education* (pp. 327–339). Sata-Foy, QC: Les Presses de l'Université Laval.

Vallerand, R. J., Pelletier, L. G., Balis., M. R., Brière, N. M., Senécal, C., & Vallieres, E. F. (1993). On the assesment of intrinsic, extrinsic, and amotivation in education: Evidence on the concurrent and construct validity of the Academic Motivation Scale. *Educational and Psychological Measurement, 53,* 159–172.

Wilson, P. S., & Chauvot, J. B. (2000). Who? How? What? A strategy for using history to teach mathematics. *Mathematics Teacher, 93*(8), 642–645.