TEACHER PROFESSIONAL LEARNING IN NUMERACY: TRAJECTORIES THROUGH A MODEL FOR NUMERACY IN THE 21ST CENTURY

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This paper reports on a year long, state wide research project that aimed to assist primary and secondary teachers to improve their teaching and learning practices through engagement with a new model of numeracy. Data collection included sequence maps of participants' development as teachers of numeracy as elements of the model became more prominent in their thinking and planning. Semi-structured interviews were also used to clarify and expand upon teachers' perceptions of their own development. Findings include a propensity to begin with the dispositions element of the model but responses showed that in most cases all elements were eventually addressed by each teacher.

Introduction

The importance of developing numeracy capabilities, in addition to acquiring purely mathematical competence, has been acknowledged in national reports (Human Capital Working Group, Council of Australian Governments, 2008), pending national curriculum documents (Australian Curriculum Assessment and Reporting Authority, 2009) and through the inclusion of contextualised mathematics problems in the assessment frameworks of international testing regimes (e.g., OEDD/PISA, 2003). Numeracy is increasingly seen as fundamental to developing students' capacities to use mathematics to function as informed and reflective citizens, to contribute to society through paid work and in other aspects of community life (Steen, 2001).

While there is a substantial body of literature devoted to the nature of and importance of numeracy education and to effective approaches to professional development in mathematics teaching (Loucks-Horsley, Love, Stiles, Mundry & Hewson, 2003), far less is known about how teachers learn about, appropriate and then create effective mathematics teaching practices. This paper reports on a year long research and development project that investigated approaches to assisting teachers to plan and implement numeracy strategies across the curriculum in the middle years of schooling (Years 6-9). The aim of this paper is to examine teachers' perceptions of their own professional learning in relation to a rich model of numeracy and to map how these perceptions changed through the duration of the project.

Theoretical framework

Numeracy, which is also known as quantitative or mathematical literacy in some international contexts, has been recognised internationally through the OECD's Program for International Student Assessment (PISA). According to PISA's definition, mathematical literacy is:

> an individual's capacity to identify and understand the role mathematics plays in the world, to make well-founded judgments, and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, 2004, p. 15)

Within Australia, increasing importance is also being placed on the need for individuals to have the capacity to use mathematics in the beyond school world. This way of using mathematics is captured in the following definition, which has gained general acceptance in Australia, "To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life" (Australian Association of Mathematics Teachers, 1997, p. 15).

While these definitions capture the broad thrust of the concept of numeracy, they lack the detail necessary for teachers to implement numeracy based approaches in practice. More recently, however, Goos (2007) has proposed a model of numeracy which encompasses four essential elements which are enacted within a perception of mathematics as knowledge-in-action. The model incorporates attention to real-life contexts, the deployment of mathematical knowledge, the use of physical and digital tools, and consideration of students' dispositions towards the use of mathematics. These elements are embedded in a critical orientation to the use of mathematical skills and concepts which emphasises the evaluative and judgemental aspects of numeracy practice, for example, the capacity to evaluate quantitative, spatial or probabilistic information used to support claims made in the media or other contexts. The elements of the model and the critical orientation within which these elements interact are described in Table 1.

Table 1: Descriptions of the elements and critical orientation of the numeracy model.

mathematical knowledge	Mathematical concepts and skills; problem solving strategies; estimation capacities.
contexts	Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings
dispositions	Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge.
tools	Use of material (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners) and digital (computers, software, calculators, internet) tools to mediate and shape thinking
critical orientation	Use of mathematical information to: make decisions and judgements; add support to arguments; challenge an argument or position.

The elements of the model are represented as the net of a tetrahedron surrounded by and bound together by a critical orientation (Figure 1, below).

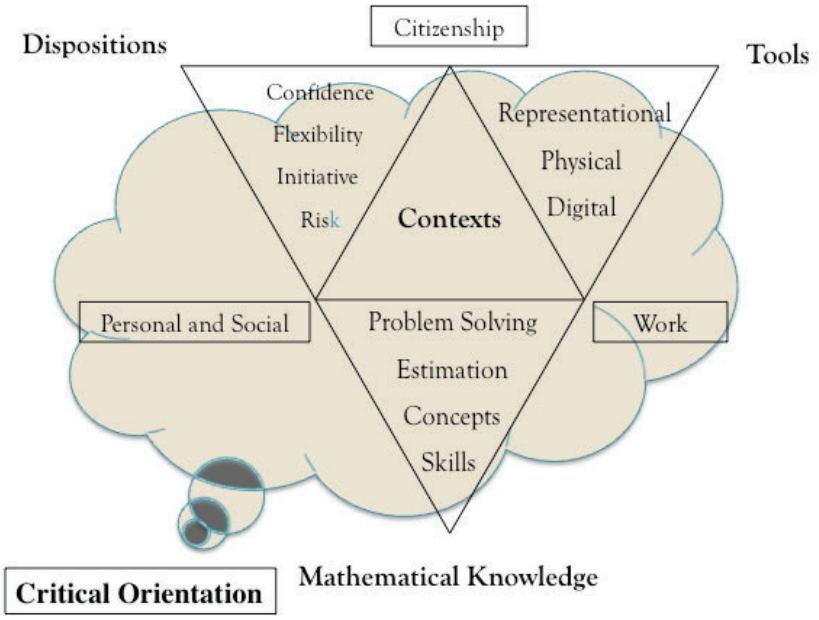

Figure 1: A model for numeracy in the 21st century (Goos, 2007).

This model offers a broad interpretation of the role of mathematics in bridging the gap between school mathematics and the wider world and has been used as a framework to audit mathematics curriculum designs (Goos, Geiger & Dole, 2010) and for analysis of teachers attempts to design for the teaching of numeracy across the curriculum (Goos, Dole & Geiger, 2010). The numeracy model was also used to promote teacher professional learning and, in particular, to assist teachers to reflect upon their own practice.

Teacher professional learning in numeracy

Ball and Bass (2003) and Ma (1999) have both identified the importance of personal and professional identities to reform in mathematics teaching. In supporting these identities and associated teacher self-efficacy issues, Millett, Brown and Askew (2004) emphasise the vital elements of time, talk, expertise and motivation. These elements were deemed as essential in promoting teachers' sense of agency when attempting to make fundamental changes to their teaching practice.

In a synthesis of literature related to effective teaching in numeracy, Muir (2008) identified the following practices as being central: making connections; challenging all pupils; teaching for conceptual understanding; facilitating purposeful discussion; maintaining a focus on mathematics; and possessing and instilling positive attitudes towards mathematics. While it is helpful to identify such practices, Muir (2008) does not attempt to describe how teachers decide to change their current practice or how to support them in the process of change. In a study of teachers' numeracy pedagogical practices in Tasmanian schools, Beswick, Swabey, and Andrews (2008) found that most teachers focused on the creation of supportive classroom environments but there was a disconnect between the aims of the mathematics curriculum and teachers' actions in relation to numeracy specific pedagogical approaches.

These studies highlight the need for ongoing research into understanding how teachers come to identify and then appropriate new pedagogical practices specific to

numeracy, especially those practices which are different from those practices specific to the teaching of mathematical skills.

Research design

Participating teachers self-identified as volunteers in response to a request for expressions of interest in a cross-curricular, middle school (Years 6 to 9) numeracy project which was distributed to every government school within a single Australian state. Participants were selected in order to provide coverage across metropolitan, provincial and remote schools and to capture a mix of primary (K – Year 7), secondary (Years 8 – 12) and area schools in rural areas (Years 1 – 12). In addition, because the focus was on the teaching of numeracy across the curriculum, efforts were made to include teachers who had specialist mathematics knowledge and those who did not. This meant that participants included generalist primary teachers who taught across the curriculum and also secondary teachers with specialised subject knowledge (e.g., mathematics, science, English) Participating schools nominated two teachers in order that these teachers could collaborate on, and support each other with, their contributions to the project.

The project was conducted between January and November 2009 and included both teacher professional learning and research components. As action research is an appropriate methodology for supporting educational reform through collaborative partnerships between participating teachers and university researchers (Somekh & Zeichner, 2009), in this case, the embedding of numeracy throughout the school curriculum, this approach was adopted for this study. A series of project meetings and school visits were conducted to support teachers through two action research cycles of plan, act, observe, reflect in order to replan and continue through the next cycle.

Teacher professional learning activities included whole project teacher meetings (March, August and November) which were led by the project researchers. In these meetings, elements of the numeracy model were explored and examples of classroom activities which embodied these elements were demonstrated. After the initial meeting, whole project meetings were also used by teachers to showcase work in progress and to seek feedback on ideas they were preparing for implementation from other project teachers as well as the researchers. Between whole project meetings, members of the research team visited each participating school and provided feedback and advice on the introduction of numeracy based approaches to teaching in their specific school contexts. This included, for example, feedback on an observed lesson using the numeracy model as a guide or providing support in assisting teachers to understand aspects of the numeracy model they were struggling to comprehend.

The research component of the project was based on data gathered during whole project meetings and school visits. During whole project meetings teachers were asked to: outline their initial conceptions of numeracy; complete a survey on teachers' confidence with numeracy teaching; and to map their personal progress in numeracy by using the numeracy model as a lens. Researchers' visits to schools involved: recording field notes for lesson observation; pre- and post-lesson teacher interviews and post-lesson interviews with students; collection of student work samples. An outline for both teacher professional learning activities and research data collection appears in Table 2.

Table 2: Outline of professional learning and research activities.

Event	Activity
1st whole project meeting	Professional Learning: Researcher led orientation to numeracy model, exemplar activities. Research: Teachers' initial conceptions of numeracy; numeracy confidence survey.
School visit 1	Professional Learning: Researcher feedback on teaching programs and lesson observations. Research: Field notes of lesson observations; student work samples; audio recorded pre- and post-lesson teacher interviews; audio recorded post-lesson student interviews.
2nd whole project meeting	Professional Learning: Evaluating implementation of the initial numeracy unit that the teachers had taught; setting goals and planning for the second action research cycle.
School visit 2	Professional Learning: Researcher feedback on teaching programs and lesson observations. Research: Field notes of lesson observations; student work samples; audio recorded pre- and post-lesson teacher interviews; audio recorded post-lesson student interviews.
3rd whole project meeting	Professional Learning: Showcase of four different professional learning trajectories. Research: Learning trajectory mapping; repeat of conception of numeracy activity and numeracy confidence survey.

As this paper is concerned with teachers' perceptions of their own professional learning, the data examined here are drawn from the learning trajectory mapping activity conducted in the final whole project meeting and the final post-lesson teacher interview conducted during the second school visit.

Teacher trajectories through the numeracy model

The professional learning trajectory activity required teachers to indentify the element of the numeracy model which represented their initial focus at the beginning of the project and also to indicate those elements that assumed greater importance to them as the project progressed. Teachers were provided with a copy of the numeracy model and asked to annotate the model in a way that indicated their professional learning journey over the duration of the project. For example, Karen annotated her copy of the numeracy model in the following fashion (Figure 2).

This annotated copy indicates that Karen began with a focus on supporting students' development of mathematical skills as she believed that, once acquired, these skills would be "naturally" adapted for use in out-of-school contexts. She comes to realise that it is the use of the skill in real-world contexts she wants to promote and not just the skill itself, which leads her to take account of the importance of the other elements of the model as time progresses.

Of the 20 teachers involved in the project, 18 completed the mapping task in the way we requested. Figure 3 shows these teachers' starting points and the direction in which they indicated they had developed as the project progressed. Of the 18 valid responses, 8 teachers indicated that they had entered the project with a concern for students' dispositions. Their annotations suggested that they were uneasy with students' negative feelings towards mathematics and wanted to devise numeracy learning experiences that would have a positive impact. Seven teachers indicated that their starting point had been

students' mathematical knowledge and skills, and their annotations suggested that they believed that if students had appropriate mathematical knowledge and skills, they would be successful in applying these as required in context. Only 3 teachers indicated that they started the project with an emphasis on contexts, stating that this approach allowed students to apply their mathematical knowledge in meaningful situations. None of the teachers said they came to the project with a primary interest in tools or a critical orientation.

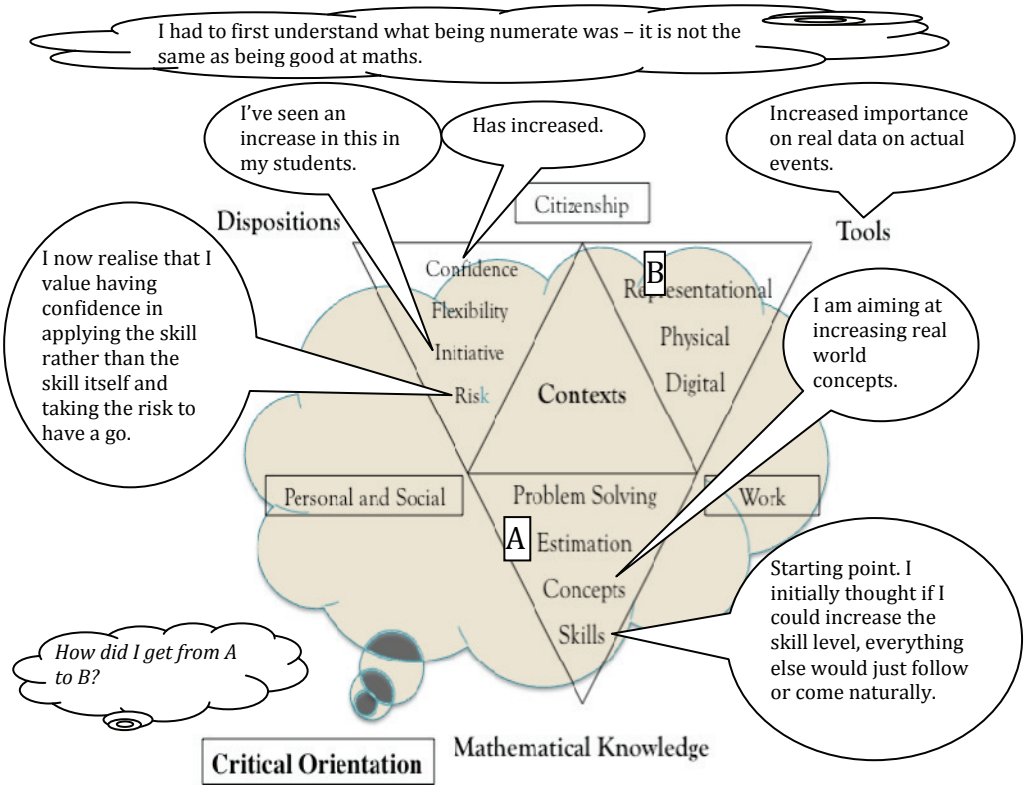

Figure 2: Karen's trajectory through the numeracy model

Although varied, teachers' trajectories through the model showed some patterns of similarity (see Figure 3). Knowledge to dispositions (K – D) and dispositions to knowledge (D – K) were common patterns, possibly indicating teachers' beliefs about the connection between success in using mathematical knowledge and a positive disposition.

Dispositions (D)	Knowledge (K)	Context (C)
D – C	K – D (2 teachers)	C – K – CO
D – C – T	K – D/C	C – K – D – T
D – C – K (2 teachers)	K – D – T	C – All
D – K/T/C	K – T – D (2 teachers)	
D – K/T – C	K – C – D	
D – K/T – C/CO		
D – K/T/C – CO		

Figure 3: Starting points and trajectories in engaging with the numeracy model.

For the latter pathway, tools were linked often with knowledge. Only four teachers indicated that they considered the critical orientation aspect of the numeracy model, and

this was their end point. One teacher, indicated by C – All in Figure 3, put the starting point as contexts, but then annotated the model comprehensively to show how integrated and equally important all these elements were.

Although the teachers identified different starting points and trajectories through the numeracy model, at least half of the valid responses to the mapping task indicated they had attended to four of the model's five components during the life of the project: 16 teachers annotated knowledge, 16 dispositions, 13 contexts, and 9 tools.

It was interesting to observe that teachers' most common starting point in engaging with the model was a concern for student dispositions. It appears that teachers may have initially paid most attention to components of the model representing student characteristics of concern to them, such as dispositions and mathematical knowledge, and then explored the use of contexts, tools, and, less commonly, a critical orientation as a means of enriching their numeracy teaching.

Vignettes

Figure 3 indicates that teachers took different directions in developing their approaches to the teaching of numeracy. These directions varied in relation to both their starting points and also the order in which they developed an appreciation for the other elements of the numeracy model. Three different teacher learning trajectory cases are now presented to illustrate the types of reasoning teachers used to make decisions about the directions they chose for their own development.

Catherine

During her final interview, the researchers asked Catherine (a middle school teacher with an English specialisation) to reflect on her changing understanding of numeracy in terms of the model presented early in the project. She explained that her desire to improve students' dispositions marked her entry point to the model, and she attempted to do this by exploring the numeracy demands of different curriculum and real world contexts. This necessitated a change in teaching practice towards a less directive and more inquiry-oriented approach, a "letting go" process that Catherine found difficult but more effective for enriching students' mathematical knowledge and promoting a critical orientation to evaluating information and answers. Once she began to give students more responsibility for their learning, she became more willing to experiment with unfamiliar tools, such as spreadsheets, for problem solving. While her entry point into enhancing her students' numeracy was through attempting to improve students' dispositions, Catherine, over the duration of the project, addressed all aspects of the numeracy model and through this process changed her approach to teaching in a fundamental way.

Maggie

When Maggie (a secondary mathematics teacher) was asked what were the key factors in developing her new understanding of teaching numeracy, she said she began with a desire to improve her teaching by increasing her focus on embedding student learning in engaging contexts. She believed this was a vital precondition before she could convince students of the need to acquire relevant mathematical knowledge. Through the course of the project Maggie noticed her increased focus on developing activities that provided a

critical orientation towards the use of mathematics. Once this element was introduced, she realized the role dispositions played in encouraging students to try approaches to solving a problem for themselves rather than expecting Maggie, as the teacher, to simply provide solutions. Finally, Maggie had increased her use of tools, particularly digital tools, through the project as she could see there were advantages in using these tools in exploring and analysing authentic contexts she could bring into her classroom. Although she had taken a different pathway from other teachers in the project, Maggie addressed all aspects of the numeracy model, resulting in a deeper understanding of what it means to be numerate and in a more targeted approach to developing numeracy capacities in her students.

Sarah

Sarah (a generalist primary teacher) came to this project as an experienced and successful classroom teacher who incorporated literacy development in her teaching at every opportunity. She knew that numeracy should also be promoted across all learning areas, but believed that numeracy stemmed from proficiency in mathematics knowledge and skills – and this had been the predominant emphasis of her mathematics program. Indeed, at the beginning of the project she said that she looked at the elements of the numeracy model and saw them as a blur, in that she knew they were all important but felt that the model had little clarity to guide her planning for numeracy. Her journey started by using a context to extract mathematical knowledge, with the result being an artificial imposition of mathematics in unnatural and irrelevant contexts (e.g., what pattern makes up the floor of the War Memorial when the focus of the unit was on history, heroism and the horror of war). Through critical self-reflection, Sarah saw how the learning area provided the context, not the topic, and through the learning activities the numeracy elements of mathematical knowledge and tools could be developed. The context also enabled students to develop a critical orientation as they explored particular topics in depth. (e.g., How many young men died serving the war? What percent of the population was this?) By developing mathematics knowledge through such meaningful contexts, Sarah noted the growth in her students' positive dispositions towards mathematics and confidence in their desire and ability to apply mathematics as required. Sarah claims she now sees the importance of all elements of the numeracy model that we presented at the start of the project, and is confident in developing units of work that integrate this vision of numeracy into her natural teaching style.

Conclusion

Even though teachers' involvement in this project began with an introduction to a single, specific model of numeracy, their own development as teachers of numeracy varied considerably in relation to elements of the model they initially chose to emphasise and in the order they chose to adopt other elements. It would appear that, as their own understanding of the nature of numeracy developed, so did their appreciation for other elements of the model. This, in turn, led teachers to incorporate other elements of the model within the duration of the project.

Teachers also noted the inter-related nature of the elements and often reported how beginning with one element led to the incorporation of another in their teaching. This was the case with Sarah, who noted how the use of an authentic context led naturally to

the incorporation of a more critical approach to the use of mathematical skills as there was a need to use quantitative methods in order to resolve important questions which arose through her unit of work.

This study has provided insight into possible approaches to assisting teachers to find their own directions in developing effective numeracy pedagogies, but further research is required into how to best support teachers in finding directions and trajectories most suited to their own circumstances.

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