CHALLENGING AND EXTENDING A STUDENT TEACHER'S CONCEPTS OF FRACTIONS USING AN ELASTIC STRIP

ROGER HARVEY

Victoria University of Wellington, New Zealand

roger.harvey@vuw.ac.nz

This study investigated the fraction content knowledge of a student teacher, and his ability to use that knowledge in a novel situation through use of a scaled elastic strip. Data indicated that using the elastic strip was effective in challenging and enriching the participant's knowledge of ordering fractions. The results suggest that use of the elastic strip could assist student teachers to develop their understanding of fractional concepts.

Introduction

This paper reports a study exploring the use of an elastic strip (Figure 1), which can be viewed as a flexible number line and acts as a manipulative for supporting and extending fraction concepts. It reports on a teaching experiment that investigated the effectiveness of the elastic strip to challenge and give support to developing the content knowledge and pedagogical content knowledge of pre-service primary teacher.

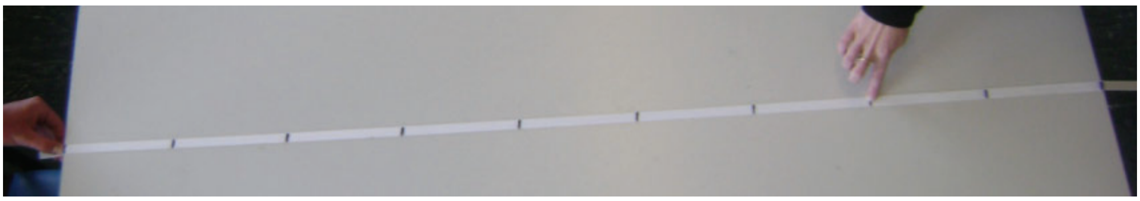

Figure 1. Elastic strip divided into 10 intervals being used to find $^{7}/_{9}$ of length of a table.

Primary teachers' knowledge about fractions

Many studies have shown that a high proportion of primary teachers lack sufficient content knowledge and Mathematical Knowledge for Teaching (MKT) (Hill, 2010) to teach the fraction concepts of primary mathematics effectively (e.g., Leinhardt & Smith, 1985; Ward, 2010). In a study in which 53 New Zealand primary teachers self-assessed their knowledge for teaching fractions, 27% rated themselves as very weak or weak (Ward & Thomas, 2007). In a study of the fraction MKT of 78 New Zealand teachers from years 1 to 9 Ward (2010) found 85% of teachers correctly ordered fractions $^{3}/_{5}$, $^{1}/_{3}$, and $^{4}/_{8}$, however just 30% were able to describe how they could support students to order these fractions using a conceptual approach.

Primary school-based studies of fraction learning

In an analysis of the strategies school students used when solving fraction tasks, Smith (1995) found that the competent performers used a rich range of approaches, well matched to specific tasks. Weaker students tended to use a narrower range of taught strategies performed in an algorithmic manner. In an Australian study, 323 grade 6 children were required to explain the reason for their selection of the larger of a pair of fractions. *Benchmarking*, whereby fractions are ordered by considering the relationship of each one to common benchmarks such as 0, ½ and 1, and use of *residual strategies* whereby fractions just less than 1 are compared by consideration of the difference of each from 1, were strategies that demonstrated good number sense that were used effectively. Neither of these strategies was familiar to many of the teachers of these children, indicating that they were likely to have been developed by individual children (Clarke & Roche, 2009).

Results from empirical studies have suggested that the teaching of fractions in primary school should be guided by the following:

- an increase in emphasis on the meaning of rational numbers rather than on calculation procedures (Charalambous & Pitta-Pantazi, 2007; Clarke & Roche, 2009; Moss & Case, 1999);
- making the process of constructing fraction equivalence more explicit in a range of fractional situations (Ni, 2001);
- explicit sharing of benchmarking (Clarke & Roche, 2009; Moss & Case, 1999) which, for example, supports the ordering of $^3/_7$ and $^{11}/_{20}$ by comparing them both with ½;
- a decrease in using pie graphs as a representation of fractions, and an increase in using other forms of visual representation (Moss & Case, 1999);
- building on children's self-developed solution strategies (Moss & Case, 1999);
- careful definition of numerator and denominator so that the improper fractions fit naturally within the definition (Clarke & Roche, 2009);
- explicit sharing of residual thinking which, for example, allows reasoning such as $^7/_8$ is greater than $^4/_5$ by comparing the amount by which each is less than one (Clarke & Roche, 2009); and
- increased emphasis on estimation and approximation when representing and operating with rational numbers, in order to develop number sense (Clarke & Roche, 2009).

Transforming primary mathematics teaching to meet these recommendations requires analysis of approaches to teaching that can support such instruction.

Models for teaching fraction concepts

There is a range of models commonly used to support fraction instruction, for example, sets of discrete objects, number lines, double number lines, and area models such as circles and rectangles. The selection of the most effective models for use in instruction is paramount (Cramer & Wyberg, 2009). An important feature of effective instruction is the explicit discussion of the attribute on which the model is based, such as relative length for linear models, relative area for two-dimensional models, and relative number in the set model (Steinle & Price, 2008).

Number lines are commonly used for fraction instruction. Effective use of number lines requires the learner to co-ordinate information provided pictorially by the marked line together with the numbers which give information about scale (Bright, Behr, Post, & Wachsmuth, 1988). Bright et al. (1988) suggested that using multiple number lines, partitioned in different ways but all showing the same fraction, would assist learners to construct richer understandings of number lines. In a similar way, Abels (1991) used a calibrated elastic strip as a tool for supporting the introduction to calculating percentage change. This tool is similar to the tool used in the current study (Figure 1).

Description of the elastic strips

The elastic strips used in the *teaching experiment* (Steffe, 1991) have graduated scales with equal intervals. The initial scale used in the study was about one metre in length and marked off in ten intervals (Figure 2). The elastic used to make the strips was able to be stretched to approximately double its un-stretched length. When the strips were used to find fractions of lengths, the physical restriction imposed by the limits on the elasticity necessitated the use of equivalent fractions to complete some tasks.

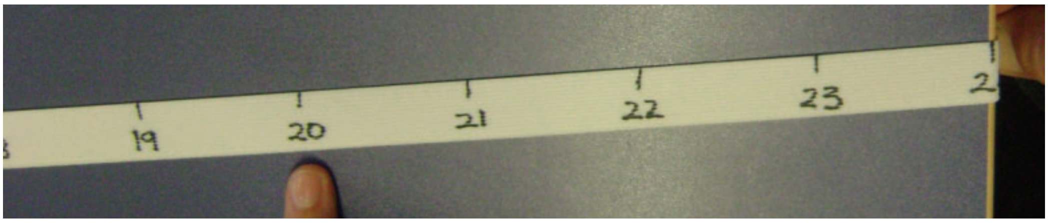

Figure 2. Elastic strip being used to find $^{5}/_{6}$ of the length of a table using the equivalent fraction $^{20}/_{24}$.

Method

The research model used was a teaching experiment in which the researcher held both participatory and data collection roles (Steffe, 1991). This paper reports on the teaching experiment conducted with one student teacher, Greg, held two months into his one year primary teacher education programme. Greg volunteered to participate as he believed his own knowledge of fractions was weak.

Greg completed a written questionnaire (Figure 3), and then participated in the teaching experiment which was informed by the results of the questionnaire. Initially Greg was questioned about his answers to the questionnaire, then shown how to use a ten-segmented elastic strip. He was then asked find points that were $^{7}/_{10}$, $^{7}/_{9}$, and $^{2}/_{3}$ of the length of the table. Intentionally, the strip was not sufficiently elastic to stretch across the table using just three segments. Other elastic strips were then introduced which had been graduated and numbered into 20 and 25 segments respectively and the tasks from the questionnaire were investigated using the strips. The physical nature of the task required Greg to give instructions to the researcher to act as partner in carrying out the tasks. When appropriate he was asked to support the instructions and actions with reasoning.

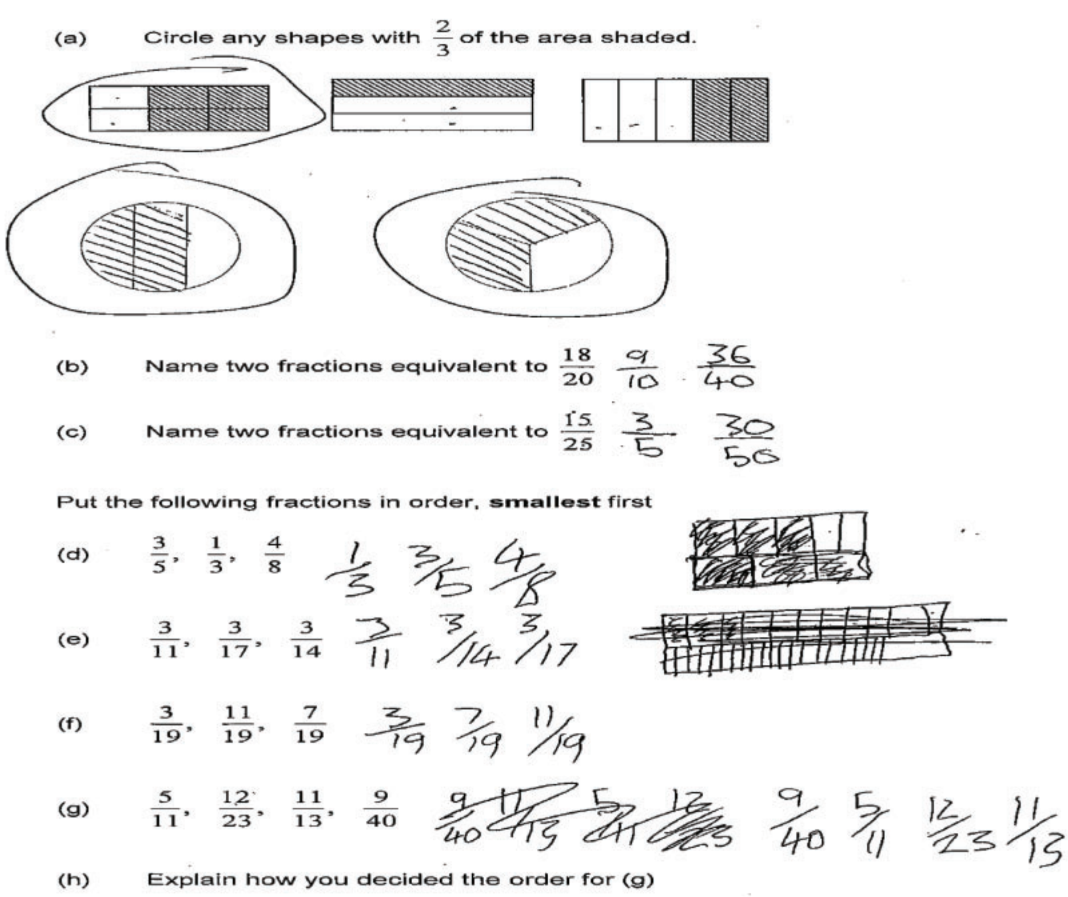

Figure 3. Greg's answers to the questionnaire.

Results

Themes emerging from the data (equivalence, ordering, and benchmarks) are presented below in order to illustrate ways in which the student teacher's knowledge was challenged and extended through the initial questionnaire and the interview.

Equivalence

Greg correctly calculated equivalent fractions in questionnaire parts (b) and (c). At the start of the teaching experiment Greg needed to find a suitable equivalent fraction for $^{2}/_{3}$. He quickly found $^{4}/_{8}$ and $^{8}/_{12}$, but neither of them was suitable to use with the ten-segmented strip. He had some difficulty realising that he could multiply both terms in the fraction by 3 to create the equivalent fraction $^{6}/_{9}$. From then on he found equivalent fractions fairly comfortably, however he was hampered at times by his lack of certainty with multiplication and division facts.

Ordering fractions with the same numerator but different denominators

Greg incorrectly ordered the fractions $^{3}/_{11}$, $^{3}/_{14}$, and $^{3}/_{17}$ as going from smallest to largest in question (e) (Figure 3). He explained his ordering:

Greg — I was thinking 3 pieces shaded out of 11, 3 out of 14, and 3 out of 17. But I think the order might be reversed. I really struggle with that one.

Later the question was revisited with the aid of the strip. Initially $^{3}/_{11}$ of the length of the table was found.

Researcher	Now we are going to find $^3/_{14}$. Before we do it, what is going to happen to each of the pieces?
Greg	It's going to stretch forward. So it is going to go (hand gesture indicating that the fraction was larger).

After the three fractions had been found using the strip, Greg proposed a rule:

Greg	So it is actually the other way round from my answer. I am just wondering when we have got the same number on top. Is that a general rule that you could follow, if you have the same number on top, and the denominator is bigger, the smaller the value?

After discussion about the ordering of unit fractions and then fractions with the same denominator, Greg was asked if the elastic strip had helped his thinking.

Greg	It just totally changed my way of thinking about fractions. It's a visual for me that I like to see.

Ordering fractions and benchmarking

Greg's incorrectly ordered the fractions from questionnaire part (d) (Figure 3) as $^1/_3$, $^3/_5$, $^4/_8$, and explained:

Greg	I tried to draw pictures to help me work it out. $^1/_3$ is quite easy to visualise. I just see one piece shaded out of three. The same with 3 out of 5, so I thought $^1/_3$ is smaller, $^3/_5$ is getting more pieces so if I have 3 pieces out of 5 shaded, I can see more pieces being shaded with less left over. And $^4/_8$ is $^1/_2$. So you are getting $^1/_2$ of something, and that is the biggest.
Researcher	Are you happy with that order?
Greg	I think it is wrong. Maybe I might change it to $^1/_3$, ½, $^3/_5$. I feel I should change it, I feel that $^3/_5$ is more than $^1/_2$, but I am not confident. This is where I really struggle.

After using the elastic strip to correctly order these fractions, Greg again commented that using the strip had helped his understanding.

In the questionnaire, Greg answered question (g) correctly as $^9/_{40}$, $^5/_{11}$, $^{12}/_{23}$, and $^{11}/_{13}$, but he was not confident about his answer.

Greg	$^9/_{40}$ is the smallest seems a ridiculous amount shaded out of 40. Littlest amount I could think of. Seems small to me. $^5/_{11}$, that is quite close to ½. $^{12}/_{23}$ is quite close to ½. $^5/_{11}$ and $^{12}/_{23}$ and seems almost the same: both close to ½. $^{11}/_{13}$ is seems quite close to $^3/_4$. There is a lot more shaded out of that proportion. If we did that stretchy thing we might actually be quite close.

Greg used the strip to locate each of the fractions in this set. He recognised that $^9/_{40}$ was approximately ¼, and after prompting to compare $^9/_{40}$ to $^{10}/_{40}$, recognised $^9/_{40}$ was just less than ¼. Similarly the strip was used to find $^5/_{11}$, and it was pointed out that 5.5 out of 11 would be ½, so $^5/_{11}$ is just less than $^1/_2$.

Researcher	These two ($^5/_{11}$ and $^{12}/_{23}$) were pretty close and you said they were both about $^1/_2$. Tell me about $^{12}/_{23}$.
Greg	$^{12}/_{24}$ would be ½. Half of 23 is 12.5, sorry 11.5. So $^{12}/_{23}$ is slightly more than ½. So it is more than this one ($^5/_{11}$) because it is slightly less than ½. So if I was to work this out again knowing this now, I could do it. Half of

> 11 it is 5.5. Half of 23 it is 11.5, and so I could compare them to a half and see that one was slightly more than half and one was just less than half.

This idea of benchmarks was discussed and consolidated by considering $^{11}/_{13}$. Greg was asked to order a similar set of fractions and then asked if he had learnt anything from the session.

> G Yes I have. I have got more of an understanding of fractions and how they work. I'd love to take this strip into an exam and sit down and stretch it out. Now I have some kind of visual measurement in my brain that I can see; I can see that is close to 1, that is close to 0, that is more than a quarter, or less than quarter. Those are now my measuring blocks. 0, ¼, ½, ¾, and 1. I can work within those boundaries to work out what the answer might be. I might borrow it now and go home and practise it so I re-illustrate it in my head again.

Discussion

In the questionnaire Greg was able to use a fraction to generate equivalent fractions. His lack of certainty about the relative size of fractions casts doubt on whether this skill was securely linked to the idea of size of fractions. After the investigation of fractions with the same numerator, Greg's tentative proposal about a rule for ordering them is encouraging. Greg found the fraction ordering tasks in the questionnaire challenging, and the diagrams that he drew to support his working (Figure 3) appear unhelpful. He had recognised $^{5}/_{11}$ and $^{11}/_{23}$ as were being about ½, but was not confident in his ordering of those two numbers. The dialogue suggests that the use of the strips had assisted him to see how benchmarks could be used to find the approximate magnitude of numbers. This development was physically supported by the strips, however later in the teaching experiment Greg reported that he was developing and using mental images to assist his ordering of fractions.

Conclusions

The study showed that the fraction strip has the potential to assist learners in consolidating and reinforcing the images of the number line. The results support previous research (e.g., Leinhardt & Smith, 1985; Ward, 2010; Ward & Thomas, 2007) showing that some student teachers have significant gaps in their fractional content knowledge, casting doubt on their ability to effectively teach these concepts in primary classrooms. These concerns are illustrated in the vignettes from the interview with Greg who appears to need ongoing support to help him develop understanding of the key ideas of primary school fraction knowledge. For Greg, use of the elastic strip was an effective activity to challenge, consolidate, and extend his fraction thinking. Specifically the teaching experiment addressed fraction concepts of equivalence (Ni, 2001), representation of fractions using a linear model (Moss & Case, 1999), and benchmarking (Clarke & Roche, 2009; Moss & Case, 1999). The novel and the physical nature of the activity made recalling rote routines less likely, and communicating in order to complete the task required Greg to re-engineer his knowledge of fractions.

When using elastic strips, care is required to keep the learner's attention on the size of the unit. It is essential that the teaching does not stop with the use of the strip, but rather starts with it and moves on to developing images of the strip, and then to the key fraction ideas.

There have been calls for the teaching of primary school mathematics to have a greater focus on concepts (Charalambous & Pitta-Pantazi, 2007; Clarke & Roche, 2009; Moss & Case, 1999). A significant number of teachers have weak knowledge of fractional concepts (Ward, 2010; Ward & Thomas, 2007). Programmes that enhance the MKT of student teachers in fractional concepts need to be considered as one way to lift the teaching skill in this area. The use of the elastic strip offers the potential to challenge and extend the knowledge of fractions of student teachers.

Acknowledgements

Thank you to Greg for taking part in the research and to Robin Averill who provided constructive feedback on drafts of this paper.

References

Abels, M. (1991). Procenten in W12-16. *Nieuwe Wiskrant*, *10*(3), 20–25.

Bright, G., Behr, M., Post, T., & Wachsmuth, I. (1988). Identifying fractions on number lines. *Journal for Research in Mathematics Education, 33*(2), 215–232.

Charalambous, C., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students' understandings of fractions. *Educational Studies in Mathematics, 64*(3), 293–316.

Clarke, D., & Roche, A. (2009). Students' fraction comparison strategies as a window into robust understanding and possible pointers for instruction. *Educational Studies in Mathematics, 72*, 127–138.

Cramer, K., & Wyberg, T. (2009). Efficacy of different concrete models for teaching the part-whole construct for fractions. *Mathematical Thinking and Learning, 11*(4), 226–257.

Hill, H. (2010). Nature and predictors of elementary MKT. *Journal for Research in Mathematics Education, 41*(5), 513–545.

Leinhardt, G., & Smith, D. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology, 77*(3), 247–271.

Moss, J., & Case, R. (1999). Developing children's understanding of the rational numbers: A new model and an experimental curriculum. *Journal for Research in Mathematics Education, 30*(2), 122–147.

Ni, Y. (2001). Semantic domains of rational numbers and the acquisition of fraction equivalence. *Contemporary Educational Psychology, 26*(3), 400–417.

Smith, J. P. (1995). Competent reasoning with rational numbers. *Cognition and Instruction. 13*(1), 3–50.

Steffe, L. (1991). The constructivist teaching experiment: Illustrations and implications. In E. von Glaserfeld (Ed.), *Radical constructivism in mathematics education* (pp. 177–194). Dordrecht, The Netherlands: Kluwer.

Steinle, V., & Price, V. (2008). What does three-quarters look like? Students' representations of three-quarters. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia* (pp. 483–489). Brisbane: MERGA.

Ward, J. (2010). *Teacher knowledge of fractions: An assessment*. Unpublished doctoral dissertation, Otago University, Dunedin, New Zealand.

Ward, J., & Thomas, G. (2007). What do teachers know about fractions? In *Findings from the New Zealand Numeracy Development Project 2006.* Wellington: Ministry of Education.