REVIEWING THE EFFECTIVENESS OF MATHEMATICAL TASKS IN ENCOURAGING COLLABORATIVE TALK WITH YOUNG CHILDREN

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This paper presents results from the *Resourcing Talking in Maths* project. The project aimed to review the resourcing, management and orchestration of collaborative mathematical tasks with young children (6 years old) and to examine the tensions involved in developing tasks that are accessible but also provide a challenge. It was found that the level of explicitness of the attended focus of the task needed to be balanced and that this balance was informed by the precision of the teacher's explanations and the definition of the mathematical relations as presented in the use of resources.

Introduction

The *Resourcing Talking in Maths* project, funded by the National Centre for Excellence in Teaching Mathematics (NCETM) in England, built on previous work that has shown the effectiveness of pupil-pupil talk on attainment in mathematics (Mercer & Sams, 2006; Murphy, 2011). It also acknowledges the difficulty teachers face in presenting tasks that encourage engagement and talk with younger lower-attaining children. Although rich problem solving tasks can overcome barriers to mathematical learning (Sullivan, 2003; Lubienski, 2000), it is seen that the 'richness' of a task depends on the teacher's management and orchestration, and that there is little guidance for teachers on this.

The project was based on a collaborative classroom teaching experiment involving two primary school teachers over one term. Classroom experiment is seen as one type of setting for a design experiment methodology (Cobb, Confrey, diSessa, Lehrer, R & Schauble, 2003) in which the researcher collaborates with the teacher to investigate instructional design. In this way pedagogical design is used to inform theory within a specific domain. Results are presented from two groups of three children (6 years old), where each group is engaged in three mathematical tasks. These six tasks and are used to develop a framework to support the development of effective collaborative tasks. Although based within a specific domain it is hoped that the framework is useful in supporting teachers in developing collaborative tasks more generally.

Background to research aims

The research is based on the assumption that children's engagement in collaborative mathematical tasks will enable children to participate actively in learning arithmetic. From a social constructivist perspective transmission of knowledge is seen to happen in the context of solving a problem where solutions are proposed and responded to (Wells, 1999). Barnes (1976) had proposed that encouraging children to talk in an exploratory way allowed them to use language as a way of thinking aloud. Exploratory talk has been further typified as "a way of using language effectively for joint, explicit, collaborative reasoning" (Mercer, Wegerif and Dawes, 1999, p. 97). The development of such talk would seem to support children's collaborative exploration of ideas and discussions within mathematics. As children engage in pupil-pupil talk they test out their understanding and applications of procedures in key mathematical ideas.

Encouraging young children to work in this way requires a different pedagogy and for teachers this may mean learning new skills. The development of exploratory talk through explicit teaching strategies has been seen to be effective in supporting children's use of talk as well as helping teachers to change their practices. The teachers involved in this study had participated previously in research on the introduction of explicit talk strategies and the children were familiar with this approach to mathematical tasks. A further element to consider is the task that the children are engaged in. Blatchford, Kutnick, Baines and Galton (2003) have suggested that in developing strategies for effective group work the learning task is a critical factor. If tasks are simplified they do not necessarily lead to success (Houssart, 2002). The difficulty would seem to be in developing rich, problem solving tasks that are accessible but also provided a challenge.

A key aim of the project was to examine the effective management of learning tasks and, in particular, to examine the balance between the precision of the explanations given by the teacher and the definition of the mathematics represented in the use of the resources. Developing the tasks within a classroom-based experiment required the teachers to be reflective and innovative and it was anticipated that the teachers' involvement would support professional development.

The study

The study involved a series of three workshops interspersed with the trial of group tasks in the teachers' classrooms. Each of the group tasks were videoed and observed in the workshops by the teachers in collaboration with the researcher. The workshops were used to analyse the way the children negotiated ideas and the way they engaged with the mathematical relations intended in the task. This analysis was used to identify the next step in instructional design. Final analysis was carried out using the video data from the group tasks to inform a theoretical framework.

Based on Nunes, Bryant and Watson's (2009) studies on key understandings in learning mathematics, the tasks aimed to help children connect their understanding of quantity with their knowledge of counting. It was decided to look at comparison as a mode of enquiry in order to make distinctions and to sort representations with regard to equivalence as a mathematical relation. The tasks were based on a type of rich task identified by Swan (2006) as comparing representations, in this case to make

connections between quantities and numbers. The teachers developed tasks that involved sorting and matching representations that included both quantities and numbers. The learning intention was that children worked with the mathematical relations rather than perceptual differences and similarities in order to find equivalent quantities (Nunes et al., 2009). Although the key purpose of each task was the same, the teachers took different approaches in resourcing, managing and orchestrating the tasks.

In this paper, six of the tasks are presented (3 tasks across 2 groups of 3 children). The tasks were developed by the two teachers, Teacher 1 and Teacher 2, and carried out with a group of three children in each of their classes. In the first task the teachers used cards with pictorial representations such as stars, cars, blocks and number lines as well as numerals (figures 1 and 2), and the children were asked to sort the quantities in relation to operations on numbers. In the second task the purpose was to find matching pairs of quantities or quantities and numbers. Teacher 1 used the same resource as task 1. Teacher 2 used cards with different calculations showing commutative pairs (figure 3). In the third task, both teachers used money to represent quantities and children were asked to find representations of equivalent amounts (figures 4 and 5).

Results

Task 1. Sorting representations

Teacher 1 gave the children a set of cards that provided a wide range of representations, each totalling 15 (figure 1). The teacher gave no initial explanation other than to sort the cards. The children negotiated ideas but sorted by perceptual similarities such as shape and colour. The teacher gave prompts such as "Why do you think some are split into two colours?" but the children did not notice the mathematical relations. Teacher 2 limited the range of representations (figure 2) on the cards that she gave the children. She did not include the number line and used fewer representations. She also gave the children cards in stages as sets according to the representations. The teacher provided a grid for organising the resources. The teacher modelled how she would sort the cards and asked questions such as, "Why have I put these two together?" The children focused on the mathematical relations but there was limited negotiation of ideas.

Figure 1. Task 1and task 2, Teacher 1: Sorting representations and matching pairs.

Figure 2. Task 1, Teacher 2: Sorting representations.

Task 2. Matching pairs

Teacher 1 used the resources from Task 1 but with representations totalling 12. Again she did not model what to do and gave no initial explanation other than to sort the cards. As the children started to sort by perceptual similarities the teacher prompted the children to find pairs, "Are there any you could match together as a pair?" The children negotiated ideas and with further prompts in finding matching pairs they sorted according to mathematical relations. Teacher 2 provided three calculations; 4+5, 5+4, 5+3, and asked for the 'odd one out'. There was limited negotiation in this initial task. Then the teacher provided a wider range of calculations to find other 'odd one out' calculations (figure 3). The children engaged in negotiation in this subsequent task and the teacher questioned the children after they had completed the task, "6+6; would that have a partner?"

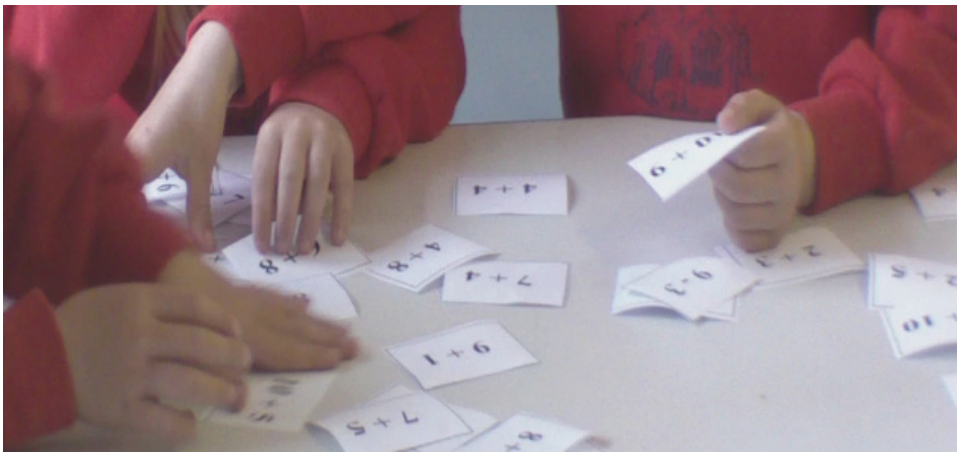

Figure 3. Task 2, Teacher 2: Matching pairs.

Task 3. Money representations

Teacher 1 modelled to the children how she would find equivalent solutions for making 10p. The children were then asked if she had found all the ways. The children were given blank cards to record further solutions (figure 4). As the children investigated other equivalent solutions the teacher prompted the children in working systematically, "Where do you think that would go in your order?" and in recognising equivalence, "Is this one different, why is it different?" The children noticed the mathematical relations

and negotiated ideas. Teacher 2 gave the children a set of cards with different amounts of money to put into pairs and a grid to organise the pairs (figure 5). The teacher allowed the children to decide how they would use the grid. The children noticed the mathematical relations and negotiated ideas. At the end of the task the teacher supported the children's organisation of the pairs on the grid.

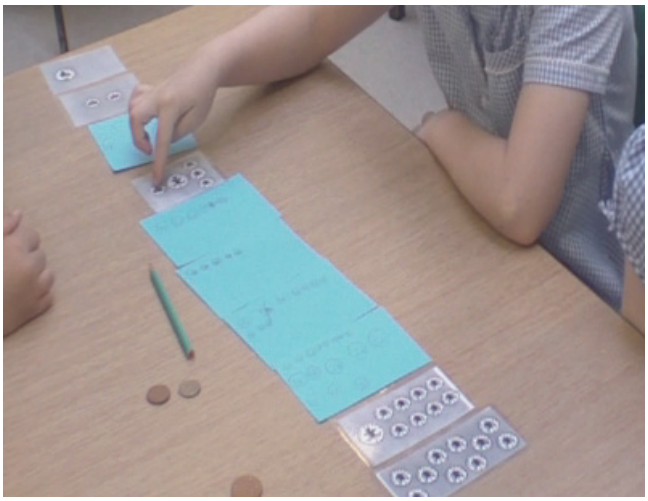

Figure 4. Task 3, Teacher 1: Money representations.

Figure 5. Task 3, Teacher 2: Money representations.

Analysis and discussion

Sfard and Kieran (2001) identified different components that children focus on as they work together on mathematical tasks; the attended focus, the intended focus and the pronounced focus. The attended focus relates to an individual pupil's focus as they attend to the process of a task. The intended focus is mainly private and relates to the experiences evoked by the other focal components. The pronounced focus is the publicly agreed focus. The relationships between these different focal components are seen to have an effect on collaboration.

The attended focus mediates between the public pronounced focus and the private intended focus. It is how the children attend to the process of the task and share their own private intended foci. In other words, how the private intended focus becomes pronounced or public. This mediation is influenced by the explicitness of the attended

focus. Various pedagogic strategies, such as scaffolding the tasks, questioning and prompting, and repetition of similar formats can make the attended focus more explicit. Also, the resources used can more or less define the mathematical relations and hence make the attended focus more or less explicit. The pedagogic strategies, along with the use of resources, can help the children attend to the process of the task, to share private intended foci and make them public. In this way the level of explicitness is related to the teacher's presentation and orchestration of the task, and how her intended focus is made sufficiently public. If the teacher's presentation is prescribed, her intended focus is made public in a precise way.

In relation to these focal components the tasks are analysed according to two factors:

1. Level of precision provided by the teacher in their explanations and organisation. This is determined by how the teacher prescribes the task.
2. Level of definition of the mathematical relations presented in the resources used in the task.

Task 1. Sorting representations

Teacher 1 did not provide any precise explanations nor did she define the use of the resources, so the attended focus was not explicit. The children may have negotiated ideas but they did not notice the mathematical relations as expected. On the other hand Teacher 2's explanations were precise and she defined the use of the resources. In this case the attended focus was very explicit. The teacher's intended focus was made public in a precise way. The children did notice the mathematical relations as expected but they did not need to negotiate ideas and there was little collaboration.

Task 2. Matching Pairs

Teacher 1 repeated the format of the previous task and in this way the use of resources became more defined. The prompts and questions focused the children on the process of finding pairs and the teacher's intervention was more precise. In this way the children were able to attend to the mathematical relations as expected but there was still a need for the children to negotiate ideas. In Teacher 2's initial task the use of resources was clearly defined, the presentation was prescribed and the attending focus was very explicit. The children did not need to negotiate ideas. However this initial task helped to define the use of the resources in the wider subsequent task and the attended focus was sufficiently explicit for the children to focus on the mathematical relations as expected and also to negotiate ideas.

Task 3. Money representations

In this task, Teacher 1 provided an initial stimulus that made her intended focus precise and defined the use of resources. This then informed the subsequent task as the children found all the solutions. The attended focus was sufficiently explicit. The children negotiated ideas and focused on the mathematical relations as expected. Teacher 2 gave no precise explanations but the resources were defined through the repeated format of finding pairs. The attended focus was sufficiently explicit to enable the children to negotiate ideas and focus on the mathematical relations as intended.

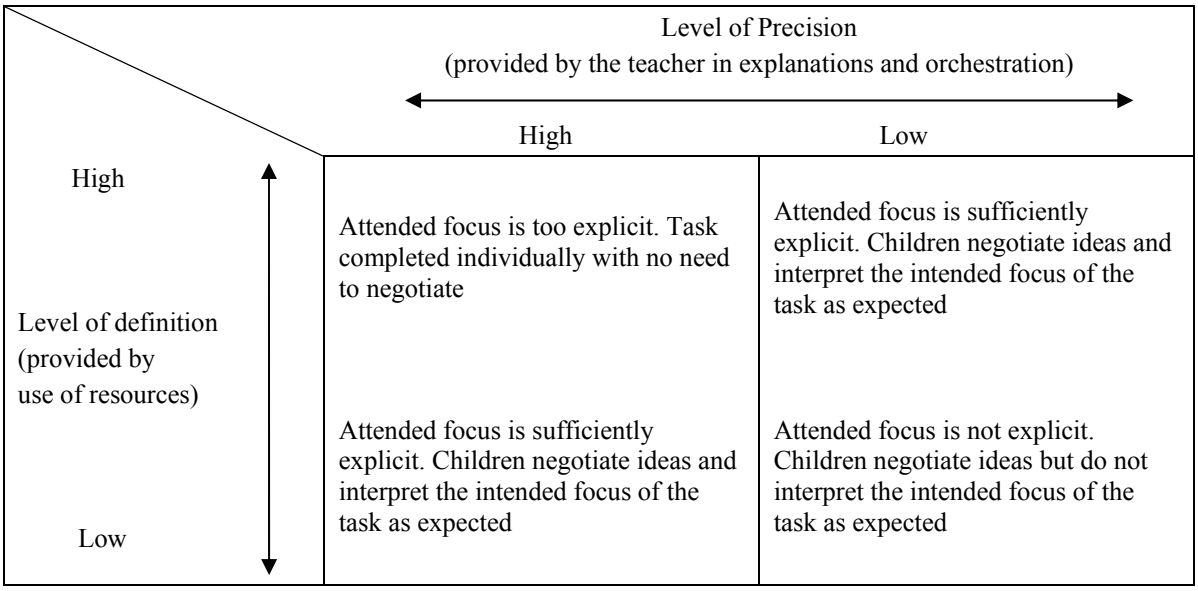

Figure 6: Balancing the level of explicitness in the attended focus of a task.

It would seem that the level of explicitness requires a critical balance in order to enable children to engage collaboratively in a task and also to focus on the mathematical relations as expected. If the mathematical relations are very well defined in the resources and the teacher's explanations are prescriptive, the intended focus of the teacher is made public in a precise way and the attended focus is very explicit. There is little need to engage in discussion or share ideas. If the resources are ill defined, and the teacher's explanations are not precise, the attended focus is not sufficiently explicit. The children may not be able to negotiate ideas, or they may negotiate ideas but interpret the teacher's intended focus in an unexpected way. Tasks that had a balance between precision and definition seemed to encourage talk and collaboration. This is summarised in Figure 6.

Conclusion

A key aim of the research was to investigate the development of mathematical tasks that encouraged talk and collaboration with young children. It was hoped that such tasks could be used to help children see the relations between number and quantity as a key understanding in arithmetic. The tension was seen to be in developing tasks that were accessible but that also provided a challenge.

The learning task was seen to be a crucial factor in enabling collaboration and talk to happen. From the analysis of the six tasks it would seem that the effectiveness of a task is determined by the explicitness of the attended focus, and that this explicitness is, in turn, determined by the precision of the teacher's intended focus through the level of prescription and the definition of the mathematical relations in the use of resources (Table 1). If the attended focus is sufficiently explicit the children are able to negotiate ideas and focus on the intended mathematical relations. This enabled the children to see the relationships between number and quantity as expected. Using this theoretical framework it would seem that the tasks that were accessible but that also provided a challenge were those tasks that had a balance between the precision of the teacher's

explanations and the definition of the intended mathematical relations as presented in the resources.

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