FORMATIVE ASSESSMENT TOOLS FOR INQUIRY MATHEMATICS

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Inquiry in mathematics is well suited to address authentic, ill-structured problems that are encountered in everyday life. However, available formative assessment tools are typically not designed for an inquiry approach. An exploratory study using Design Research aimed to understand and improve assessment practices of mathematical inquiry. Data collected from one classroom provided detailed examples of these assessment practices in action. Results from the initial stage and future directions of the project will be presented.

Reform efforts have for several decades worked to improve the curriculum, pedagogy and assessment of mathematics in schools. An inquiry-based approach shows particular promise for improving student learning in mathematics. Duckworth (2006) contends that the most wonderful ideas and understandings of children are revealed when adequate time to explore is provided and learning activities are designed to allow conflict and reconsideration of ideas. There is a professional need to capture those learning moments and to use the evidence to enhance student understandings and extend their new and emerging ideas. Little is known about how assessable information from these activities can be better identified, stored and used to inform future teaching and learning experiences. In order for inquiry to be used more widely, there is a need to recognise ways to assess students that value multiple types of understandings, rather than focusing and reporting only on narrow content. One potential resource for addressing this problem is a framework from The Programme for International Student Assessment (PISA) (OECD, 2009). PISA identifies a reflective cluster of competencies to assess mathematical literacies that value processes used by students to solve open-ended problems. These hold promise as a design framework for making assessable information from inquiry available to educators.

This paper will discuss results of a pilot PhD study from one teacher's experiences in trialling and implementing innovative assessment tools. The case study builds an initial foundation for supporting teachers in adopting new assessment tools. The study addresses an important gap in the field as little is known about the kinds of formative assessment tools that can support mathematical inquiry. Until the field is able to capture and record students' mathematical learning of inquiry processes, there is little hope of inquiry becoming a normative practice in school mathematics.

LITERATURE

Inquiry

Makar (2007) discusses the shift in paradigm in the teaching and learning of mathematics away from a primary emphasis on skills, facts, and procedures in isolation. An increased stress is on integrating these within the development of children's mathematical conceptions, and proficiency at applying mathematical tools to new situations, in particular, open-ended, complex and everyday problems. Her work describes cycles of statistical and mathematical inquiry as investigations that immerse learners in open-ended problems where phases of investigating and reporting are repeated to refine understandings through improved knowledge tools (e.g., statistical concepts, technology support). Ill-structured problems in mathematics inquiry, like real-life problems, can generate discussion to identify characteristics of a phenomenon and how to capture its possible qualities (Makar & Fielding-Wells, in press). Learning contexts that provide such rich problems, such as in an inquiry classroom, have the potential to provide rich assessment opportunities that reveal students' level of mathematically literacy.

Assessment

An understanding of how assessment is used in a primary classroom will help readers to better understand that many assessment methods do not match teaching and learning experiences in an inquiry classroom. The Queensland Curriculum, Assessment and Reporting Framework (QCAR) (Queensland Studies Authority, 2009) includes assessment development as a process to support the planning of teaching and learning experiences and offers guidance on how and when to provide feedback. With such an importance placed on assessment, and with so many opportunities available in a math inquiry classroom, effective assessment tools should be fore-grounded and consideration given in how to capture these learning moments.

Assessment can inform in two ways, summatively and formatively. When evidence is used to adapt the teaching to meet student needs it becomes formative assessment. Wiliam (2007) highlights how formative assessment can support learning and even refers to this type of assessment as assessment *for* learning. Teachers must assess their students while learning is in progress in order to adapt instruction so that it is successful in helping students achieve learning goals (Black & Wiliam, 1998; Furtak & Ruiz-Primo, 2008). Black and Wiliam (1998) point out that there is convincing evidence that formative assessment can raise standards of achievement, this being an important educational priority.

Challenges arise in a primary mathematics inquiry classroom when using formative assessment. Furtak and Ruiz-Primo (2008) analysed formative assessment prompts for their effectiveness in eliciting valuable assessment information. They categorised timeliness in how student responses were collected, and teacher-responses shared, to complete a successful feedback loop. Formal or informal prompts can be recorded through student writing, eliciting students' conceptions and offering an opportunity for students who are less sure of answers to share their ideas (Furtak & Ruiz-Primo, 2008). Yet in a classroom context, analysis of these reflections can result in a delayed teacher response. The PISA assessment framework may assist in faster analysis and turnaround time to ensure a successful feedback loop.

PISA Framework

The PISA (OECD, 2009) mathematics framework is provided to describe and illustrate the PISA mathematics assessment. In PISA, mathematisation is used to describe the process students use to transform complex real-life problems into ones which can be solved with mathematics. In order to engage successfully in mathematisation, students need to possess a number of mathematical competencies. Eight of these have been identified and can be possessed at different levels of mastery.

The mathematical competencies are further organised according to three clusters reflecting conceptual categories of broadly increasing cognitive demand and complexity, summarised below in Figure 1. Categorising competencies into these three clusters offers a description of the cognitive activities students undertake when completing the mathematical problems. The reproduction cluster highlights those basic mathematical processes, knowledge and skills of common problem representations, commonly required on standardised assessments and classroom tests. Students can build on these skills and apply them to situations that are not routine as part of the connections cluster. Assessment items that require integrating, connecting, and an extension of practised material measure the connections cluster. In the reflection cluster, students reflect on the processes required and may have to plan solution strategies for unfamiliar problem settings.

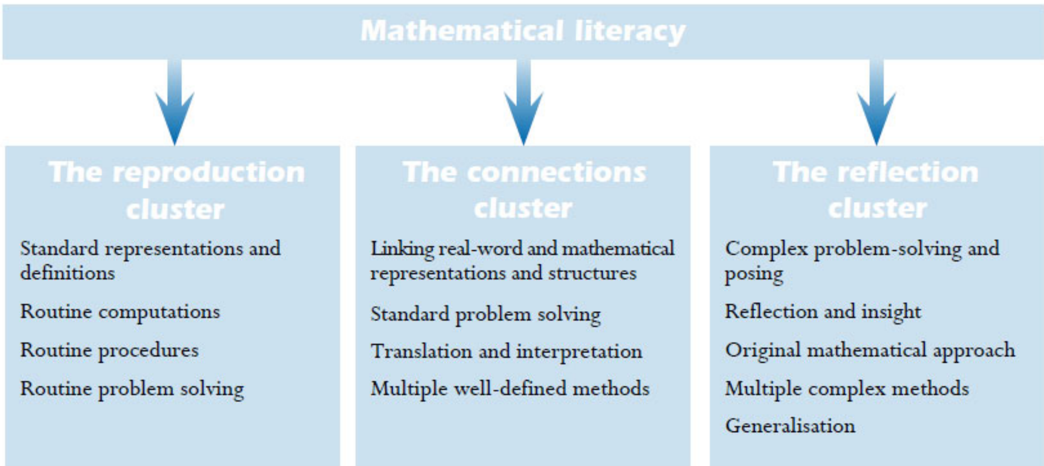

Figure 1. PISA Competency clusters to assess mathematical literacy (OECD, 2009, p. 115).

METHOD

Participants and context

This pilot study uses a case study method (Merriam, 1998) to examine the learning environment, student artefacts, and observations from a primary, inquiry mathematics classroom. The pilot study involved a laptop class from a middle-class suburban primary school with 28 Year 6 students (age 10-11) taught by the author. The students were confident in using their own laptops while working in mathematics inquiries. The study aimed to improve, record, and reflect on the assessment practices during mathematical inquiry. In the data presented here, the students were engaged in a mathematical inquiry entitled *How much is 1m³?* (Figure 2) as part of the normal activities and assessment practices within an inquiry-based classroom. In this inquiry, students had to demonstrate their understanding of a cubic metre and its relationship to 1000 litres and to one tonne. The ambiguity of 'How much?' lends itself to the open-endedness of the question as students could choose multiple pathways to present this.

How much is 1m³?
You need to show:
- The cubic metre you make,
- How you made the cubic metre,
- And how you worked out that it was a cubic metre.

Figure 2: Inquiry question.

Data collection

Students were given many opportunities to reflect on their learning throughout the inquiry. Whichever stage of a mathematical inquiry students are working in, reflection can offer valuable insight. In this inquiry, students were encouraged to use a *Quickwrite* (Dodge, 2009) strategy to record their reflections in a timed two-minute written reflection that can take place at any time within the learning experience. These included thoughts, questions, and ideas about the topic, as well as reflections on what had been achieved in the lesson or a prediction of what the lesson might look like. The students individually recorded these Quickwrite-inspired ideas and conceptions using their laptops over time as an electronic learning journal.

The nature of the electronic learning journal made the information easy for students to access and manipulate. Students could delete, backspace, and insert thoughts and ideas without 'rubbing' any work out. They could also use menu options such as Synonyms, Look up (Online dictionary), or Images (internet or clipart) to better understand relevant vocabulary or to improve their own explanations. Enthusiasm meant that students would eagerly format and edit their work with a purpose to reflect on their mathematical understandings. As the inquiry progressed, additions were made to their original thoughts and were dated to help show the audience progression and development of ideas. Different choices in fonts, text sizes, and colours also made the progression clearer. Students were adept at saving their work in a number of places to ensure access in any physical location.

As well as working individually on the computer, students worked in small groups of no more than three to solve the problem, *How much is 1m³*? Many of the ongoing ideas formulated in groups were recorded on A3 sheets of poster paper. These were a second set of important artefacts of group collaboration with a focus on communicating this information later to others.

Finally, a research journal was used by the teacher to record discussions with students and observations of their work. Teacher reflections were also created to record the context and circumstances at iterations within the inquiry. For the teacher as researcher, reflection on these experiences could provide evidence of categories that may require further investigation. This reflective tool assisted the teacher to inform and improve upon teaching and learning.

Analysis

The electronic learning journal became important documentary evidence of students' learning yet how the teacher was to best interpret and use this information to inform teaching and learning was not initially clear. The PISA assessment framework was trialled as a way to analyse and assist in making teacher judgements of student work in the journals. This would highlight the mathematical language, modelling and problem-solving skills used, as organised in terms of competency clusters. Processes or thinking skills particular to this inquiry were Thinking and Reasoning, Problem posing and Solving, Communication and Modelling. Although examples of other competencies could be found, they are not a focus for this paper. Examples of the processes or competencies were identified in the students' reflections and rated according to each competency cluster: Reproduction, Connections, or Reflection with a 1, 2, or 3 respectively (Table 1). In the PISA assessment framework, there is considerable overlap in the processes or competencies students will use to work mathematically, as is common when working through mathematical inquiries in the classroom. Identifying which cluster students are working in can provide evidence of higher order thinking skills used, highlight gaps or topics that may need further explanation, and can inform teaching practice.

Peer analysis of the electronic learning journals occurred in the classroom also as students defended each stage or juncture in the inquiry. An interactive whiteboard allowed groups to display their reflections, calculations and ideas and to edit this work while on display. As ideas were shared, other students posed questions that critiqued the work. Also, other students analysed their own efforts and the effectiveness or appropriateness of their ideas.

Table 1: Rubric comparing competencies and clusters.

			Cluster		
			1 Reproduction	2 Connections	3 Reflection
Process/Description	1	Thinking and Reasoning			
	2	Argumentation			
	3	Communication			
	4	Modelling			
	5	Problem posing and solving			
	6	Representation			
	7	Using symbolic, formal and technical language and operations			
	8	Use of aids and tools			

Evidence of students working collaboratively can be found in their journals, but also in the written work completed together. The A3 poster papers recorded the inquiry process throughout all iterations and were used to also communicate findings and generalisations with other groups. Using a rubric (see Table 1) comparing competencies with clusters, again using the PISA framework, analysis could capture evidence of mathematical processes used and when learning moments moved between clusters.

Teacher reflections of learning moments provided further explanation of the findings in the electronic journals and poster papers. Discussions between teachers and students, and students with other students, were analysed using the PISA framework to find further evidence of competencies that had been developed through mathematical inquiry, and opportunities for competencies to be further developed if gaps were identified.

Results

Electronic journals

When the class began their mathematics inquiry unit on the cubic metre, it was clear some students were not confident in their understanding of volume and the relationship to mass. Students had previously completed a one-week planned unit of work exploring this concept and had experienced a range of activities that included manipulating and viewing materials and practise of routine operations in particular contexts. The kinds of answers or conceptions evident in the inquiry often did not match those in the previous unit of work, where both aimed to develop the same mathematical understanding of a cubic metre. The cognitive mathematical competency (PISA) of *thinking and reasoning* was identified in the reflections in the electronic learning journals where students demonstrated an ability to pose mathematical questions and have knowledge of the kinds of corresponding answers mathematics can offer. When asked to pose a question where the answer was $1m^3$ (a pre-assessment task), one student wanted to explore a problem about filling an area with grass and a flower bed. This conception of volume did not match the *thinking and reasoning* already hoped to have been developed in the previous unit of work. *Thinking and reasoning* and *problem-solving* skills used by the student were only based in the Reproduction Cluster, where the student was working with contexts familiar to them. They were still developing their understanding of practised routine procedures regarding area and were not yet making links to less familiar, real-world contexts. Nor were they solving problems using independent problem-solving approaches. Later in the inquiry, the same student changed their mind to explore how much cement would be needed to fill and build the slab under a shed; their understanding of volume was beginning to develop.

Identification of the thinking processes students use can be of interest when students who perform reasonably well in pencil and paper tests (in the previous unit of work) display low thinking and reasoning ability, generally in the Reproduction cluster, when applying the understanding to an inquiry context. For example, reflections in one student's electronic learning journal indicated an additive understanding of a cubic metre as opposed to a multiplicative understanding ($1m \times 1m \times 1m = 1m^3$). A problem was posed by the teacher (see below) at the beginning of a lesson to orientate the students to the next phase of the mathematical inquiry. This student, "added all of the measurements together to check if it was over, under, or equal to 3m". This student had earlier produced results that indicated a sound understanding of volume.

> A child's wading pool measures 1.3m wide, 1.5m long and 75cm deep. What is the volume of the pool? How much water is needed to fill it up?

A3 poster paper

Aiming to present findings to the class, students used A3 sheets of poster paper to record mathematical conceptions and ideas collaboratively. Evidence of *thinking and reasoning, modelling,* and *problem posing and solving* as competencies were identified as being used in this process. The rubric (Table 1) was used to rate evidence of learning in each competency to gain an initial label of one through to eight. A rating of one to three was a second score earned which indicated in which cluster the students were working. Working in the Reproduction cluster received a score of one, whereas, evidence of working in the Reflection cluster received a score of three. Scores therefore indicated which process was evident, and the level of thinking e.g. a score of (1, 2) would indicate evidence of the first competency, *thinking and reasoning*, and that the student was working in the Connections cluster.

As students began to work on the inquiry topic, evidence of problem posing and solving was rated in the Reproductions cluster as students reproduced standard, closed problems, which could typically be solved in the one way. For example, one group had recorded on their poster the following question: "I had $6m^3$ and I had another $5m^3$ and subtracted them and got a result of $1m^3$." This lower-order thinking was not challenging the students to think beyond the problems already practised in class. To push students beyond these questions, the teacher was then able to discuss with the class what types of questions display a mathematical understanding of the concept. A basic equation like $1m \times 1m \times 1m = 1m^3$ would demonstrate some basic understanding of volume, but the equation was not highly creative nor did it demonstrate a good mathematical understanding of the concept. A new criterion was jointly constructed to guide students to think about how their responses might be more creative or reflect a good mathematical understanding of the concept. Solutions began to move away from the reproduction-style problems that practised standard problem solving. Assessing which cluster students are working in can highlight areas for improvement and can help students to use more rigorous mathematical processes.

Researcher field journal

Identifying where students are working can help teachers know where to go next. For students who do not like to write, posters did not reflect much thinking. It was useful to then look at the individual student's reflections in their electronic learning journal, where an enthusiasm for computers meant that more effort was made in recording their ideas. Many assessment opportunities are missed when students are working in groups and discussion is not recorded. In this inquiry, one group of students who generally did not like to record their reflections in writing became a focus for the teacher to record anecdotal evidence. This information was recorded in the teacher journal. Anecdotal observations of student work have long been an assessment method valued by teachers with notes being recorded in different ways and being stored in various locations for later use at report card time. Using the PISA assessment framework, the teacher in this instance was able to reflect on the teacher journal to identify clusters students were working in and areas to improve teaching and learning. One reflection noted how students moved between clusters when *modelling* their ideas. Initially, the group had sketched a box with the dimensions $0.5m \times 0.5m \times 0.5m$ with an assumption that this was half a cubic metre. Once the students were encouraged to use the cubic metre model in the class to show this they were able to visualise that about eight of the boxes would be able to fit into one cubic metre! Students were interpreting back and forth between models and their results, and this communication was evidence of students now working in the Connections cluster. Knowing where these students were working using the PISA assessment framework provided the teacher with direction and focus.

Discussion

The student-centred nature of a mathematical inquiry unit of work can mean that the direction learning travels is not always as expected. Mathematical inquiry is complex and open-ended (Makar, 2007) and intended content areas are not always realised. This makes it a difficult task for teachers to foreground assessment. Often new and exciting mathematical topics need to be introduced to assist in answering students' questions. Using the PISA assessment framework to analyse student work made it possible for the teacher to quickly check the cognitive level the students were working in and provided some direction in how to push their reasoning beyond those experiences. A faster turn-around of feedback on rich, written reflections ensures a more successful feedback loop (Furtak & Ruiz-Primo, 2008). It offers an alternative to earlier pencil-and-paper-style assessment with tasks that give a clearer indication of the level of a student's mathematical reasoning. In this inquiry, there were three levels of evidence to show how students were working: individually through the use of electronic learning journals, collaboratively by analysing poster sheets student groups worked on, and with a focus on differentiation as the teacher reflected and added anecdotal notes to their journal.

In analysing these three areas, the framework provided useful feedback and information to both students and teachers. In the electronic learning journals, the feedback was for the individual student as it identified which clusters the student was working in or developing. It allowed opportunities for teachers to formatively assess their own teaching practice and to offer feedback to students that was meaningful and could guide them further in their inquiry. For small groups, analysis highlighted which competencies students were working in. This information could be used formatively to provide feedback to the whole class, encouraging groups to use higher order thinking skills. Identifying the competencies and clusters in the teacher journal further informed teaching and learning pedagogy and could assist the teacher to make more informed judgements of the level of students' mathematical reasoning.

Although this case study was not a typical classroom, the results provide insight into ways to capture and analyse assessment opportunities in a primary classroom using mathematical inquiry. Using this information can inform teachers and students how to move away from lower-order thinking processes of posing and solving familiar and practised problems (Reproduction cluster). Teachers and students can aim to apply problem-solving processes, knowledge and skills to situations that are not routine (Connections cluster) or with an element of reflectiveness (Reflection cluster) with problems that contain many elements and may be more unfamiliar.

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