# AN EVALUATION OF THE PATTERN AND STRUCTURE MATHEMATICS AWARENESS PROGRAM IN THE EARLY SCHOOL YEARS


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This paper reports a 2-year longitudinal study on the effectiveness of the Pattern and Structure Mathematical Awareness Program (PASMAP) on students' mathematical development. The study involved 316 Kindergarten students in 17 classes from four schools in Sydney and Brisbane. The development of the PASA assessment interview and scale are presented. The intervention program provided explicit instruction in mathematical pattern and structure that enhanced the development of students' spatial structuring, multiplicative reasoning, and emergent generalisations. This paper presents the initial findings of the impact of the PASMAP and illustrates students' structural development.


Mathematics learning that focuses on pattern and structure can not only lead to improved generalised thinking but can create opportunities for developing mathematical reasoning commensurate with the abilities of young learners. Pattern has been described as any predictable regularity involving number, space or measure; and structure, as the way in which various elements are organised and related (Mulligan & Mitchelmore, 2009). Over the past decade a suite of studies with four- to nine-year olds has examined how children develop an Awareness of Mathematical Pattern and Structure (AMPS), found to be common across mathematical concepts (Mulligan, 2011; Mulligan, English, Mitchelmore, & Robertson, 2010). An assessment interview, the Pattern and Structure Assessment (PASA) and a Pattern and Structure Mathematics Awareness Program (PASMAP) focuses on the development of structural relationships between concepts. Tracking, describing and classifying children's models, representations, and explanations of their mathematical ideas—and analysing the *structural* features of this development—are fundamentally important.

Our goal is a reliable, coherent model for assessing and describing structural development with aligned learning and pedagogical frameworks. In this paper we focus on the development of the Pattern and Structure Assessment (PASA) interview and a

Rasch modelled scale for measuring student growth over time. An exemplar of the qualitative analysis of structural development is provided.

## Background

Previous studies have examined independently, counting, grouping, unitising, partitioning, estimating, and notating as essential elements of numerical structure (Thomas, Mulligan & Goldin, 2002); multiplicative concepts (Mulligan & Mitchelmore, 1997); combinatorial thinking (English, 1993); and spatial structuring in geometric figures and arrays (Battista, 1999). Recent studies of young children's mathematical reasoning have provided complementary evidence of the importance of early patterning skills, analogical reasoning and the development of structural thinking (Blanton & Kaput, 2005; English, 2004; Papic, Mulligan, & Mitchelmore, 2011).

There is also increasing evidence that early algebraic thinking develops from the ability to see and represent patterns and relationships such as equivalence and functional thinking in early childhood (Warren & Cooper, 2008). Recent initiatives in early childhood mathematics education, for example 'Building Blocks' (Clements & Sarama, 2007; Clements & Sarama, 2009), 'Big Maths for Little Kids' (Greenes, Ginsburg, & Balfanz, 2004), and 'Curious Minds' (van Nes & de Lange, 2007) provide research frameworks to promote 'big ideas' in early mathematics education. Papic's assessment of preschoolers using an Early Mathematical Patterning Assessment (EMPA) show that children are capable of abstracting complex patterns before they start formal schooling (Papic et al., 2011). Thus in designing PASMAP and an accompanying assessment, we focussed on the relationships between children's patterning skills, structural relationships and the big ideas in mathematics.

## Method

A purposive sample of four large primary schools, two in Sydney and two in Brisbane, representing 316 students from diverse socio-economic and cultural contexts, participated in the evaluation throughout the 2009 school year. At the follow-up assessment in September 2010, 303 students were retained. Two different mathematics programs were implemented: in each school, two Kindergarten teachers implemented the PASMAP and two implemented their standard program. The PASMAP framework was embedded within but almost entirely replaced the regular Kindergarten mathematics curriculum. The program focused on unitising and multiplicative structure, simple and complex repetitions, growing patterns and functions, spatial structuring, the spatial properties of congruence and similarity and transformation, the structure of measurement units and data representation. Emphasis was also laid on the development of visual memory and simple generalisation (for details see Mulligan et al., 2010). A researcher/teacher visited each teacher on a weekly basis and equivalent professional development for both pairs of teachers was provided. Incremental features of PASMAP were introduced by the research team gradually, at approximately the same pace and with equivalent mentoring for each teacher, over three school terms (May-December 2009). Implementation time varied considerably between classes and schools, ranging from one 40-minute lesson per week to more than 5 one-hour lessons per week.

Students were pre- and post- tested with I Can Do Maths (ICDM) (Doig & de Lemos, 2000) in February and December 2009 and September 2010; from pre-test data two 'focus' groups of five students in each class were selected from the upper and lower quartiles, respectively. These 190 students were interviewed by the research team using a new version of a 20-item Pattern and Structure Assessment (PASA1) in February 2009, a revised 19-item PASA2 in December 2009 (n=184), and the PASA2 and "extension" PASA in September 2010 (n=170).

Focus children (n=190) were monitored closely by the teacher and the research assistant collecting detailed observation notes, digital recordings of their mathematics learning and work samples, and other classroom-based and school-based assessment data. These data formed the basis of digital profiles for each student.

In summary, the qualitative analysis of the focus students' learning is complemented by the quantitative analysis of the ICDM and the PASA data presented here as a scale (see Figure 1). Further analysis of students' level of structural development at the three assessment points on selected PASA items supports the quantitative analysis. (For methods see Mulligan, 2011). For example, 190 students drawn representations for selected items were systematically coded for one of five levels of structural development. This enabled the description of developmental features (see Mulligan & Mitchelmore, 2009). Other evaluation data includes the implementation of PASMAP and teachers' views of the impact of the program on student learning and their own professional learning.

**The development of the PASA assessment items**

The assessment interview sought to complement interview-based numeracy assessment instruments such as the SENA (NSW DET, 2002) by extending counting and arithmetic strategies (addition and subtraction) to multiplicative reasoning. Our earlier studies highlighting the relationship between multiple counting and patterning, the development of composite units and unitising, base ten structure, partitioning and multiplicative reasoning influenced the design of the items [Items 4, 5, 6, 9, 10, 11, 12, (Mulligan & Mitchelmore, 1997; Thomas, Mulligan & Goldin, 2002)]. This included the work of English on combinatorial thinking and problem solving (English, 1993). Particular attention was paid to representations of 2-dimensional and 3-dimensional arrays (Items 7, 8, 18) and understanding the relationship between unit size and number of units (Outhred & Mitchelmore, 2000). The patterning tasks (Items 1, 2, 15) were based on our earlier studies with Kindergarten and Year 1 students and Papic's studies with preschoolers. These were extended to include an item integrating multiple counting and emergent functional thinking (Blanton & Kaput, 2005; Warren & Cooper, 2008). The ability to subitize was considered fundamental in developing visual memory and pattern recognition (Bobis, 1996; Hunting, 2003; Wright, 2003). The subitizing tasks extended those in the Schedule for Early Number Assessment 1 (SENA 1) (NSW DET, 2002) as it was considered important to compare responses on this item with those elicited on other patterning items. The inclusion of items on analogical reasoning (Item 13) and transformation (Item 14) was inspired by the work of English (2004), based on the notion that there were strong links between analogical reasoning and spatial patterning. As well Item 14 served to inform our assessment of students' transformational and sequencing skills. Further, several items required students to draw and explain representations such as the structuring features evident on a clockface. We included this item and another on drawing a ruler (in the extension PASA) based on our previous analyses of structural development (Mulligan & Mitchelmore, 2009). Additional items such as composite units in 2- dimensional shapes, the structure of ten frames, hundreds charts and counting patterns, the pattern of squares, equivalence and commutativity, and unitising length were formulated as an extension PASA.

**The development of a PASA scale**

Although the project focused on descriptive analyses of students' structural development, we complemented these by producing measures of students' ability that could define and assess growth (growth is defined as the difference between a student's performances at two points of time). The PASA data was analysed to construct a unidimensional scale that incorporated graded items along a continuum, for students aged 4.5 to 7.5 years. In order to establish the integrity of these items within a single construct, 'Pattern and Structure', it was advantageous to conceptualise these items on a linear scale. The main advantage of using Rasch analysis for constructing the PASA scale was that it could be used to link different versions of the PASA containing different subsets of items (see Looveer & Mulligan, 2009). As well students' performance on the ICDM, also using a Rasch scale, could be later integrated into the one scale to give a broader view of mathematical growth across the three assessment points. In order to measure this growth, ability estimates of students' location on the continuum could be determined and changes in students' ability locations could provide measures of growth. Rasch Unidimensional Measurement Models (RUMM) computer software (Andrich, Lyne, Sheridan & Luo, 2001) was used to generate scale scores for PASA items and student measures for the construction of the PASA scale. Item analysis was used to discard items not functioning well in PASA1 to reformulate PASA2 and the extension PASA. Following this, a separate Item map produced for the ICDM scores was integrated into the PASA scale (each scale can be viewed separately).

## Results

Figure 1 shows an integrated ICDM and PASA scale. The distribution of ICDM (code I), PASA2 (code P) and extension PASA (code E) items and students. The right-hand side of Figure 1 shows 73 item locations; Item P3b (PASA2) is the hardest item and Item I2 (ICDM 2) the least difficult. (The PASA 3b item was difficult because the students were required to visualise and calculate the number of items in 5x5 array from memory; the ICDM item 2 required students to indicate the longer of two lines.) On the left-hand side of Figure 1, each o represents 2 students. The scale extends from -3 logits to 6 logits, representing students' ability measured in March of Kindergarten to September of Year 1. The item map indicates that the items and the students were reasonably well matched; the PASA and extension PASA together performed reliably with several items challenging students beyond 2.0 logits. In comparison, the ICDM items at the lower end of the scale did not sufficiently challenge the majority of students, although some more difficult ICDM items fill a gap in the scale between 3.0 and 4.0 logits. Taken separately, the extension PASA also performed reliably.

*Figure 1. Item map of integrated PASA and ICDM scale.*

The scale's order of item difficulty on PASA items provided a measure of pattern and structure that reflected the students' overall level of AMPS. For example, items that challenged the most able students (Items P14 and P8) clearly assessed their visualisation and spatial structuring; and discrimination between simple repetition and recognition of a transformational (rotational) pattern respectively. (Item 14: Provide a net of an open box (2cm x 2cm x 2cm) and one multilink cube. *Imagine this shape folded up to make a box. How many cubes like this would fill the box without any spaces left?* Item P8: Show three arrows in a sequence (pointing upwards, sidewards, and downwards). *Show the way you think the arrow will go next? And which way after that? Tell me why you think that?*). Thus a conceptual analysis of the item and its position on the scale reflected the complexity of the task in terms of pattern and structure as well as the reasoning required to complete it successfully. What we aimed to achieve with the scale was an indicator of AMPS aligned with student ability.

We now present some general findings from the quantitative analysis to date. The ICDM scores were analysed as a standard measure of mathematical competence for all students at each assessment point. We did not expect ICDM to provide evidence of students' development of pattern and structure. Rather the ICDM served as a measure to validate the sample with Australian norms for Grade 1 (Doig & de Lemos, 2006) and to assign students initially to 'high-achieving' or 'low-achieving' ability focus groups. Figure 2 indicates that the sample's mean ICDM scores were slightly above the ICDM norms. There were no significant differences found on ICDM scores between PASMAP and regular students at any of the three assessment points but there were significant differences found between states.

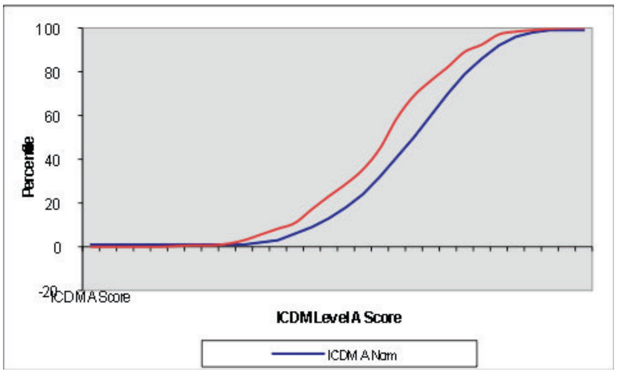



*Figure 2. ICDM norms compared with the sample (n=316).*

Both groups of students made substantial gains on the ICDM and the PASA1 across the three assessments with PASMAP students' overall mean scores consistently higher than the regular group (see Table 1). However, these were not significant ($p=0.105$). An analysis of variance revealed significant differences between states ($p=0.035$) and between schools ($p=0.040$) with NSW students outperforming Queensland students at each assessment point.

| | | | | 2009 | | 2010 | | |
|---|---|---|---|---|---|---|---|---|
| | | PASA1 | *n* | PASA2 | *n* | PASA2 | E-PASA | *n* |
| NSW | PASMAP | 9.40 | *40* | 15.05 | *38* | 14.14 | 9.09 | *35* |
| | Regular | 9.74 | *50* | 14.66 | *50* | 12.19 | 7.79 | *42* |
| QLD | PASMAP | 11.25 | *51* | 14.42 | *50* | 12.00 | 7.45 | *47* |
| | Regular | 10.94 | *49* | 15.67 | *46* | 10.80 | 7.07 | *46* |

*Table 1. Mean scores for all PASA assessments.*

## Discussion

Clearly these data showed consistently that NSW students were more advanced in their general mathematical competencies than the Queensland students. Queensland students had not necessarily experienced a preparatory curriculum and 2009 was the first year of a formal mathematics curriculum for 5 year olds. Nevertheless PASMAP students in Qld demonstrated growth in structural development in similar ways to the NSW students once they participated in the PASMAP program. Although we found consistently higher mean scores for PASAMP students, we expected that this finding may not necessarily prove to be statistically significant. We interpreted these findings in light of one confounding factor; the amount of time that individual PASMAP teachers devoted to the program implementation which had differential effects on students' learning outcomes. The time devoted to PASMAP ranged between one 40-minute lesson to more than 5 x 1 hour lessons per week. Some PASMAP teachers completed only half of the program components while others completed almost the entire program and revisited concepts regularly. Qualitative analysis of the NSW students' profiles and the classroom observation data showed stark differences in the way that the PASMAP students developed their knowledge and reasoning skills. Because the program focused intently on developing structural relationships, only the PASMAP students made direct connections between mathematical ideas and processes, and formed emergent generalisations. For example students began to link simple multiple counting to more complex multiples, arrays and multiplicative structures through their experience of the notion of unit of repeat in patterning, partitioning, in spatial tasks and in measurement contexts. Able students used particular features of pattern and structure to build new and more complex ideas. Regular students could also solve tasks requiring multiplicative thinking but these were considered as separate mathematical ideas, i.e., these students could not explain what was similar or different, what was the connection between ideas, or form simple generalisations.

**Categorising responses for stages of structural development**

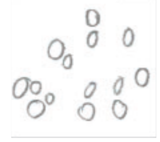
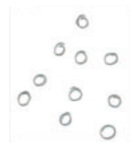
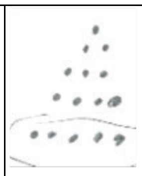
The analysis of PASA assessment interviews indicated marked differences between groups in students' levels of structural development (AMPS) at the second and third assessments. Students participating in the PASMAP program showed higher levels of AMPS than the regular group, made connections between mathematical ideas and processes, and formed emergent generalisations. Students' drawn responses and their explanations, at the three assessment points, were categorised using the levels of analysis from previous studies (Mulligan & Mitchelmore, 2009) as follows:

- *Pre-structural*: representations lack evidence of numerical or spatial structure
- *Emergent (inventive-semiotic):* representations show some relevant elements but numerical or spatial structure is not represented
- *Partial structural:* representations show most relevant aspects but are incomplete
- *Structural:* representations correctly integrate numerical and spatial structural features

An independent coder categorised each response for level of structural development with reference to each interview script (reliability of 0.91). We present an exemplar of the analysis of structural development drawn from 600 responses for Item 15 including approximately 10% as "second attempts". The coding was consistent with that used in previous studies but allowed for comparison of more challenging items.

Figures 3 to 8 show typical examples of developmental features of students' AMPS in response to Item 15. Figure 3 presents a circular border of dots and a random formation in the centre. There is some perception of outer and inner dots but it is largely idiosyncratic and depicts a 'crowded' image. Figures 4 and 5 show some awareness of triangular formation but there is little structural extension of the pattern. Figures 6, 7, and 8 represent the correct formation but vary in structural complexity. Note that Figure

7 depicts a simple repetition rather than a growing pattern. In Figure 8, the student explains the growing pattern numerically and as a simple generalisation.

| | | | | | |
|---|---|---|---|---|---|
|  |  |  |  |  |  |
| *Figure 3. Pre-structural* | *Figure 4. Emergent* | *Figure 5. Emergent* | *Figure 6. Partial* | *Figure 7. Structural features* | *Figure 8. Advanced* |

## Conclusions and Implications

The study produced a valid and reliable interview-based measure and scale of mathematical pattern and structure that revealed new insights into students' mathematical capabilities at school entry. Clearly young students were able to solve a broad range of novel mathematical tasks, including repetitions and growing patterns, and multiplicative problems, not usually asked of students of this age. Generally all students were able to construct and use counting and arithmetic strategies up to 20 and beyond. About 25% of PASMAP students recognised complex number patterns effectively on a hundreds chart in Kindergarten. The ICDM measures could be integrated with the new PASA scale to provide a comparative measure, although it assessed numeracy in traditional ways and did little to complement the PASA data.

PASMAP explicitly focused on the promotion of students' awareness of pattern and structure (AMPS): the analysis of students' learning showed that it had achieved its aims. Particular gains were noted in the related areas of patterning, multiplicative thinking (skip counting and quotition), and rectangular structure (regular covering of circles and rectangles). As expected, a focus on pattern, structure, representation, and emergent generalisation advantaged the PASMAP students. However, students in the regular program were also able to elicit structural responses but had not been given opportunities to describe or explain their emergent generalised thinking that may have been developing. Thus, it was not possible to determine whether more advanced examples of structural development could be directly attributed to the program or innate developmental advances of more able students. One of the most promising findings was that the focus students categorised as low ability were able to develop structural responses over a relatively short period of time. Further analysis of the impact of PASMAP on structural development must consider individual teacher effect and school-based approaches to evaluate the program's scope and depth of achievement.


## Acknowledgements

The research reported in this paper was supported by Australian Research Council Discovery Projects Grant No. DP0880394, *Reconceptualising early mathematics learning: The fundamental role of pattern and structure*. The authors express thanks to Dr Coral Kemp; assistants–Susan Daley and Deborah Adams; teachers, teachers' aides, students, and school communities for their generous support of this project.

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