WE CAN ORDER BY ROTE BUT CAN'T PARTITION: WE DIDN'T LEARN A RULE

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Improving numeracy performance of all students across Victoria is a government priority. A key element of this initiative lies with tertiary institutions that are responsible for adequately preparing pre-service teachers for teaching primary mathematics. This paper examines data from a larger, longitudinal study of primary pre-service teachers' mathematical content knowledge and focuses on responses to fraction tasks by nine pre-service teachers in the study who are in the final year of their course. Two dimensions were used to categorise their responses. The majority of these pre-service teachers did not demonstrate a fluid and flexible knowledge of fractional numbers; half demonstrating a regression in their knowledge of this topic since the beginning of the course. These pre-service teachers will be challenged when working with students who have a wide range of numeracy experiences and abilities.

Introduction

The aim of the larger study is to identify when, what and how primary pre-service teachers' mathematical content knowledge (MCK) develops during their course as there have been few longitudinal studies completed on how teachers' mathematical knowledge changes over time (Ball, Bass, & Hill, 2004). Teachers need, use and develop their MCK to understand mathematical concepts and processes as they teach (Chick, Baker, Pham, & Cheng, 2006; Hill, Ball, & Schilling, 2008; Ma, 1999; Rowland, Turner, Thwaites, & Huckstep, 2009; Shulman, 1986). Two important dimensions teachers need to know in order to teach are foundation knowledge, that is the content primary mathematics' teachers must draw on, as well as an understanding of how to make connections within and between topics (Rowland et al., 2009). It is important to examine issues relating to the development of pre-service teachers' MCK in order to plan and improve pre-service teaching as a means for also improving school students' numeracy outcomes.

Background

Forms of teacher knowledge

Teachers require a range of knowledge to draw on when teaching. For the past two decades research on mathematics teaching has included a focus on the knowledge teachers' use and need for their craft of teaching (Grossman & McDonald, 2008). Shulman (1986) was one of the first to categorise the characteristics for distinguishing teacher knowledge: content knowledge, pedagogical content knowledge (PCK) and curricular knowledge. Since then scholars have continued to understand and build on this work (Chick, Baker et al., 2006; Hill et al., 2008; Ma, 1999; Rowland et al., 2009).

The focus of this study is MCK or content knowledge. Shulman (1986) described content knowledge as the amount and organisation of knowledge in the mind of the teacher. Later studies have expanded on Shulman's definition and unpacked its complexities. Ma's (1999) study of Chinese teachers identified a form of content knowledge as a thorough understanding of the mathematics, having breadth, depth, connectedness and thoroughness, referring to this as Profound Understanding of Fundamental Mathematics (PUFM). Demonstrating mathematical connections and fluency of concepts and procedures is a key feature of PUFM.

According to current literature, MCK includes three facets: common content knowledge (CCK), specialised content knowledge (SCK) and knowledge at the mathematical horizon (Ball, Thames, & Phelps, 2008; Hill et al., 2008). CCK is simply when someone is able to calculate an answer and correctly solve a mathematical problem whereas SCK is unique to teaching (Ball et al., 2004; Ball et al., 2008). Teachers use SCK for identifying a range of solutions and mathematical connections when working with students, planning lessons and evaluating students' work (Chick, Pham, & Baker, 2006; Schoenfeld & Kilpatrick, 2008). Advanced content knowledge is evident when the teacher demonstrates a broad understanding of the complexities of MCK, for example; how mathematical ideas connect to the mathematics they are teaching, demonstrating peripheral vision of the curriculum. When a teacher models this advanced knowledge they are said to have an understanding of knowledge at the mathematical horizon (Ball & Bass, 2009; Hill et al., 2008).

Knowledge of content matters for teaching and excellent teachers of mathematics demonstrate a sound, coherent knowledge of mathematics appropriate to the students they teach (Australian Association of Mathematics Teachers [AAMT], 2006; Ball et al., 2004; Ma, 1999; Schulman, 1986, 1987). However, the literature continues to report on teachers' and pre-service teachers' gaps and weaknesses relating to their MCK (Fennema & Franke, 1992; Goos, Smith, & Thornton, 2008; Rowland et al., 2009).

Newton's (2008) review of the literature reported studies of elementary (primary) pre-service teachers' fraction knowledge to be limited and studies had focussed mainly on division, for example Ma's study (1999). Her study of 85 American elementary pre-service teachers' included administrating a written test at the beginning and at the end of a semester-long course designed to increase their knowledge of fractions. Results showed an improved knowledge of the four operations with fractions in the post test responses, but there was little flexibility demonstrated in the methods used when solving problems in both the pre-test and post-test. Further studies, including longitudinal studies could contribute to this gap in research.

A framework for Mathematical Content Knowledge

Rowland et al.'s (2009) *The Knowledge Quartet* was implemented when working with pre-service teachers (trainee teachers) as a tool for identification and discussion of four important dimensions for describing the types of MCK required to teach mathematics well: foundation, transformation, connection and contingency (see Table 1). Foundation is described as the knowledge possessed and the other three dimensions rely on conceptual connections for teaching.

Table 1. The codes of the Knowledge Quartet (Rowland et al., 2009, p. 29).

Foundation	Adheres to textbook Concentration on procedures Overt subject knowledge Use of terminology	Awareness of purpose Identifying errors Theoretical underpinning
Transformation	Choice of examples Demonstration	Choice of representation
Connection	Anticipation of complexity Making connections between procedures Recognition of conceptual appropriateness	Decisions about sequencing Making connections between concepts
Contingency	Deviation from agenda Use of opportunities	Responding to children's ideas

Methodology

The study and selecting participants

This paper reports on part of a larger study that includes a longitudinal qualitative component that explores the on-going learning of mathematics of 17 pre-service teachers in the various settings they encounter during the Bachelor of Education programme. This cohort is completing a Bachelor of Education Prep to Year 12 teaching course and will have qualifications to teach in primary and secondary schools. Their secondary qualification is aligned to particular discipline specialisations studied during the course and may or may not include mathematics. The 17 pre-service teachers had volunteered to participate in this longitudinal study.

For this study data collection involved qualitative analyses for nine pre-service teachers' responses to fraction items undertaken in the second-year of the course and again in the fourth-year of the course. Five of these pre-service teachers were not Mathematics majors and four were Mathematics majors. This study compared their results and responses to fraction items: one question selected from a Mathematical Competency, Skills and Knowledge Test (MCSKT) and responses to four items answered two years later during an individual interview. The remaining seven pre-service teachers were not selected for this study because they were studying at a different campus and had completed a different MCSKT.

Instruments

The pre-service teachers were given two questions, the first during second-year and the second at the end of fourth-year of their course. These fraction problems were selected to investigate their thinking used to solve two similar but different fraction problems; both were ordering tasks.

Second-year fraction test question. During the second year of the course all pre-service teachers (including pre-service teachers in the longitudinal study) completed a MCSKT to assess their mathematical knowledge of mainly number topics, for example fractions, decimals, percentage and ratio. The MCSKT consisted of 49 questions; pre-service teachers provided short answers using words or symbols and were encouraged to record their working out. All test items ranged in difficulty examining mainly procedural knowledge to a Year 8 standard. No calculator was permitted. For Question 19 pre-service teachers were asked to order a set of (fractional) numbers (0.42, two fifths, 4/9, 0.44 and one third) from least to greatest (Table 2).

Fourth-year fraction items. During the fourth year of their course the nine pre-service teachers in the longitudinal study completed a one-on-one interview with the researcher. In order to analyse their development of MCK of ordering and partitioning, they answered four items relating to common fractions. Each pre-service teacher was given three pairs of fractions and asked to identify the largest common fraction (Item 1: 3/5 and 2/3; Item 2: 3/5 and 3/4; Item 3: 3/5 and 5/8). For Item 4 pre-service teachers were asked to place common fractions (2/3, 3/4, 3/5 and 5/8) onto a number line. For each item they were asked to explain their reasoning.

Before commencing this interview the pre-service teachers were not aware they would have to demonstrate their MCK therefore were not given an opportunity to revise their knowledge of fractions. All interviews were digitally recorded and transcribed for later analysis.

Identifying methods and coding

Correct and incorrect responses for the second-year MCSKT Question 19 were entered into a spreadsheet. Table 2 summarises the responses of the nine pre-service teachers (their pseudonyms) in this study, listing their responses and an indication of whether the answer was correct (tick) or incorrect (cross), a description of the method used, and an indication to show whether the pre-service teacher was a mathematics major (tick) or not a mathematics major (cross).

A second spreadsheet was prepared for the fourth-year pre-service teacher data recording the four items, the number of correct and incorrect responses by Mathematics majors and non-Mathematics majors. For Items 1, 2 and 3 the method and the total number of pre-service teachers who used each method was coded: (known) fact, drew a linear (strip) model to compare the two fractions, converted to equivalent fractions in order to compare, converted to equivalent decimal and/or percentage to compare fractions, used number sense, or made a correct guess (Table 3). For Item 4, the number of correctly ordered responses was recorded for Mathematics majors and non-Mathematics majors. The number of pre-service teachers who demonstrated proportion when partitioning and placing the numbers onto the number line are also recorded in Table 3. Rowland and colleagues' (2009) qualitative framework (Table 1) was then used to code strategies and draw conclusions of pre-service teachers' MCK for the fraction items (Table 2 and Table 3). The aim was to compare the results of second-year and fourth-year data to identify foundation and or connections. Contingency and transformation were not used for this study as they linked to knowledge in action and what a teacher does during teaching.

Results and discussion

Table 2. Pre-service teachers' responses (N=9) to Question 19, Second-year MCSKT.

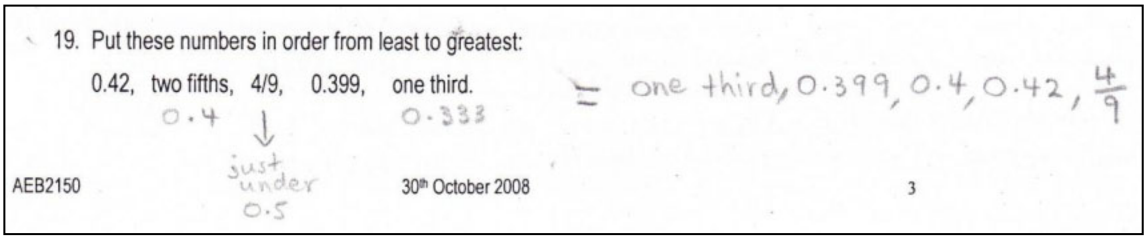
Name	Response	Answer	Method	Math Major
Lisa	One third, 0.42, 4/9, 0.399,	x	Converted to hundredths incorrectly	x

	two fifths			
Peter	One third, 0.399, two fifths, 0.42, 4/9	✓	Converted to decimal	x
Michael	One third, 0.399, two fifths, 0.42, 4/9	✓	Converted to decimal	x
Elizabeth	One third, 0.399, two fifths, 0.42, 4/9	✓	Converted to decimal and used a proportional strategy	x
Con	One third, 0.399, two fifths, 0.42, 4/9	✓	Converted to decimal	✓
Kerri	One third, 0.399, two fifths, 0.42, 4/9	✓	Converted to decimal	✓
Janette	One third, 0.399, two fifths, 0.42, 4/9	✓	Converted to decimal	x
Sean	One third, 0.399, two fifths, 0.42, 4/9	✓	Converted to decimal	✓
Shelly	One third, 0.399, two fifths, 4/9, 0.42	x	Unable to convert 4/9 to decimal; correctly converted others to decimals	✓

The responses of Question 19 from a second-year MCSKT indicated that most (7) of the nine pre-service teachers could order the numbers correctly: one third, 0.399, two fifths, 4/9 and 0.42 (Table 2). All correct responses showed some working out and recording, converting the fractions to decimals. This method concentrated on procedure and is foundation knowledge with some knowledge of the connections between the concepts of common fractions and decimal fractions. For example they demonstrated two fifths as equivalent to four tenths. There may have been further connected knowledge but Question 19 did not provide enough scope to identify this. Interviewing pre-service teachers after the test would have provided further probing of MCK.

Elizabeth's correct solution for Question 19 demonstrated a proportional strategy and was coded as demonstrating connection (Rowland et al., 2009). She was trying to make sense of the numbers so rather than converting 4/9 to 0.44 she recorded that 4/9 was "just under 0.5" (Figure 1). She was most likely using half as a reference point and knew the other numbers were "more than just under" one half. This example provided the most evidence of connected knowledge. However, it did not demonstrate if she knew how to change a common fraction (4/9) to a decimal fraction.

A range of experiences would have assisted pre-service teachers to prepare for their MCSKT contributing to the number of correct responses for Question 19. Before completing their MCSKT the pre-service teachers had just completed two education units during the second semester of the second year of their course. Both units focused on developing understanding of the primary mathematics curriculum as well as teaching and learning numeracy. All pre-service teachers had access to a sample MCSKT as a method of preparation for this assessment task. They also attended a primary school placement where they observed, participated and taught primary mathematics lessons. Similarly, they may have brought this knowledge to the course as foundation knowledge learnt during first year of the course or from their own mathematics education at primary or secondary school.


19. Put these numbers in order from least to greatest:

0.42, two fifths, 4/9, 0.399, one third.

0.4 ↓ just under 0.5 0.333 = one third, 0.399, 0.4, 0.42, 4/9

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Figure 1. Elizabeth's response to MCSKT Question 19.

Shelly, a Mathematics major, was unable to convert 4/9 to a decimal fraction. There is evidence of working out which has been rubbed out. Her recording shows 1/9 = 0.102 which is incorrect. She may have drawn on procedural knowledge to convert two fifths to a decimal and used a known fact for one third but lacked procedural fluency by not knowing a method for converting 4/9 to $0.\dot{4}$. Demonstrating procedural fluency is knowing procedures to use and performing them flexibly, accurately, and efficiently (National Research Council, 2001).

Lisa was one of the other pre-service teachers from the sample who was unable to provide a correct response for Question 19 and incorrectly converted the numbers to, hundredths. For example she recorded 4/9 as 45/100 and one third as 23/100. It is difficult to identify her errors since she did not record her thinking.

For Item 1 and Item 2, all responses were correct: 2/3 (Item 1) and 3/4 (Item 2). About half the pre-service teachers knew this as a fact because they were able to record the answer using quick mental methods. During the interview they explained how they knew this by drawing a model or by comparing equivalent fractions, decimals and/or percentages. This response was coded as foundation, common content knowledge as they could demonstrate an accurate method for ordering common fractions and explain their thinking or procedure.

Table 3. Fourth year pre-service teachers' responses (N=9) to 4 fraction items[1].

Item	Question	Number of Correct Responses: Maths Majors (N=4)	Number of Correct Responses: Non-maths Majors (N=5)	Correct proportion (N=9)	Method (N=9): Fact	Linear model	Equivalent fractions	Decimals and/or percentage	Number Sense	Correct Guessed
	Which one is larger, …									
1	3/5 or 2/3?	4	5		3	2	4			
2	3/5 or 3/4?	4	5		4	1	4		1	
3	3/5 or 5/8?	4	5		0	0	6	1	1	1
4	Record these fractions on a number line 3/5, 2/3, 3/4 ,5/8	2	2	3						

[1] Space does not permit the inclusion of the names of pre-service teachers.

Elizabeth drew a linear or strip model for all three items in order to shade and then compare these fractions. She did not elaborate on this method and lacked foundation knowledge and appeared to have forgotten any methods she had demonstrated two years earlier for Question 19 (Figure 2). She said, “This is the easiest way of me thinking about this stuff… obviously if I could do it with a ruler it would be a lot more accurate.” She correctly guessed the answer for 3/5 and 5/8 because the models looked similar. Again, she was not able to change a common fraction to a decimal fraction, as she had done two years earlier to compare fractions, or draw on her MCK to seek a suitable method.

For Item 1 and Item 2, four pre-service teachers chose to convert the fractions to equivalent fractions to compare their size. Janette used number sense by looking at the numerators and denominators. She selected 3/4 as larger than 3/5 and knew that quarters were larger than fifths. Her response was coded as ‘connection’, making connections between concepts.

All answers to Item 3 were correct. The most common method used to solve Item 3 was demonstrated by six pre-service teachers. They drew on a rote procedure, making equivalent fractions to compare 3/5 and 5/8 as 24/40 and 25/40. They performed procedural knowledge using step by step procedures and thus demonstrated foundation knowledge.

To compare and order fractions, students should develop a range of strategies (Petit, Laird, & Marsden, 2010). Only one pre-service teacher had the confidence to use their knowledge of fractions, decimals and percentage that demonstrated extending fraction ideas. Con was a Mathematics major and estimated the correct answer using connections with rational numbers as well as number sense. For Item 3 he said, “It is close. This [5/8] has to be more than point six [0.6] because one eighth is equal to more than ten percent. One eighth has to be bigger than ten percent. Four eighths is 50 percent or half or whatever and this [3/5] is sixty percent… so 50 plus more than ten percent is equal to 61 point 8 [61.8%]. I think it is point 888[0.888%] maybe something like that… I just know it is more than ten percent.” This explanation drew on extended rational number knowledge by partitioning the fraction and breaking the problem down into steps that helped him justify the answer and demonstrate ‘connection’.

For Item 4, only four—or less than half the pre-service teachers in the sample (N=9)—were able to correctly place the fractions in order on the number line even though they all correctly compared pairs of these fractions. Of the four pre-service teachers who answer correctly, two were Mathematics majors and two were not. Peter who was not a Mathematics major did not place the numbers in proportion on the number line. This is Foundation knowledge.

Michael, Con and Shelly were able to record their fractions on the number line in order and in proportion, using other numbers such as zero, half and one as bench marks. Making conceptual connections with the number line and with other representations of fractional numbers is evidence of connection or rational number sense (Lamon, 2005).

Conclusion

Rowland’s et al. (2009) framework was useful for identifying foundation knowledge when analysing the fraction question and items (tasks), as many pre-service teachers drew on known procedures in their responses. However, the items were closed questions. This was a disadvantage in the study when examining connection as most responses demonstrated one method of solution making it difficult to identify breadth and depth of pre-service teachers' MCK. Item 4 provided the best opportunity for providing evidence of connection as it was a multi-step problem involving known facts and procedures for comparing and ordering and partitioning and sense making strategies for representing common fractions with different denominators on a number line.

The results for the nine pre-service teachers MCK can be summarised as follows:

- One pre-service teacher lacked foundation knowledge in both second-year and fourth-year as she was unsuccessful with both ordering tasks; further investigation is needed to explore why she has not improved her MCK.
- Three pre-service teachers demonstrated success with Item 4 during fourth-year and demonstrated foundation and connection knowledge; they are maintaining and/or improving their MCK.
- Nearly half the sample, four pre-service teachers, could order the fractional numbers in second-year but could not order a similar set of common fractions correctly in fourth-year, demonstrating less foundation knowledge. This is a concern.
- One maths major demonstrated extended rational number knowledge. Other aspects of the longitudinal study will focus on how he transforms his knowledge and explains mathematics to primary students.

These results will be used to provide directions for probing more deeply into these pre-service teachers' MCK. They indicate that there is the need to engage pre-service teachers in tasks that promote understanding of specialised content knowledge to foster development of mathematical connections and not merely foundation knowledge. Tasks should be designed to assess their connection knowledge. Teaching experiences in various settings need to be designed to further connect knowledge and the other dimensions of specialised content knowledge they need to draw on when teaching. *The Knowledge Quartet* can be used as tool for supporting this development.

If the findings of this study are widespread and graduating teachers have gaps in their foundation knowledge and demonstrate narrow connected knowledge they will struggle when working with students within a classroom. Students have a wide range of numeracy experiences and abilities and their teachers need to draw on their MCK to build capacity in numeracy teaching and learning.

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