Effect of the Different Syntactic Structures of English and Chinese in Simple Algebraic Problems

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This paper compares the performance of first-language Chinese and English secondary students in Hong Kong on a set of items requiring the students to express simple relationships algebraically. It is suggested that the different *types* of response produced arises from the distinct syntactic structure of a given expression when translated from English into Chinese. The results are discussed in terms of the construction of mental models and evidence is also considered from interviews conducted with a small sample of the students.

Background

The analysis reported in this paper arose unexpectedly from the findings of a research study on the causes of the 'reversal' error in algebra (Lopez-Real, 1995). The nature of the reversal error and its possible causes have been well documented (Clement, 1982; Davis, 1984; Mestre, 1988; MacGregor, 1991) and two particular cases can be identified. These may be described as the 'multiplicative' case and the 'additive' case. An example of the former is the problem:

> "In a college there are six times as many students as professors. If the number of students is S and the number of professors is P, write down an equation that describes this relation"

One correct solution is the equation S = 6P, whereas the equation 6S = P is typical of the reversal error. An example of the additive case is the problem:

> "In a classroom there are 6 more girls than boys. If the number of girls is G and the number of boys is B, write down an equation that describes this relation"

Again, a possible correct solution is G = B+6 and the typical reversal error is G+6 = B. The purpose of the study was to investigate whether different syntactic structures affected the facility levels of a range of items. For example, the effect of 'contiguity' was tested with the following pair of items (where the pupils were asked to write an equation for the relation between p and q):

(i) p and q are numbers. p is 6 more than q.
(ii) p and q are numbers. 6 more than q is the same as p.

In item (ii) the phrase '6 more than' has been contiguously linked with the q rather than the p, as in item (i). The study showed that the performance of students on such paired items differed at a statistically significant level. The occurrence of the reversal error was also significantly reduced by using different syntactic structures. Johnson-Laird (1983) suggested that human comprehension and reasoning is based on the construction of 'mental models' that are independent of the syntax of the proposition being processed. MacGregor (1993) supported this theory of semantic models with reference to children's comprehension of algebraic relations. The results of the Hong Kong study suggested that no clear dichotomy of semantics and syntax is evident but rather that the mental models constructed by children are likely to be a combination of semantic and syntactic processing.

The above study was effectively concerned with analysing the *errors* made by children, particularly the reversal error. With respect to *correct* responses, the focus was purely on determining the facility levels of the items and hence the *nature* of these correct responses was not attended to. However, during the recording of the data, some striking patterns in the correct responses to item (i) above, became very evident. Clearly there are a variety of possible correct answers to this item and these can be classified into three types: a) $p = q + 6$ (and variations such as $p = 6 + q$; $q + 6 = p$)
b) $p - q = 6$
c) $p - 6 = q$

Although these equations are mathematically equivalent, the syntax of the question strongly suggests that the first category is far more likely to occur and that the expressions in (b) and (c) would occur rather infrequently. These three categories may be contracted into two classes which could be described as a 'sum-model' and a 'difference-model' of the solution. The impression while recording the data was that the difference-models were appearing almost as frequently as the sum-model. Since this was contrary to intuitive expectation, it was decided to obtain precise quantitative values for the occurrence of the difference-models in similar items of the test.

Difference-Model Responses

Apart from the items previously described, there were two other items on the test that were designed to examine the occurrence of the reversal error in the 'additive' case. For convenience, the three items are listed as Q1, Q2 and Q3 below, although they were numbered differently on the test papers.

Q1: p and q are numbers. p is 6 more than q.
Write down an equation that describes the relation between p and q.

Q2: In a classroom there are 6 more girls than boys. If there are N boys and M girls, write down an equation that describes the relation between N and M.

Q3: The following table shows the weight of a suitcase packed with different amounts of clothing:

Wt of clothing (in kg)	8	10	12	14	
Wt of suitcase *with* clothing (in kg)		13	15	17	19

If the weight of the clothing is Q kg and the weight of the suitcase *with* clothing is P kg, write down an equation that describes the relation between Q and P.

A total of 221 Form 2 secondary pupils answered these three items. The facility rates were Q1 (80%), Q2 (64%) and Q3 (61%). The breakdown of correct responses is shown in Table 1.

***Table 1**: Frequency of Alternative Correct Responses*

Item	Response	Frequency	%
Q1	$p = q + 6$	94	53
	$p - q = 6$	21	12
	$p - 6 = q$	61	35
Q2	$M = N + 6$	100	70
	$M - N = 6$	5	4
	$M - 6 = N$	37	26
Q3	$P = Q + 5$	112	84
	$P - Q = 5$	11	8
	$P - 5 = Q$	11	8

The combined figures (as a percentage of the correct answers) for the difference-model responses on each question are thus Q1 (47%), Q2 (30%) and Q3 (16%). There are large variations here but clearly the figure for Q1 confirmed the impression, during the original recording, that this type of response was almost as frequent as the sum-model response.

Another aspect of interest that was easily obtained from the data was to consider the *total* number of sum-model and difference-model responses and to identify what percentage of these responses were correct. In other words, given that a student tried to use a sum-model description say, what was the likelihood that this produced a correct answer? The results are shown in Table 2 (where S refers to Sum-model responses and D to Difference-model responses).

***Table 2**: Comparison of Sum-model (S) and Difference-model (D) Responses*

Item	Total S	Correct S	%	Total D	Correct D	%
Q1	121	94	78	85	82	96
Q2	151	100	66	48	42	87.5
Q3	137	112	82	26	22	85

For Q1 and Q2, the differences in the proportions of correct responses, given a particular model, are significant at the 1% level. That is, where a student expressed the answer using a difference-model there was a significantly higher probability that the answer would be correct than where a sum-model was used. However, there was no statistically significant difference for Q3.

The results shown in Tables 1 and 2 raise a number of interesting questions. For example, from Table 2, why does the difference-model response produce such a high success rate for Q1 and Q2, compared to the sum-model? And from Table 1, we may first ask why the difference-model response varies so much for the three items and second, are these rates indeed unusually high and, if so, why?

The reason for asking the last question is perhaps better understood when one is reminded of the sample of students being tested. These students are all first-language Chinese speakers being taught in English-medium secondary schools in Hong Kong. Such schools are 'Banded' according to performance levels on entry from primary school and the schools used in this study were Band 1 to 3 (i.e. above-average to average). Hence, although the standard of English of these students is good (and their mathematics textbooks are also in English) nevertheless, English is still their second language. It is therefore quite possible that many students will mentally translate a word problem into Chinese before they try to solve it. To take Q1 as an illustration, the relevant translation is shown below. (A dozen colleagues were independently asked to write down a translation and, with very minor variations, all produced essentially the same form. In particular, the p,q,6 sequence was consistent throughout.)

p is 6 more than q
(Insert Chinese translation here)

Translating back into English on a literal character-by-character basis, this reads: "p compared to q is larger by 6". As can be seen, the syntactic structure of the sentence is now quite different to the original English. The crucial phrase in Q2 (there are 6 more girls than boys) when translated, also gives rise to precisely this same structure. That is, "the number of girls compared to the number of boys is larger by 6". In Q3, since the relationship is 'embedded' within the table, there is no comparable phrase to be translated. Given this form of the sentence for Q1 and Q2, it now appears intuitively more likely that the difference-model may be used when solving the problems. In order to investigate this hypothesis, it was decided to conduct the same test with a sample of first-language English speakers. In the following sections, first-language Chinese or English speakers are designated as C1 and E1 respectively.

Comparison of C1 and E1 Responses

The test containing the three items discussed above was administered to 84 pupils at an international school. All the pupils tested claimed that English was their first language of communication although many were bi-lingual. (In fact, the sample included some ethnic Chinese pupils too). The focus of the analysis below is on the number of correct difference-model responses as a proportion of the total number of correct responses. The comparison between C1 and E1 students for the three items is shown in Table 3. (As before, D refers to Difference-models).

Table 3: *Comparison of Correct Difference-model Responses for C1 and E1 Students*

Pupils	Q1			Q2			Q3		
	Total Correct	Correct D	%	Total Correct	Correct D	%	Total Correct	Correct D	%
C1	176	82	47	142	42	30	134	22	16
E1	53	6	11	48	5	10	58	5	9

Here also, there is a very significant difference (at the 1% level) between the proportion of C1 students successfully using a difference-model compared to the E1 students for Q1 and Q2. There is no statistically significant difference for Q3. (These significant differences also hold true when the *total* number of difference-models used is compared; not just the successful cases). The fact that no translation effect would logically apply for Q3 lends strong support for the hypothesis that this is indeed the cause of the high occurrence of the difference-models in Q1 and Q2. It appears that the syntactic structure of the Chinese form of the questions emphasises first the *comparison* between the two numbers (p,q or M,N) and then identifies the amount by which they differ. This in turn leads to the difference-model being constructed rather than the sum-model. However, this does not imply that it is simply a *sequential* syntactic process that is being used. If this were so, one would expect answers of the form $p - q = 6$ and $M - N = 6$ to be the more common difference-model expressions. On the contrary, as can be seen from Table 1, the forms $p - 6 = q$ and $M - 6 = N$ were far more prevalent. This indicates that the pupils are constructing a particular mental model (in this case the difference-model) based on a combination of semantic and syntactic information in the problem. This also reinforces the conclusions drawn from the analysis of errors in the original study (Lopez-Real, 1995). In order to test further this suggested reason for the high occurrence of the difference-model, it was decided to interview a small sample of C1 and E1 students.

Interviews

The interviews focussed in particular on Q1 of the test paper. Since there was at least a three month time lapse between the original testing and these interviews, students were first asked to solve the problem again, rather than simply being shown their original solution. Ten C1 and five E1 students were selected for interview, all of whom had used a difference-model expression for Q1 on their test paper. The main purpose of the interviews was i) to try to identify the thought processes that led to the construction of this model, ii) to see whether any visual images played a part in the process, and iii) for the C1 students only, to see whether any translation process had been involved. It is notoriously difficult to analyse one's own mental processes, especially in a situation where there is ostensibly just a single step from problem to solution, and this is illustrated in some of the following examples. Nevertheless, a number of students *were* able to articulate their thinking and such cases can be very illuminating.

For the C1 students, all ten again wrote correct responses although three of them were different to their original answers. In two of these cases the response was now the sum-model expression $q+6=p$, while in the third case the student still gave a difference-model response, albeit an alternative model to the original (i.e. $p-q=6$ instead of $p-6=q$). The first thing this illustrates is that the construction of a mental model is not necessarily stable from one occasion to another. Neither of the two students giving the sum-model were able to explain their thinking. As one of them put it: "That's how it came in my head. I don't know how." Both of them claimed that they did *not* translate the problem into Chinese (e.g. "I can answer in English. I don't need to translate").

Of the 8 students who gave a difference-model response, 6 of them said that they *did* translate while the remaining two were unsure. (Being bi-lingual, the author can sympathise how difficult it is to know whether or not one has translated a given sentence). The following exchange illustrates this. (In the extracts below, I stands for Interviewer and Sn designates one of the students.)

I: Do you translate the problems into Chinese in your head?
S5: Sometimes I do. Um ... yes, I think so.
I: Did you translate *this* problem?
S5: I'm not sure. I don't know. I think I don't have many time.

In their attempts to describe how they arrived at their answer, two of the students (S2 and S8) were unable to give any explanation. Student S7 wrote p-q=6 and explained her thinking as follows: "Because I think .... the equation .. normally the number is here (*pointing to the right-hand side*) and p must be bigger, so it must be p minus q equals six". The striking feature about the remaining five students was that they all began with some reference to the relative size of p and q. Two examples are illustrated below. (The mixture of slightly ungrammatical English followed by a precise mathematical description, as in S4's answer, was very typical of student explanations.)

S4: First of all I know that p is bigger than q. And p is more .... uh ..is more six than q. Therefore p minus q is equal to six.

S9: Well, p and q .... p is bigger by 6. That means I have p ... I must go down six. Then it equals q. (She had written p-6=q)

Naturally, the E1 students were able to express themselves better but the problem of articulating one's *thinking* was still evident and one student was completely unable to do so: "I don't know how I did it. It's just how it came". As with the C1 cases, two of the students emphasised the initial comparison of size. For example:

S11: First it said that p is 6 more than q, so I started to think p is larger than q and it's 6 more so then I wrote p-6=q.

Student S14 gave the most explicit description involving a subtraction construct. (His written answer was p-q=6).

S14: I don't know ..... whenever I .... like .... think of subtraction, I always think of the larger number take away the smaller number.
I: What made you think of subtraction?
S14: Because when I think of left-overs I think of subtraction.
I: And what made you think of left-overs?
S14: I'm not sure .... because there were six more left over.

As mentioned earlier, the students were also asked whether they used any mental images to help them. This question arose naturally with S15 in the course of her attempt to describe her solution process for p-q=6:

S15: Because I think .... I'm not very sure actually. I think it's because I imagined like p, 6 and q and arranged them in my mind.
I: I'm not sure what you mean by 'arranged them in your mind'.
S15: Oh .... OK. Um ... I'm not sure.
I: It sounds as if you have some kind of picture in your mind.
S15: No .... I don't think so.
I: Have you ever used the number line in maths?
S15: Yes ... but because when I was learning algebra I didn't really learn to use it. I didn't use it here.

In fact, not a single student thought they had any visual image in their mind even when their description, as with S15 above, suggested there might be. Another example is the phrase used by S9 (quoted earlier) when she says "I must go down to six". Later in the same interview she was asked to solve the following related problem: "b and c are numbers. b is 4 less than c. Write down an equation that describes the relation between b and c". (As before, the literal Chinese translation is "b compared to c is smaller by 4"). Her solution was given as "b + 4 = c" and she described her thinking process thus: "First I know that b is less than c. It's 4 less. So I start at b and I must go up 4 then I get c". Her up/down metaphors suggest a strong visual image but she too, claimed to have no such picture in the mind. (The ubiquity of Up/Down metaphors in language generally is well illustrated by Lakoff & Johnson, 1980). All ten C1 students were asked this problem. Four of them gave the answer above, two wrote 'c – b = 4' and the other three correct answers were 'b = c – 4'. (One student wrote the typical 'reversal' error b-4=c).

When the students were asked to draw a 'picture' of their "p is 6 more than q" solution, even if they were not conscious of using one, the prevalent image was of two discrete sets of objects with one set having 6 more elements than the other. Perhaps unsurprisingly, a number of these pictures represented the objects as fruit! It was difficult not to avoid the impression that these were retrospective attempts to represent their solutions pictorially and played no part in their mental constructions. A number of the students were explicitly asked if they ever used the number line in mathematics and whether this could be a suitable way to picture the relationship. The responses clearly suggested that they associated the number line only with specific topics in mathematics and did not see it as a useful generic image. For example: "I only think of the number line when we do negative numbers".

Discussion

The very significant differences between the C1 and E1 students in constructing difference-model expressions for their algebraic solutions strongly suggests that a 'translation effect' is one of the causal factors. In particular, this is supported by the fact that *no* significant difference was found for Q3 which was effectively 'translation-independent' (the relation being represented in tabular form). Since the syntactic structure of the English and Chinese sentences have quite distinct patterns, this also suggests that the syntax itself plays a part in determining the mental model constructed. For example, the emphasis on first *comparing* p and q (in the Chinese version) may make it more likely that a difference-model is constructed. The interview evidence also supports this view. The terms 'sum-model' and 'difference-model' have been used in this paper as a convenient descriptive shorthand. However, it would perhaps be more accurate to use a phrase such as 'sum-expression' since what is being identified here is the symbolic representation of the relationship, rather than the mental model itself. Indeed, it is probably impossible to determine precisely what is the mental model constructed by an individual. All that can be determined with certainty is whether or not the symbolic representation correctly describes the relationship. As mentioned in the previous section, the C1 students were also asked during the interview to describe the situation 'b is 4 less than c'. Given the syntax of the question, the 'expected' solution might be 'b=c-4' and the other responses may again arise from an initial comparison of b and c initiated by a translation of the question. Five out of the six students giving the 'alternative' expressions claimed to be 'translators'.

Whatever the precise nature of constructing a mental model, the argument in this paper is that an interaction of semantic and syntactic processing is involved. (And in the written solution, a further syntactic process must occur in order to express the relationship symbolically). The suggestion from the data is that the different syntax of the Chinese translation has directly influenced the type of description in the symbolic representation. When translation does occur this implies *interpreting* the initial statement and producing an alternative way of expressing that statement. It is this kind of flexibility (i.e. alternative descriptions) that may lie at the heart of constructing a suitable mental model. This is similar to the idea of re-writing a problem in one's own words as a useful problem-solving strategy (Charles & Lester, 1984). Clearly, for most pupils, who are working in only one language, the translation effect is not a relevant factor. However, simply re-expressing a statement in the same language is a comparable process. As Haylock & Cockburn (1989) point out, with reference to early mathematical experiences, comparison problems are more often presented using a 'greater than' expression as opposed to a 'less than' expression, and hence children need the experience of re-writing such phrases using the equivalent alternative form. (Interestingly, they also point out that there are more words in English for describing 'greater' qualities than there are for 'smaller'). Flexible approaches to interpreting a given statement might also involve using some form of visual imagery and this was the reason for examining this in the interviews. Given the ubiquity of the number line image in many mathematics textbooks, it is surprising to find (at least, from the interview evidence) that it is not transferred to the context of algebraic relationships as tested in this study. It would seem to be a particularly potent image for this situation. This is not to say that it is the *only* useful visual image available but it appears that *no* imagery (at least, consciously) was used by the students.

Finally, as a teaching strategy, it is suggested that whenever pupils need to transform a word statement into a symbolic mathematical statement, they should be encouraged first to try re-stating the word expression in an alternative form. The very act of performing such a re-formulation may in itself help in the construction of the appropriate mental model.

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