PROMOTING AN UNDERSTANDING OF MATHEMATICAL STRUCTURE IN STUDENTS WITH HIGH FUNCTIONING AUTISM

MAUREEN FINNANE
Bardon Counselling Centre, Brisbane
maureenfinnane@optusnet.com.au

In a study of Year 2 students, performance on a novel open-ended Make 10 task was one of two strongest predictors of students diagnosed at risk of mathematical learning difficulties (MD) on the Queensland Year 2 Diagnostic Net (Finnane, 2007). Students who performed poorly on this task produced few combinations, gave counting sequences or figurative responses featuring physical embellishments of the numeral 10, compared to a range of flexible responses by normally achieving students. This paper demonstrates the application of the Make 10 task to facilitating the conceptual and skill development of a Year 4 student with high functioning autism who was facing significant mathematics anxiety and pervasive mathematical leaning difficulties.

Introduction

In a special issue on mathematics in the *Journal of Learning Disabililities,* authors of a research review on early identification and interventions for students with mathematical learning difficulties described the field as “in its infancy” (Gersten, Jordan, & Flojo, 2005). Gersten et al. stressed the importance of identifying the best predictors of early difficulties in mathematics as a guide to designing effective interventions for struggling students. Mazzocco (2005) further drew attention to the need to fully understand the nature of the mathematics learning difficulties of students with other significant cognitive and processing difficulties.

One group of students whose mathematics learning needs may prove challenging to teachers are students with high functioning autism. While a proportion of students with high-functioning autism (HFA) may have outstanding mathematical abilities, research suggests that up to half of students with HFA may face significant difficulties in learning mathematics (Chiang & Lin, 2007; Mayes & Calhoun, 2006; Reitzel & Szatmari, 2003). Chiang & Lin (2007) raised the need for assessments which can adequately measure the strengths and weaknesses of students with HFA. The cognitive profiles of students with HFA suggest that they may have difficulty in detecting patterns and distinguishing relevant details, and may find it difficult to conceptualise numbers as abstract concepts of comparative quantities. In addition, students with HFA are prone to anxiety (Attwood, 2007), which may further disrupt their ability to make mathematical connections. On the other hand, students with high functioning autism may be expected to show strengths in sustained attention and ability to master facts (Sansoti, Powell-Smith, & Cowan, 2010).

There is little available research to guide effective interventions for students with HFA and mathematics difficulties. Research on assisting academically low achieving students in mathematics focuses on the importance of developing number sense and rich mathematical concepts (Dole, 2003; Gersten & Chard, 1999; Gersten et al., 2005; Woodward, 2006). There is a danger that promoting rote learning by students with high functioning autism might inhibit the development of a meaningful understanding of mathematical concepts.

Australian researchers have identified important developmental frameworks and constructs which are helpful in establishing priorities for intervention for students with mathematics learning difficulties. Wright, Martland, and Stafford (2000) highlighted the critical role of mastery of forwards and backwards counting and fluent numerical identification skills in developing essential concepts of numbers as composite units and efficient strategies for solving basic additions and subtractions.

Mulligan, Mitchelmore, English, and Robertson (2010) have further demonstrated that progress in students' mathematical understanding depends on an understanding of underlying mathematical structure. Using the construct of Awareness of Mathematical Pattern and Structure (AMPS), Mulligan, Mitchelmore, and Prescott (2005) have shown that low achieving students have more difficulty in perceiving and representing visual patterns and mathematical structure and, most importantly, that these problems may be associated with weaknesses in multiple counting, partitioning, equal grouping and equal units of measure.

The present paper aims to contribute to the continuing growth of the field of early identification and interventions for mathematics learning difficulties by:

1. Presenting the results of a research project on early predictors of mathematical learning difficulties.
2. Applying the research findings to an intervention to support the mathematical development of a Year 4 student with high functioning autism, anxiety problems, and pervasive mathematics difficulties.

Method

The paper is presented in two parts: Part 1 identifies early predictors of mathematical learning difficulties, while Part 2 reports a case study of an intervention with a Year 4 student.

Part 1. Early predictors of mathematical learning difficulties

As part of a large study exploring early indicators of mathematics learning difficulties, a comprehensive set of mathematical, memory and processing tasks was administered to 68 students (mean age 7.1 years) in three Year 2 classes in metropolitan Brisbane, Queensland (Finnane, 2007). The mathematical tasks included forwards and backwards counting, numeral identification and strategy use for solving basic additions and subtractions, as assessed on the Learning Framework in Number (Wright et al., 2000). On the *Make 10* task that is the focus of this paper, students were asked: "How many different ways can you make the number 10?" The task was administered after the author had modelled making a variety of number combinations for the numbers 7 and 9 using smaller numbers.

A subset of students (n = 17) was identified at risk of mathematical learning difficulties, determined by an independently administered state-based assessment process—the Queensland Year 2 Diagnostic Net (Education Queensland, 2007). The author has previously presented the results of *t*-test comparisons which showed significant differences in verbal memory capacity between students identified in the Net and normally achieving students (Finnane, 2008). Regression analyses were used to determine which of the measures were the best predictors of students who would be assessed at risk of mathematical learning difficulties on the Year 2 Net, with a view both to identification and intervention.

Results

Make 10 was one of two strongest predictors of the 17 students who were caught in the Queensland Year 2 Diagnostic Net as at risk of mathematical learning difficulties (Finnane, 2007). The other best predictor was the stage of early arithmetic learning (SEAL) as assessed on the Learning Framework in Number (Wright et al., 2000). Student responses on *Make 10* depicted different levels of number understanding, which were consistent with their stage of arithmetic learning. Net students tended to provide a number sequence 1, 2, 3, …, 10 or to focus on figural features of numerals only. Figure 1 shows the response of a student caught in the Year 2 Net, who depicted 10 as the initial two numerals of 4-digit numbers, with differing physical embellishments. Similarly, at a figural level, this student said you could “make 9” by painting a 9, or you could “make 7” by two people lying down, one horizontally to form the top ¯ and the other diagonally to form /. This student was assessed at SEAL 2 (Figurative stage) on the Learning Framework in Number, where he was using his fingers and a count-all strategy to solve basic additions with sums less than ten, but did not know how to solve combinations (e.g. 9 + 6) with sums greater than ten.

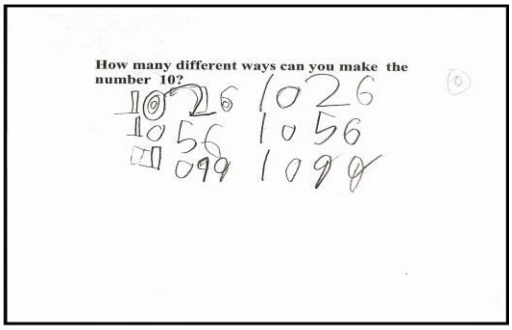

Figure 1. Make 10 *Figural level response of a Year 2 Net student.*

Normally achieving students showed a range of responses involving partitioning skills (see Figure 2) or a flexible use of operations (Figure 3), indicating a more advanced concept of numbers as composite units (Fuson, 1988). The responses of these students were consistent with their further progression to more fluent counting, numeral identification and advanced strategy use on the Learning Framework in Number.

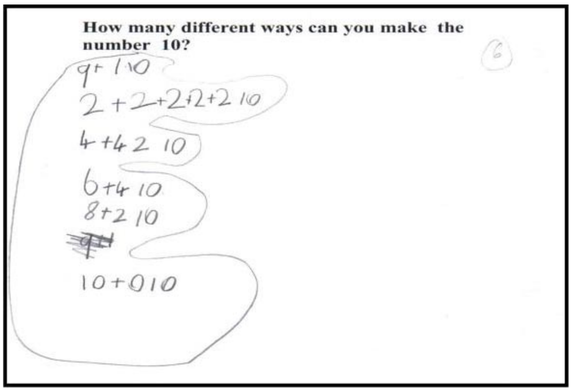

Figure 2. Make 10 *response of a normally achieving Year 2 student showing partitioning skills.*

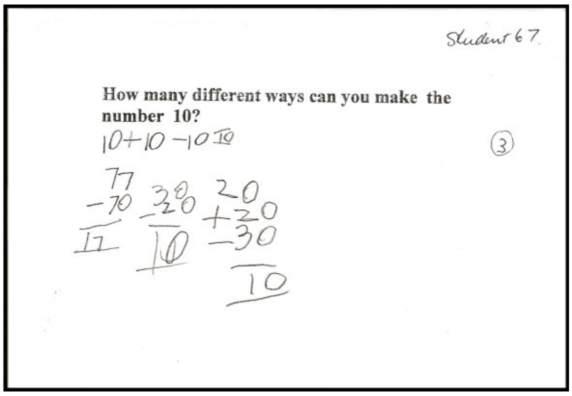

Figure 3. Make 10 *response of a normally achieving Year 2 student showing flexible use of operations.*

Part 2. Case study intervention

Assessment.

A Year 4 student aged 9.8 years with diagnoses of high-functioning autism, anxiety and learning difficulties was referred to the author by his Paediatrician for a mathematics assessment. The student will be referred to as Will, a pseudonym. Will was showing signs of significant stress during mathematics lessons within the classroom. His mother reported that he showed particular distress in relation to problem solving, did not seem to understand the concepts needed, and would cry on the way home from school in anticipation of his mathematics homework. The student was assessed using the mathematics subtests of the Wechsler Individual Achievement Test - Second Edition (WIAT-II), the Learning Framework in Number (Wright et al., 2000), and the *Make 10* task (Finnane, 2007).

Will performed in the Well Below Average range for his age on both the Mathematics Reasoning (1st percentile) and Numerical Operations (8th percentile) subtests of the WIAT-II. In Year 4, he was still unable to solve 2-digit additions and subtractions with regrouping and expressed a very high level of anxiety in relation to written number questions. Will showed persisting confusions between teen and -ty numbers (e.g., 13/30) in both oral sequence counting on the Learning Framework in Number and in written algorithms. Will's responses showed that he had developed only an initial concept of 10, where he focussed on the individual items that make up ten rather than ten as a unit. He was unable to match a quantity meaning to 2-digit algorithms, or to interpret and solve word problems.

Intervention

An intervention was designed for Will to address his mathematics anxiety, and to build his number sense, place value understanding, addition and subtraction concepts and problem solving skills. Will attended sessions on a weekly or fortnightly basis, with activities to complete between sessions. Will's parents were very supportive, providing assistance as needed. During the course of the intervention, Will's class teacher reduced the level of his set word problems from Grade 4 to Grade 2 level. This paper reports only one aspect of the follow-up intervention with Will, involving the *Make 10* task described above, and related number tasks (e.g. *Make 20*, *Make 100, Make 120*). *Make 10* was chosen as an integral part of the intervention, as it was observed in the assessment that Will enjoyed this task and he showed an initial level of familiarity with the Ten facts as components of 10.

The *Make 10* task was used on a repeated basis during the intervention to encourage Will to explore number composition and base-10 structure. After providing Ten Facts and multiplicative solutions to *Make 10* as he had done before, Will was excited when he thought of a subtractive solution 11–1 = 10. Figure 4 shows how Will maintained his attention on the pattern of subtracting the Ones from the teen numbers in order up to 19-9, and then expended considerable effort to subtract Tens up to 100.

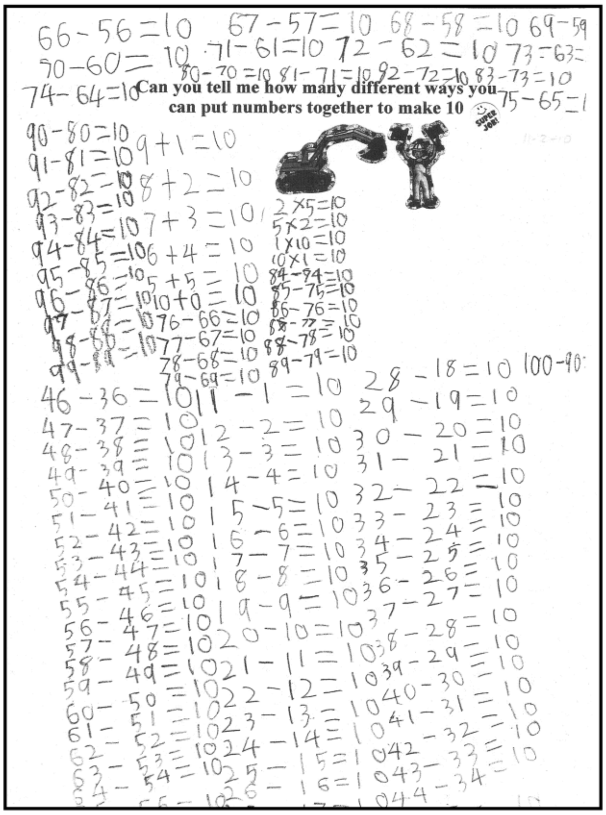

Figure 4. Will's response on the Make 10 task when he first discovered a subtractive solution.

This was an important session for Will. By using order to produce his responses in sequence, he gained a better understanding of the tens/ones composition of 2-digit numbers. Will was later able to generalise this ordering strategy for constructing ten to enable him to construct 3-digit numbers.

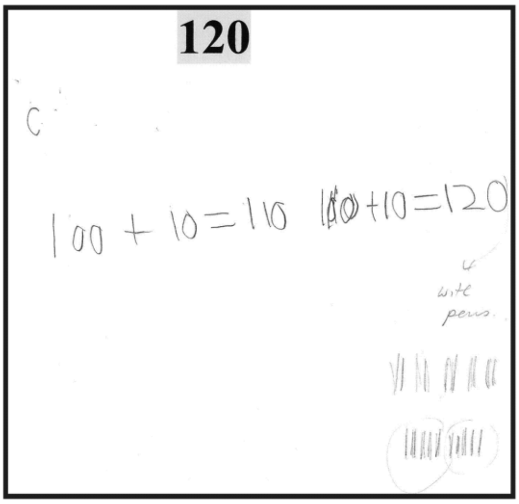

Figure 5. Will's initial response when asked to Make 120.

Figure 5 shows that, initially, Will found it very difficult when he was asked to *Make 120*. He became stuck and needed prompting to explore the composition of the missing quantity, after adding one 10 to 100 to make 110. Will understood this was not a sufficient response, but he could not work out what was missing. When I suggested he could use pens to find the missing amount, Will explored the component parts of 10 by dividing the pens into 5 groups of two, and then 2 groups of five. This partitioning enabled him to realise 10 was the missing part or addend of 120 he needed (Figure 5).

In the following session one week later, Will used order to produce several combinations to *Make 120*, after first taking 1 away from 120 to give the parts of 119 and 1 (Figure 6). He had started to see the inverse relationship between addition and subtraction more clearly. By sustaining his attention on the whole (120), Will was able to shift his attention to producing the different component parts, and he enjoyed depicting the decrementing and incrementing relationships. To finish, Will reproduced the combination 110 + 10 that he had initially struggled with in the previous session.

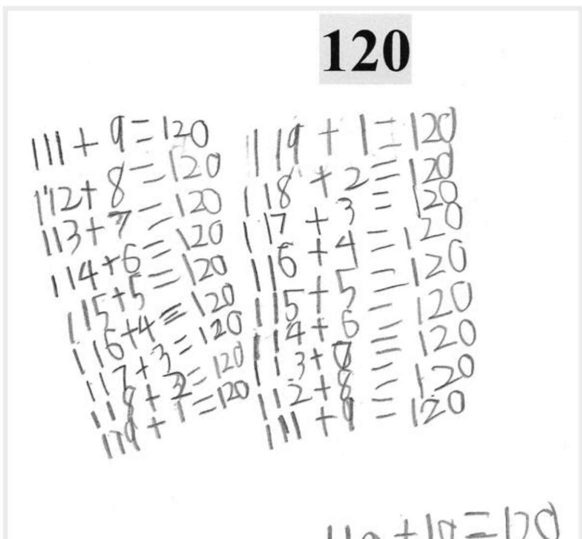

Figure 6. Will's second response when asked to Make 120.

An example from the next session (see Figure 7) illustrates Will's increasing flexibility in deconstructing and constructing numbers. Given a 2-digit number *Make 20*, he is no longer reliant on order to increase and decrease his responses in a unitary manner.

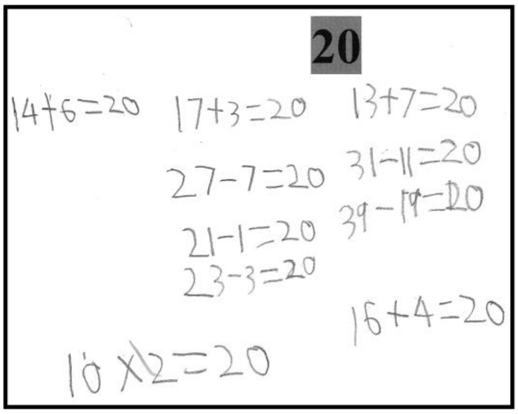

Figure 7. Will's flexible solutions when asked to Make 20.

Will's part-part-whole number concept development is further illustrated in Figure 8. Here he was able to use mental computation successfully to decompose numbers in multiple steps to perform a variety of subtractions. On the first item, Will explained that he broke up 800 into 700 and 100 and subtracted 80 from 100 to give 20, then added back the 700. By this stage, Will had developed the conceptual understanding of numbers as composite units which enabled him to carry out this mental computation confidently, and also to manage regrouping in subtraction up to 4-digits. By mid-year 5, after 20 intervention sessions, Will was in a mainstream mathematics class 90% of the time and achieving at Year 5 level. His teacher reported that "his whole mathematics has changed".

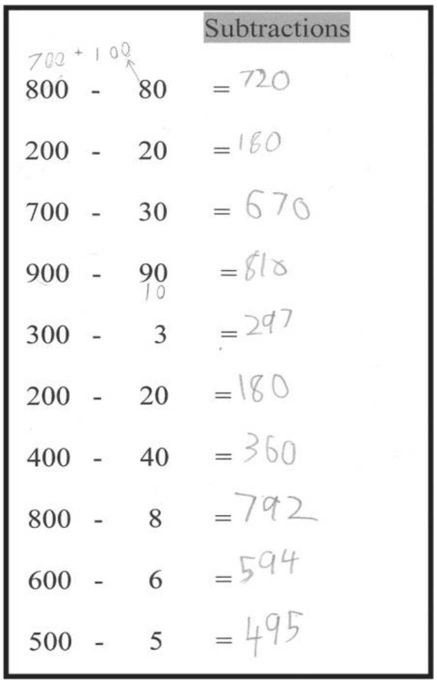

Figure 8. Will's responses to a subtraction task where he used partitioning and mental computation.

Discussion

The research described in this paper confirms the validity of counting stage on the Learning Framework in Number (Wright et al., 2000) as a predictor of performance on the Queensland Year 2 Net, and describes another discriminating task *Make 10* (Finnane, 2007) which has significance for interventions with students facing significant mathematics learning difficulties.

The paper illustrates how an open-ended assessment task (Make 10) assisted a Year 4 student with high functioning autism to explore mathematical structure and part-part-whole relationships in a way he had previously been unable to do. The opportunity to make the number 10 in as many ways possible on multiple occasions allowed the student to gradually discover and apply his understanding of addition and subtraction as inverse relationships. This understanding enabled him to develop a schema he could apply successfully to additive and subtractive problem solving (Xin & Jitendra, 2006). The intervention also had a marked impact on reducing the student's anxiety and in significantly reducing familial stress associated with mathematics homework. It is argued that the open-ended nature of the task together with a specific limited instruction was empowering for the student in accessing his existing knowledge and allowing him to make new connections to this knowledge.

While students with high-functioning autism might be able to learn facts by rote, particular attention should be paid to their level of conceptual understanding of number. The *Make 10* task can provide a useful intervention tool for facilitating students' development from a unitary concept of number to a flexible understanding of part-part-whole number structure. Future research can determine whether the open-ended nature of *Make 10* provides a useful tool for lowering anxiety in relation to a range of written mathematics topics in highly anxious students.

References

Attwood, T. (2007). *The complete guide to Asperger's syndrome*. London: Jessica Kingsley Publishers.

Chiang, H., & Lin, Y. (2007). Mathematical ability of students with Asperger syndrome and high-functioning autism. *Autism, 11,* 547–556.

Dole, S. (2003). Questioning numeracy programs for at-risk students in the middle years of schooling. In L. Bragg, C. Campbell, G. Herbert, & J. Mousley (Eds.), *Mathematics education research: Innovation, networking, opportunity: Proceedings of the 26th annual conference of the Mathematics Education Research Group of Australasia* (Vol 1, pp. 278–285). Melbourne: MERGA.

Education Queensland (2007). *Queensland year 2 diagnostic net.* Brisbane: Education Queensland.

Finnane, M. (2007). *The role of fluency in mathematical development: Factors associated with early learning difficulties in mathematics.* Unpublished doctoral thesis, University of Queensland, Brisbane.

Finnane, M. (2008). Addressing verbal memory weaknesses to assist students with mathematical learning difficulties. M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions: Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia*. Brisbane: MERGA

Fuson, K. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag.

Gersten, R. & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *The Journal of Special Education, 33*(1), 18–28.

Gersten, R., Jordan, N. & Flojo, J. (2005). Early identification and interventions for students with mathematical difficulties. *Journal of Learning Disabilities, 38(4),* 293–304.

Mayes, S. D., & Calhoun, S. L. (2006). Frequency of reading, math, and writing disabilities in children with clinical disorders. *Learning and Individual Differences*, 16, 145-157.

Mazzocco, M. (2005). Challenges in identifying target skills for math disability screening and intervention. *Journal of Learning Disabilities*, *38(4)*, 318-323.

Mulligan, J. T., Mitchelmore, M., & Prescott, A. (2005). Case studies of children's development of structure in early mathematics: A two year longitudinal study. In H. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 1–8). Melbourne: PME.

Mulligan, J. T., Mitchelmore, M. C., English, L. D., & Robertson, G. (2010). Implementing a pattern and structure mathematics awareness program (PASMAP) in kindergarten. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education*: *Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia* (pp. 796–803) Fremantle: MERGA.

Reitzel, J., & Szamari, P. (2003). Cognitive and academic problems. In M. Prior, (Ed.), *Learning and behavior problems in Asperger syndrome* (pp. 35-54). New York: Guildford Press.

Sansoti, F., Powell-Smith, K., & Cowan, R. (2010). *High-functioning autism/ Asperger syndrome in schools: Assessment and intervention*. New York: Guildford Press.

Woodward, J. (2006). Making reform-based mathematics work for academically low-achieving middle school students. In Montague, M. & Jitendra, A. K. (Eds.), *Teaching mathematics to middle school students with learning difficulties* (pp. 29–50). New York: Guildford Press.

Wright, R. J., Martland, J., & Stafford, A., (2000). *Early numeracy: Assessment for teaching and intervention.* London: Paul Chapman Publishing.

Xin, Y. P. & Jitendra, A. K. (2006). Teaching problem-solving skills to middle school students with learning difficulties: schema-based strategy instruction. In M. Montague & A. K. Jitendra (Eds.), *Teaching mathematics to middle school students with learning difficulties* (pp. 51–71). New York: Guildford Press.