# SOME LESSONS LEARNED FROM THE EXPERIENCE OF ASSESSING TEACHER PEDAGOGICAL CONTENT KNOWLEDGE IN MATHEMATICS

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For the past three years, the authors have been using questionnaire items to assess the pedagogical content knowledge (PCK) of primary teachers involved in a multi-faceted professional learning program in Catholic schools in Victoria. We will describe the challenges of developing and coding items which assess PCK in mathematics, levels of performance on various items, and the extent to which change over time was evident. We will also share insights about areas for which professional learning programs might give greater emphasis, arising from the data.

## Introduction

Shulman (1986) first introduced the notion of pedagogical content knowledge, which he described as "the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others" (1986, p. 9). Since this time, researchers have attempted to conceptualise and measure the mathematical knowledge needed for teaching (Ball & Bass, 2000; Chick, 2007; Hill, Ball, & Schilling, 2008; Hill, Sleep, Lewis, & Ball, 2007). Barton (2009), in reflecting on the phrase *pedagogical content knowledge* with respect to teaching mathematics suggested that it "includes knowledge about how mathematical topics are learned, how mathematics might best be sequenced for learning, having a resource of examples for different situations, and understanding of where conceptual blockages frequently occur, and knowing what misunderstandings are likely" (p. 4).

In studying teacher knowledge, some researchers developed frameworks (Ball, Thames, & Phelps, 2008; Chick, Baker, Phum, & Cheng, 2006). Ball and her colleagues proposed a model with several categorises encompassing Shulman's Subject Matter Knowledge and Pedagogical Content Knowledge. They include all of these under the domain of *mathematical knowledge for teaching.* Also Ball, Hill and Bass (2005) differentiated between two types of mathematical content knowledge: "We defined mathematical content knowledge for teaching as composed of two key elements: "common" knowledge of mathematics that any well-educated adult should have *and* mathematical knowledge that is "specialised" to the work of teaching and that only teachers need to know" (p. 22).

Some researchers have investigated teachers' PCK associated with a particular domain of mathematics, such as proportional reasoning (Watson, Callingham, & Donne, 2008), area and perimeter (Yeo, 2008), fractions (Watson, Beswick, & Brown, 2006), chance and data (Watson, 2001) and decimals (Chick, et al., 2006), while utilising different instruments of assessment (e.g., multiple choice items, open response items, interviews, and classroom observations).

## Background to CTLM

The Contemporary Teaching and Learning of Mathematics Project (CTLM) is a professional learning and research project that will involve 82 Catholic primary schools in Victoria (Australia) between 2008 and 2012. Each school participates in a two-year program with Australian Catholic University (ACU), consisting of 10 to 12 full days of teacher professional learning (including workshops, professional reading, and between-session activities), along with in-classroom support from the research team. Each year a new cohort of schools begins their first year of the project with ACU (i.e., Intake 1 in 2008; Intake 2 in 2009; Intake 3 in 2010 and Intake 4 in 2011). This cycle of professional learning continues until 2012 when the final intake completes their second year. One of the project aims is to enhance teacher pedagogical content knowledge, prompting a need to measure improvement in PCK over time.

### The PCK framework

Prior to constructing PCK items, the authors developed a framework that would underpin their construction. There were three considerations that were helpful in this: frameworks for mathematics PCK developed by other researchers; the mathematical content focus of the CTLM Project for the respective cohorts in the given year (whole number, rational number, structure, measurement, space, chance and data); and some of the key aspects of the teacher role on which the project was to focus.

In light of the three considerations above, our current framework has the following components:

*Pathways*: Understanding possible pathways or learning trajectories within or across mathematical domains, including identifying key ideas in a particular mathematical domain.

*Selecting:* Planning or selecting appropriate teaching/learning materials, examples or methods for representing particular mathematical ideas including evaluating the instructional advantages and disadvantages of representations or definitions used to teach a particular topic, concept, or skill.

*Interpreting:* Interpreting, evaluating and anticipating students' mathematical solutions, arguments or representations (verbal or written, novel or typical), including misconceptions.

*Demand:* Understanding the relative cognitive demand of tasks/activities.

*Adapting:* Adapting a task for different student needs or to enable its use with a wider range of students.

The authors stress that the framework was not intended to be exhaustive, and clearly is not as broad as some others mentioned previously.

## The questionnaires

We developed six questionnaires each year, one each for teachers of Grades Prep–2 (teachers of 5 to 8 year-olds), 3–4, and 5–6, for each cohort. Typically, during the teachers' first year of the project the questionnaires contained between four and six items focusing on Whole Number, (and Rational Number and Structure in years 5-6) and three items in the second year focusing on Measurement, Space, and Chance and Data. Each questionnaire involved items intended to reflect the broad content focus of the CTLM professional learning program, but also provide data on teachers' capacities in each element of the framework, with several items addressing more than one component of the framework. To date, 774 teachers have completed the same questionnaire twice in a single year (119 in 2008, 321 in 2009 and 334 in 2010). 632 teachers completed their first questionnaire this year.

## Constructing PCK items

Apart from the considerations above, several decisions were made in our attempt to construct items that assessed teachers' PCK and these were:

- It was intended that the items reflect some classroom scenario to which the teacher had to respond hopefully illuminating knowledge used in the practice of teaching.
- In order to show change over time, most items were chosen to be relatively challenging for teachers early in the year.
- Initially, items were scored out of three points, but as will be discussed later, this was changed in 2010 to scores out of six.
- The questionnaire was intended to take no longer than 40 minutes to complete.

### Challenges of constructing and coding PCK items

In order to highlight that this process was difficult and evolving, we note that in the period 2008 to 2010, 39 "new" items were developed and administered. Seven items were administered only once and then dropped. Fifteen items were administered twice (in the same year) and then dropped. Some items had minor changes to wording or format and were not deemed "new" items, while others had substantial rewrites and were considered "new" items. Some difficulties faced in creating PCK items and coding teachers' responses are now discussed.

*Highly rated responses early in the year*

A reason for some items being administered only once (usually in February/March) was the high quality of responses, indicating teachers were generally proficient in this area and retesting would not show growth or positive change (for example, see Fig. 1). In respect of our framework we categorised this task as related to the Interpreting and Pathways elements of our framework.

In March 2008, most teachers in P-2 who were given the item called "Molly's Method" were able to identify a less sophisticated response, e.g., counting all by ones; and a more sophisticated response, e.g., multiplying 3 x 4.

**Molly's Method**

Molly, in Year 2, is asked this assessment question:
Here are three cherries in one bunch. How many cherries would there be in four bunches?
Molly says: "There would be 3, 6, 9, 12. That's 12 cherries altogether."
Describe 2 **other** ways a Year 2 child might correctly solve this problem; one **less** mathematically sophisticated and one **more** sophisticated than Molly's.
Less sophisticated:
More sophisticated:

*Figure 1. Molly's Method (P-2 questionnaire 2008).*

*Assessing content only*

For some content areas, in particular Space, we had difficulty devising an item that was not an assessment of *common content knowledge* (CCK) "in disguise". (e.g., Fig. 2). However, with some alterations we chose to continue with the use of items like these for Space for four reasons: results from 2009 indicated that many teachers had difficulties with the terminology and attributes of 2D and 3D shapes and we felt an improvement in their content knowledge might have a positive effect on their ability to provide worthwhile classroom examples; the items could be consistently coded; this content *per se* was a key feature of some of the professional development days; and the item could be easily adjusted for the different questionnaires for the various year levels.

**Can you do it?**

Mr Magoo's Year 6 students were having fun trying to describe shapes and solids which actually don't exist. Mr Magoo made a list of some of his favourites.
(a) Place one tick in each row to indicate whether it is "possible", "impossible", or "I'm not sure."

| | *Possible* | *Impossible* | *I'm not sure* |
|---|---|---|---|
| A trapezium with no lines of symmetry | | | |
| An equilateral, right-angled triangle | | | |
| A rectangle that is not a parallelogram | | | |
| A cone that is a prism | | | |

(b) For one response which you labelled "impossible", please explain why.

*Figure 2. Can you do it? (5-6 questionnaire, 2009).*

*Disadvantages of multiple choice items*

The item "Decimal Diversity" (Fig. 3) was dropped after being administered twice in 2008 to teachers in Years 5-6, for three reasons. It was deemed to be assessing only mathematical content and not PCK; teacher knowledge related to decimals was assessed better in 2009 by another item called "Ordering Decimals"; and most importantly, we made a decision at this point not to use pure multiple choice items where the teachers were not asked to explain their reasoning (see reasons below).

**Decimal Diversity**

Mrs Mason asked her Year 6 students to give her another name for this number. **0.125**
Her students came up with a variety of answers; some correct, some incorrect.
Please circle all of the following which are **really the same** as 0.125. (More than one is correct).

A. One tenth, two hundredths and five thousandths
B. One hundred and twenty-five tenths
C. One eighth
D. Twelve hundredths and five thousandths
E. 0.125%
F. 12.5 percent
G. One hundred and twenty-five thousandths
H. 1 tenth and a quarter of a tenth

*Figure 3. Decimal Diversity (5-6 questionnaire, 2008).*

We know that multiple-choice items for students without the opportunity to elaborate their decision making are not very useful and can provide deceptive information about understanding (see, for example, Clements & Ellerton, 1995). In our opinion, this is also a major weakness of much of the work of Ball and her colleagues with respect to teachers (e.g., Ball & Bass, 2000). In 2010, 202 teachers of Prep–6 were asked “Is this shape a rectangle?” (see Fig. 4). We categorised this task as Pathways, in particular the “key ideas” aspect of this element of the framework. In February, 33.0%, 45.1% and 64.2% of teachers in P-2, 3-4, and 5-6 respectively, indicated “Yes”. However, only 2.7%, 4.9%, and 3.0% of Prep–2, 3–4 and 5–6 teachers respectively could provide an appropriate explanation for children. Some examples of inappropriate explanations provided by teachers who circled “yes” were:

*A rectangle is a square by definition therefore a square is also a rectangle*
*A rectangle is a four sided shape therefore a square is technically a rectangle*
*It has two sets of parallel sides*
*A rectangle has four straight sides with opposite sides of equal length*

During class discussions, there was much debate about whether this shape (with equal length sides and equal angles) is a rectangle.
Is this shape a rectangle? …………….Yes / No ***[please circle one]***
Please say how you would explain your reasoning to children.

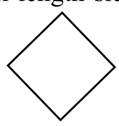

*Figure 4. Is this shape a rectangle? (P–6 questionnaires 2010).*

It is clear that if those teachers had been scored according to their correct answer (without any elaboration expected), the data would have been misleading.

*Limitations of pencil and paper assessment*

While we believed it was important to have teachers explain their thinking or justify their choices, sometimes these responses were not well articulated. This sometimes led the coder into making inferences about the correctness of a response or the potential of a described activity about which they were not fully comfortable, making the coding difficult and at times unreliable and ultimately the item unusable. It could be argued that a teacher’s written response may not match their intended practice and it is sometimes difficult to know how the teacher intended to enact their idea, without observing the teaching or at least interviewing them to enable elaboration of their response.

We acknowledge that providing a written test to assess PCK is not ideal and that a more comprehensive view of a teacher's knowledge would be obtained through a series of assessments such as observing teacher's practice in the classroom and using interview protocols to supplement questionnaire data. These data would more likely take the form of a case study approach which provides information about a small number of teachers, a small number of lessons, limited mathematical content and possibly only a few aspects of a PCK framework. However, while not as comprehensive as possible, our data provide information about a large cohort of teachers and some aspects of their content knowledge and related PCK.

*Constructing items for Chance and Data*

Overall we had the most difficulty coming up with items we felt were appropriate for assessing elements of our framework for Chance and Data. In particular, we had difficulty creating challenging items for teachers of P-2, while focusing on Chance and Data content for those grade levels. In 2011 we chose not to include items that assess Chance and Data.

*Creating scoring rubrics and applying them*

The two main difficulties in creating a rubric for each item were creating a rubric that could be consistently applied across coders (see previous discussion) and one that captured evidence of change over time, if it existed. In 2009, each item was coded out of three and the data from that year indicated limited change over time. It was decided that this *may* be partially due to the coarse rubric. In 2010, we decided to code most items out of six, as we hoped a more fine-grained rubric for the PCK items would help us identify changes that were not evident in the coarser 3 point rubric. Depending upon the item, the results were mixed as to whether the fine-grained rubric was helpful.

For most items, one coder who was proficient with the rubric scored each teacher's responses to items. Where there was any confusion or uncertainty about assigning a code, a second coder was used to confer or check codes. For some items all responses were checked by a second coder. Where two researchers (one of the authors and a PhD student) coded all items independently, they obtained 96% agreement, a very high level of inter-rater reliability (Roche & Clarke, 2009).

## Teachers' improved performance on PCK items

Although space does not allow us to provide detailed data for all questionnaires, Figure 5 provides a sample of the results from 2010. These data indicate the mean results per item, for February and October, for Year 5-6 teachers in 2010. As anticipated, the scores for items administered early in the year were generally low and for most items the improvement was also small.

For each task, the element(s) of the framework we judged to be reflected in the item were as follows: Comparing fractions (Selecting and Pathways); Up the garden path (Interpreting); Division stories (Selecting and Pathways); Ordering decimals (Interpreting and Pathways); and Closed to open (Selecting and Interpreting and Adapting).

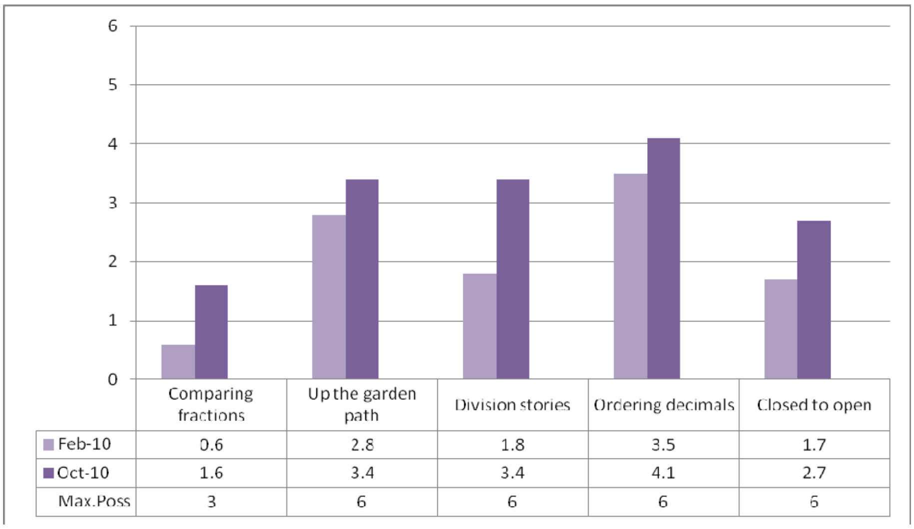


*Figure 5. Mean score per item for Year 5/6 teachers (Feb and Oct 2010).*

The item for which there was the greatest mean improvement for teachers in P–2 and 5–6 was Division Stories (see Roche & Clarke, 2009 for a description of this item and the scoring rubric). The teachers were required to name the two forms of division, draw a simple picture and write a story problem that represented 12 ÷ 3 for each form of division. The results showed that teachers were more familiar with partition division than quotition division and were generally unaware that quotition division was helpful for making sense of division by a fraction less than one.

In 2010, we added a question within the Division Stories item whereby teachers were asked to solve 8 ÷ 0.5. While the added question is assessing *common content knowledge (CCK)* and not PCK, we would argue that improvement in teacher knowledge about the two forms of division might improve a teacher's content knowledge. Table 1 provides the percentage of teachers who, in February and October, could correctly name quotition division as the form of division most helpful for making sense of 8 ÷ 0.5 and could explain why. It is reasonable to argue that the professional learning program was responsible for the teachers' improvement in their knowledge of the two forms of division.

*Table 1. Percentage of teachers who were successful in naming and describing why quotitive division is helpful for division by a fraction less than one (P-6 questionnaires 2010).*

| | *Feb* | *Oct* |
|---|---|---|
| P-2 (*n* = 63) | 8.7% | 30.4% |
| 3-4 (*n* = 33) | 6.6% | 23.0% |
| 5-6 (*n* = 36) | 5.5% | 30.9% |

Table 2 shows the percentage of teachers who could correctly solve 8 ÷ 0.5 in February and October in 2010. It should also be noted that some teachers appeared to use the "invert and multiply rule" to solve 8 ÷ 0.5 as evidenced by their scribbles next to the calculation. This apparent need to use a rule may be one reason why the percentage of success in Table 2 is much higher than for Table 1. Not surprisingly, teachers of Year 5-6 were more successful with this content than teachers of lower grades.

*Table 2. Percentage of teachers who solved 8 ÷ 0.5 correctly (P-6 questionnaires 2010).*

| | *Feb* | *Oct* |
|---|---|---|
| P-2 ($n$ = 63) | 44.4% | 60.3% |
| 3-4 ($n$ = 33) | 57.6% | 72.7% |
| 5-6 ($n$ = 36) | 77.8% | 83.3% |
| P-6 ($n$ = 132) | 56.8% | 70.5% |

### Possible areas of emphasis in professional learning programs arising from our data

After slightly more than three years assessing teachers' PCK using our classroom scenarios approach, there have been several themes that have emerged, which point to possible extra emphasis in professional learning programs. We note the difficulty many teachers have in changing a closed question into an open one, being able to interpret student-invented alternative algorithms; in articulating the nature of a very high quality response to a given mathematics task; understanding alternative methods and solutions in Structure (early algebraic thinking); and creating a story problem to match a particular equation (e.g., 12 ÷ 3 in Division stories task). We have already acted to embed a greater emphasis on these aspects in our 2011 professional learning program.

## Conclusion and recommendations

The process of creating questionnaires related to our framework, while challenging, has helped us to think about what we value in PCK and more importantly reflect on our professional learning program and to make adjustments as necessary along the way. It has highlighted the very complex nature of teacher knowledge and the difficulties in defining and assessing all those elements that constitute the act and art of teaching.

We have acknowledged some of the difficulties in measuring teachers' mathematical PCK, such as the limitations of pencil and paper items; designing items that we believed assessed faithfully a teacher's PCK for mathematics; creating rubrics that could be applied consistently; making choices about on which content to focus; and ultimately finding evidence of change over time, if it exists. We note that it may be difficult for teachers to show greater improvement, given the breadth of content of the professional learning program and the limited ability to make an impact on such a large group of teachers. However, we also acknowledge that some improvement was evidenced by our PCK items and that improving teachers' PCK appeared to improve teachers' CCK.

### Acknowledgement

We acknowledge gratefully the support of the Catholic Education Office (Melbourne) and that of Gerard Lewis and Paul Sedunary in particular in the funding of this research.

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