INSTRUCTIONAL COHERENCE: A CASE STUDY OF LESSSONS ON LINEAR EQUATIONS

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In this paper we examine the nature of the instructional coherence across a series of lessons on linear equations. Using video and interview data from a Year 9 class in the New Zealand component of the *Learner's Perspective Study* (LPS) we explore how the teacher's pedagogical strategies associated with the selection and enactment of tasks and the action of 'sowing seeds' were key factors in establishing instructional coherence. We provide excerpts from classroom episodes to illustrate how instructional coherence supported students' learning of mathematics.

Introduction

It is widely agreed that raising achievement, especially for those groups of students who are currently underserved in our classrooms, is a priority focus for educational reforms. In New Zealand we have an array of policy initiatives, supported by a 'standards agenda', that are designed to identify and address underachievement. Underlying these policy initiatives is the belief that teacher quality—and thus classroom instruction—is a major determinant of student progress in schools.

We know that effective pedagogy can take many forms. Anthony and Walshaw (2009) in their research review of practices relevant to New Zealand education context offered ten pedagogical principles. However they note that "any practice must be understood as nested within a larger network that includes the school, home, community, and wider education system" (p. 6). In arguing that teaching is a holistic and complex endeavour, it is clear that other synthesis, especially those related to East Asian classrooms, may offer different combinations of key principles that define effective pedagogical approaches.

'Coherence' is one such factor that features in cross-national comparative studies. As reviewed by Chen and Li (2009), coherence is promoted as an important characteristic of mathematics classroom instruction in Asian countries. A review of earlier studies led these researchers to conclude that "coherent mathematics lessons can help lead to students' better mathematics learning with connected and coherent conceptual understanding" (pp. 711-2). But what does a coherent mathematics lesson—or unit of lessons—look like, and how might coherent instruction support student learning?

Instructional coherence

Instructional coherence is not a descriptor that is specific to the mathematics classroom. Indeed, Finley, Marble, Copeland, Ferguson and Alderete (2000) proposed that any teacher who brings "the components of the system—curriculum, instruction, assessment, external mandates, and community context—together intentionally with a focus on student learning" (p. 4) creates instructional coherence. Coherent instruction, they claim, supports teachers to make instructional decisions by using both the information collected in the classroom and information from external sources about what is important for students to learn.

Aligned with this perspective, existing studies on instructional coherence in mathematics classes have tended to explore the connectedness or integration of instructional elements. For example, Wang and Murphy (2004) defined instructional coherence as activities or events that are casually linked in terms of the structure of instructional content and the meaningful discourse reflecting the connectedness of topics. Schmidt (2008) argues that topics in mathematics "need to flow in a certain logical sequence in order to have coherent instruction" (p. 23)—a characteristic of mathematics curricula of top-achieving countries. As described by Fernandez, Yoshida, and Stigle (1992), lesson events that are coherent relate to each other in ways that allow students to infer relationships among events

Existing studies on coherence are largely sourced from cross-cultural comparative studies or Asian countries (e.g., Cai & Wang, 2010; Chen & Li, 2009; Shimizu, 2009). An impetus for studies in Asian countries originated in the widely disseminated finding by Stigler and Perry (1998) that found that both Japanese and Chinese mathematics lessons were structured more coherently than American lessons. It was noted that students in Japan would frequently spend an entire lesson studying one or two problems, a feature that was different to classes in American schools. Additionally, Hiebert et al.'s (2003) analysis of lesson in the TIMSS 1999 video study highlighted the explicit linking within the Japanese classroom that formed an identifiable lesson pattern. The lesson pattern was typically organised around (i) review of the previous lesson; (ii) presenting the problem(s) for the day; (iii) students working individually or in groups; (iv) discussion of solution methods; and (v) highlighting and summarising the main learning. Shimizu (2009) claimed that the 'pulling together' of the main points of the lesson, the 'Motome', was a key instructional factor to the effectiveness of the lesson.

Some studies suggest that coherency involves more than lesson structure as represented by sequencing of lesson events—it also involves the coherency of discourse that frames these events. For example, discourse associated with learning objectives may well serve to guide students learning in productive ways (Chen & Li, 2009). Conversely, when learning objectives override opportunities for students to build on their own thinking and reasoning, such discourse may limit opportunities for sense making (Askew, 2004). Discourse when viewed as a pedagogical tool also enables teachers to provide opportunities for students to participate in mathematical practices of argumentation. Such practices can help students build on their former mathematical knowledge, connect with their new knowledge, and comprehend their mathematical knowledge more deeply (Walshaw & Anthony, 2008).

Utilising a coherence lens, Sekiguchi (2006) characterised the effectiveness of a Japanese-style lesson by four aspects of the teacher's classroom discourse management.

Rhetorical' management, or coherence between goals and discourse production, is organised by the lesson's "four-phase script" (p. 84) comprising the introduction, student working independently and in groups, student explanations, and teacher summary. Thematic' management involves the coordination of the related topics within and across lessons that comprise a particular mathematical theme. Referential management refers to the ways that the discourse participants—the students and teacher—keep track of referents during discussions. Strategies include the deliberate use of processes such as "naming, symbolizing, drawing, reviewing, summarizing, and using textbooks, blackboards, worksheets, notebooks, and projectors" (p. 86). "Focus' Management deals with strategies that direct students' attention to see the 'point' of the lesson, including the use of comparison and contrast, discussion, and summary.

Teacher knowledge is also a key factor in developing coherence across linked lesson events or activities. Ma's (1999) comparative study of teacher knowledge has been influential in highlighting the knowledge teachers were able to draw on as related to particular content topics, and how such knowledges influenced their instructional sequences for developing ideas and their access to students' thinking.

We see from the literature that whilst there is agreement that instructional coherence is desirable, there is also evidence that what defines coherence, or is key to obtaining coherence, may in some instances be culturally prescribed. While educational systems and curricula may support coherence (Leung, 2005), it is clear that individual teacher's enactment of pedagogical strategies in relation perceived students needs, mediated by individual teacher knowledge in its many forms, may serve to influence levels and qualities of instructional coherence.

The case study

As part of our participation in the New Zealand component of the *Learners Perspective Study* (LPS) the authors had sustained access to three different secondary classrooms. For one classroom our research team was particularly struck by the apparently seamless flow of the lessons across the unit of 10 lessons—creating a sense of coherency. It is difficult to describe the feeling of watching these lessons—for us as observers there was sense that learning was happening in a continuous fashion—that is, the learning trajectory seemed to evolve continuously rather than in discrete units defined by specific lessons objectives. The mathematics lessons of this classroom, we felt, could usefully be analysed using an instructional coherence lens. We were concerned to discern those key elements of the teacher's pedagogical practice and knowledge base that determined the observed instructional coherence across the sequence of lessons.

Case study context

The case comprises a unit of lessons from a New Zealand Year 9 (Grade 8) classroom in a large coeducational urban school, catering for students from, in the main, the middle socioeconomic sector. The classroom teacher, Dave, with 4 years experience, was identified by the local mathematics community as an effective practitioner. His class of 30 students was one of two extension classes at the Year 9 level in the school.

In an interview following the sequence of lessons, Dave explained his teaching goals for the unit as twofold: (i) for students to be able to solve and understand linear equations of the form ax ± b = cx ± d; and (ii) for students to develop an understanding of the meaning of equality (=). The sequence of lessons is summarised as follows:

- L1: Revisions of order of operations and algebraic notation and manipulation.
- L2: Solving 1-step linear equations of the form x ± b = c [using a working backwards model].
- L3: Solving linear equations of form ax ± b = c.
- L4: Solving linear equations of form ax ± b = c using function boxes.
- L5: Solving linear equations of form ax ± b = c [introduced fractions, decimals].
- L6: Real world applications of solving linear equations.
- L7: Review of definition of equations and refocus on the meaning of the equal sign, introduction of balance model to solve 3x + 4 = 2x + 9.
- L8: Use of balance model to solve ax ± b = cx ± d.
- L9: Solving equations of form ax ± b = cx ± d. Introduction to systems of equations with infinite or null solution sets.
- L10: Real world applications of forming and solving equations.

Data collection and analysis

The teacher and his students agreed that our team could collect video, interview, and observe across a sequence of 10 lessons that represented a unit on algebra—focused on solving linear equations. Video capture involved three cameras: focused on the teacher, a group of students, and the whole class. Post lesson video-stimulated recall interviews involving the teacher and students generated further data. Triangulation of the video and interview data was enhanced by reference to researcher classroom observation notes, photocopies of written work by the focus students, photocopies of class activities, and teacher questionnaire data (for a description of LPS research design see Clarke, 2006).

We adopted a three-pronged analysis of instructional coherence. Firstly, we examined coherence across the intended and enacted curriculum across the lesson sequence. The analysis of lesson content and specified teacher objectives for lessons was the main data source. Then we tracked the connections of mathematical knowledge within and across the lessons, looking closely at the nature of tasks and at links between previous learning/knowledges and new knowledge construction. Lastly, we used the post-lesson student interviews (two for each lesson)—focused on students' perspectives of their learning and the teaching process—to examine coherence in terms of zones of proximal development (Vygotsky, 1986).

Establishing instructional coherence

As we have seen in the literature review instructional coherence involves a combination of factors related to curriculum sequence, making connections within and between topics, creating clear organisational patterns and establishing social and mathematical norms, attending to and building on students' existing understandings and knowledge, to name a few. But what is not so clear is what specific pedagogical actions a teacher might take within a lesson sequence to ensure that coherence is developed and maintained. In this section we discuss two distinct pedagogical strategies that we claim significantly contributed to the observed instructional coherence: (1) springboard tasks, and (2) sowing the seeds. We provide excerpts from three lessons to demonstrate how these instructional approaches helped lead students to develop effective mathematical practices and sound mathematical understandings.

Episode 1: Introducing methods for solving linear equations

As was typical, Dave started lesson one (L01) with a set of student problems (see Figure 1). Some problems required students to access previous knowledge in the form of consolidation/practice tasks; others required students to use existing knowledge to move towards new knowledge. The intention of the latter task was that these would act as a springboard for new learning.

1. 5 x 14 - 9 2. 13 + 2 x 17 3. 3 + 8 x 7 - 10	4. $5 + 3^2$ 5. 111 - (17 x 3) 6. $(9 + 2)^2 + (3 + 2^2)$	7. 120 ÷ 12 x 5 8. 3 + (21 - 6 x 3) 9. 2.4 + _ x 2 = 5.8

Figure 1. L01 task.

Dave's instruction to the students that they should show their working reaffirmed the shared mathematical obligation (Cobb, Gresalfi, & Hodge, 2009) that was evident within the classroom environment:

> So all I want to do with this lesson is do a little bit of revision of the work that is going to be crucial to your understanding of the next topic. … the answers aren't necessarily the most important thing, but the process of how we get the answer is really important.

While students were engaged in this activity, Dave walked around the class providing individual assistance. Updating on progress, Dave remarked to the whole class: "I don't expect everyone to finish number nine. ...But if you have had a go at number nine we will get some answers of those."

In the whole-class discussion requests for solutions to problem 9 resulted in two alternative approaches. The first approach offered was: "I minused 2.4 from 5.8 and then what was left I divided by 2. Another student demonstrated his solution as follows: "2.4 plus 0.5 because 2.9 is half of 5.8." Dave's invitation for students to offer reasons why they might have two solutions, and whether or not both were correct, prompted students to refer back to the BEDMAS rules of operations. In summarizing their contribution Dave remarked:

> Okay, so another reason to be careful of BEDMAS is even if we don't know what this is, it is a number that I have smudged. But it is still a number and normal rules work—times before plus.

Utilising the student contributions, Dave drew attention to the new idea of 'working backwards' remarking that, "We are working backwards. Why did he undo the plus first when BEDMAS says do times first?" He provided justification for the method that would be revisited in Lesson 2 in the context of solving one-step linear equations:

In the post-lesson interviews one student confirmed the expectation they have a go at the springboard problems:

> Pat: Question 9 was easy to understand. I just subtracted the 2.4 and went on from there.
> I: What made you subtract?
> Pat: I reversed what I would usually do and it just worked out.

I: So is the reversing thing not a foreign thing for you to do in maths even though you haven't really been formally taught?

Pat: It was quite unusual but I just automatically tried to reverse it and see how it would work out, and it worked out.

However, the second student, Vanessa, reported that she was unable to work out how to do number nine in the first instance. In the post-lesson teacher interview it was apparent that the choice of task was planned to support on-going learning linked to previous learning. Dave offered a metaphor of "sowing the seed" as follows:

> I just really wanted to make sure they were comfortable with that before the second day which was the start of solving equations. … so I sowed the seed in the first lesson with the idea of working backwards of solving an equation.

Episode Two: Introducing a balance model

The second lesson (L02) began with a discussion of the strategies for solving a set of puzzles that had been set as homework in L01. One of the puzzles is shown in Figure 2.

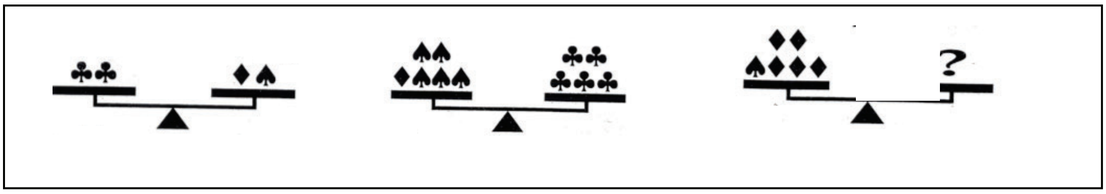


Figure 2. The starter learning task in L02.

Early in the discussion it became apparent that some students had used a trial and error method to balance the scales. After further discussion the teacher drew the students' attention to the strategy of keeping the scales balanced:

> What I want you to be thinking about is the strategy of keeping the scales balanced. That is how Henry and Jack and some of you others managed to work it out. And that is a theme that we are going to revisit over the next few

In the post-lesson interview Dave indicated that the use of the balanced scale puzzles was a precursor to extending their understanding of the meaning of the equal sign:

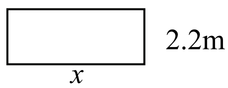
> Later on they are going to start doing complicated equations and they are going to need to understand that the equal sign doesn't just mean calculate and up until now most of them think the equal sign means 'works out to be' or 'I get this'. .. I am going to have to adjust their view of what the equal sign means and think of it in terms of balance scale.

The post-lesson interview with the two students Pat and Ruth confirmed their struggle with the puzzle activity. Sowing the seeds early in the sequence of lessons possibly was an indication that the teacher had a strong sense of his students' need to revisit these ideas over an extended sequence of lessons. The idea of the balance model to solve equations was not formally introduced until lesson 7.

Episode three: Using equations in solving practical problems

The fifth lesson began with students working on a problem set that included two challenge problems:

> Challenge 1: The perimeter of this rectangle is 15m.
> Form and solve and equation to calculate the length of this rectangle.



Challenge 2: A rectangle has an area of $72m^2$. Its length is twice its width. Calculate the perimeter.

After a brief review of the first four problems, Dave invited one student Charley to present his thoughts about the fifth problem. While Dave accepted Charley's solution method, he pressed his students to apply their current learning to the challenge problem:

Charley: I did 2.2 times 2 is 4.4 and then I did 15 minus 4.4 which is equal to 10.6 so then I divided it by 2 and got 5.3.
Dave: What does 5.3 represent?
Charley: The length.
Dave: The length. Who agrees that that is the length? Anyone like to suggest another method that uses that x? Henry.
Henry: I did 2x plus 2.2 times 2. Like the way it is up there and so I worked backwards.

Even though Henry tried to use x to solve the unknown Dave engages in further probing aimed to highlight the formation of an equation:

Dave: Where did you get your 2x from?
Henry: The fact that x equals the two lengths.
Dave: Right, opposite sides of a rectangle have the same length. If that is x, that's x, and that's 2.2, and this might be 2.2. What does perimeter mean?
Henry: All of the sides added together.
Dave: And what does all of that [$2x + 2.2 \times 2$] equal if we add them all together?
Henry: I don't know.
Dave: Have we been given more information in that question?

The students' preference to use pre or partial algebraic process to solve the unknown was again observed in the discussion of the sixth problem. Here, Dave required a student to explain his solution of $6 + 6 + 12 + 12 = 36$ to the sixth problem as follows:

Dave: So how did you get that one must be twelve and one must be six?
S: Because it said that the length must have been twice the width.
Dave: Good so did you use the x at all?
S: No, I didn't.
Dave: Okay, that is a really good method the guess and check … I would like if you have had a go at this to try and form an equation like we did for the last one and solve that equation to get this answer which is the perimeter.

The intent to introduce a new method with this springboard task was confirmed by the teacher in the post-lesson interview:

> The main goal was partly to cement their ideas of how to solve equations really and also to introduce the idea of using equations to solve problems. …I deliberately try and push them as far as I could today. …But I don't mind doing that because when I go back on Monday …I am going to give them exercises out of [text] and after what I have done with them today they are hopefully going to find it straightforward.

Dave's pedagogical decision to return to the problem was informed by his knowledge about his students' learning potential and current understandings:

> I felt at this stage if I take it any further I was going to lose some of them, I was hoping someone would come with ... we choose 72 so that when they divide by 2 they are going to get 36 and they would be able to spot that if 'x square' is 36 they will spot 6 they wouldn't have necessarily gone onto explanation on how we got that 6. So I was hoping someone would have come up with that equation in which case I would have followed it through. But I felt at this stage the only solution I got were the guess and check points, so

> I thought I would sow the seed that there's an algebra method there but I felt if I was going to carry on too far I was going to lose too many of them.

Discussion and implications

In the foregoing section we provided three episodes to elucidate the coherence of Dave's instruction. These episodes were selected to exemplify two instructional approaches that were regularly observed in the 10 lesson sequence: one concerns the nature and enactment of the tasks; the other is the teacher's action of "sowing seeds".

The teacher regularly posed a set of tasks for students to work on prior to new instruction. Typically, the first a few problems acted as revision or consolidation activities, while the last few problems were challenging. These 'springboard' tasks generated new ideas that were central to learning goals that were more fully developed later in the lesson, and or revisited in subsequent lessons. We claim that this instructional approach supported coherence as characterised by the connections of students' existing (and prior) knowledge to new knowledge. The success of the linking was supported by the opportunities for student to work independently on the problem set prior to the more formal introduction of new knowledge and the subsequent obligation for students to explain and justify their thinking through class discussion.

In our exploration of Dave's metaphor 'sowing the seeds', the 'seeds' correspond to the multiple layers of new knowledge and methods embedded in the intended curriculum. It appears that Dave sowed seeds with a careful consideration of the distance between the actual and potential development of individual students. These seeds assisted Dave to plan a logical sequence of knowledge construction that builds and links to students' existing and emergent ideas. Such orientation to and anticipate of further learning, we claim, is a hallmark of coherence.

In combination, sowing seeds and the use of springboard tasks supported the provision of appropriate challenge for students and affirmed the expectation that struggling with the task was the norm and that not all students would be immediately successful. We hypothesize that Dave's expectation that students needed to experience struggle as a natural way of learning was indicative of his awareness of the need to establish appropriate zones of proximal development for his students. As Dave reflected in a post-lesson interview: "I deliberately try and push them as far as I could today (to introduce the idea of using equations to solve problems)". The press for students to engage and resolve springboard tasks, combined with the sowing of seeds, appeared to be a conscious way the teacher scaffolded students within their zone of proximal development. However, an overriding question remains concerning the reasons for the implementation and effectiveness of the two instructional approaches: How effective would these strategies be in other classes, especially those classes which contain mixed or predominantly low-achievement levels?

This analysis is limited in that it offers an insight in to one classroom. But as noted earlier, this classroom stood out for us in terms of the seamless, almost invisible, way that learning and teaching appeared to be structured. Attending to features of coherence, a concept that is more typically applied to Asian classrooms studies, has provided a useful lens to look at how this teacher promoted mathematical learning.

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