Area integration rules

Gillian Kidman
Centre for Mathematics and Science Education,
Queensland University of Technology, Kelvin Grove, Australia

This paper investigates the use of area integration rules. 36 children in Grades 4, 6 and 8 were given area judgement tasks, using rectangles of varying areas and perimeters. Information Integration Theory procedures revealed both additive and multiplicative judgement rules that determined the children's responses. It was found that judgement rules change intra-individually but there does not appear to be a relation between judgement rule and Grade level. The form of presentation of the rectangle was found to be important.

Although area is one of the essential concepts of mathematics instruction, it is a concept that textbooks make little attempt to define, and many (for example, Blane & Booth, 1989) discuss area with the apparent assumption that students already understand it. Textbooks aimed at the Year 8 level of schooling in Queensland appear to limit their approach to the concept of area to two ways: a combination of basic pre-formula exercises, statements of formulae and exercises using the formulae (e.g., Blane & Booth, 1989; Duffy & Murty, 1988); and directly into statements of formulae and exercises using the formulae (e.g., Clark, Clark, Burza & Conway, 1988; Priddle & Davies, 1989).

This apparent lack of definition and emphasis on formulae seems to be attributing, in part, to well documented misconceptions of the concepts of area in both primary and secondary school aged children (Kidman & Cooper, 1996a; Outhred & Mitchelmore, 1996; Clements & Ellerton, 1995; Bell, Costello & Kuchemann, 1983; Bell, Hughes, & Rogers, 1975). Research (e.g. Hirstein, 1981; Hirstein, Lamb and Osborne, 1978) has shown that one of the major misconceptions is confusion between area and perimeter. In particular, as Kidman and Cooper (1996b) found, students' have difficulty with the process of obtaining a shapes' measurements, determining which dimensions to consider and how to integrate these dimensions when calculating either area or perimeter. Research on student teachers (Baturo & Nason, 1996; Simon & Blume, 1994) has revealed possible inadequate knowledge of the concept of area. The student teachers are able to apply procedural formulae, but they confuse area and perimeter and use linear rather than square units. They show very little conceptual understanding of the relationship between area and side length.

As argued by Kidman & Cooper (1996a) and Wolf (1995), Information Integration Theory (IIT), a functional measurement technique (Anderson & Cuneo, 1978) used to identify the rules applied by children to integrate dimension information, and cognitive algebra offers an excellent opportunity to explain the process of area concept development in children. According to IIT, "all behaviours reflect a blend of stimuli, and a response is the consolidated resultant of multiple causal forces" (Kidman & Cooper, 1996a, p. 340). The methodological counterpart of IIT, called functional measurement, allows diagnosis in simple algebraic terms, "... of the rules which govern integration of information about perceived stimuli." (Wolf, 1995, p. 49-50).

At some stage between the age of 5 and 12, a child is expected to make the transition from an additive integration rule to the normative multiplicative integration rule (Kidman & Cooper, 1996a; Wolf, 1995; Schlottman & Anderson, 1994; Lautrey, Mullet & Paques, 1989; and Silverman & Paskewitz, 1988). The general consensus of these and many other recent studies is that students' judgements of area obeyed two-dimensional rules. It appears that 8 year old students were in a transitional stage between the additive and multiplicative rules. In general, students have been provided with different rectangular shapes and asked to place their area on a linear scale.

This paper describes an investigation to determine the judgement rules used by students in Grades 4, 6 and 8 in a private college in Queensland and reports on student responses to experiments to explore how the students integrated length and width dimensions to judge area of rectangular or near rectangular shapes. The purpose of the investigation was to:

- identify the way in which children integrate stimuli to determine area; and
- determine if integration rules change intra-individually.

The experiments was based on the body of literature and the functional measurement methodology stemming from the work of Anderson and Cuneo (1978).

The study

The study used a multi-method design where the quantitative methodology of functional measurement was combined with the qualitative methodology of semi-structured clinical interview.

Participants. The sample consisted of 36 students, 12 students from each of the three grade levels, and a range of mathematical abilities, one third each of below average, average, and above average, from each grade.

Instruments. The instruments used were three experiments and an interview. The first experiment contained 16 rectangular wooden pieces (Fig. 1a) painted to represent chocolate and with dimensions corresponding to the factorial combinations of 3, 6, 9, and 12 cm. The pieces were to be presented to students who would be asked to judge the area of the rectangular pieces in relation to two end anchors. To obtain measures of the children's area judgements, the children were provided with a 19 point scale with two end points. Two special pieces of dimensions 2.7 x 2.7 cm, and 15.8 x 15.8 cm were used as end anchors.

The second experiment used 16 rectangular pieces identical in dimensions to the first experiment, but with a rectangular corner 'bitten' off producing a figure of equal perimeter, but less area (fig. 1b). The dimensions of the 'bitten' off corner were all one third of the width and one third of the height of the rectangular stimulus. The third experiment again used 16 rectangular pieces identical to the first experiment, but this time they had a semi-circular 'bite' out of one side producing a figure with less area but greater perimeter (fig. 1c). The 'bite' was centred along one dimension with the radius of the 'bite' one third of the length of the shortest dimension.

The interview was short and semi-structured and asked each student to describe the method they used to rate each piece. They were quizzed as to whether they were aware of any changes they had made to their method over the course of the three interviews. Diagrams of identified methods were sought, from the students. At the conclusion of the interview, the students understanding of both area and perimeter was discussed, and the student was then asked to identify if he/she had employed either or both of these concepts to rate the chocolate pieces.

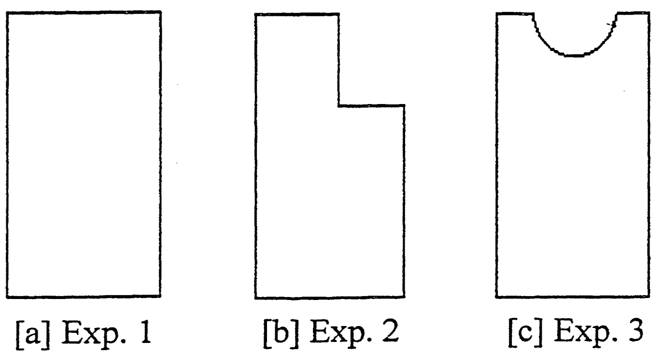

Figure 1. Experimental chocolate shapes

Procedure. The students were withdrawn from their class and the three experiments and the interview were administered in a separate room. The experiments and the interview were videotaped. The diagrams were sought at any time during the experiment, and the interview followed the third experiment. It took no longer than 30 minutes.

For each experiment, the children were first familiarised with the end anchors which were presented as corresponding to the end points of the scale. The scale had a smiling face at one end and a frowning face at the other. The small end anchor was presented as a piece of chocolate that the child would be unhappy to receive while the large end anchor was presented as a piece the child would be happy to receive. The students were then asked to judge how happy someone would be to receive each of the 16 pieces if they were chocolate to eat. The pieces, each of equal thickness, were presented individually, and judgement was expressed on a 19-point response scale (see Anderson & Cuneo, 1978, for more details). The presentation of the chocolates was randomised, and a practice phase preceded the test phase. The children judged three replications of the chocolate stimuli in each experiment.

Analysis. The experiments were analysed using Anderson and Cuneo's (1978) functional measurement methodology. This methodology uses algebraic rules as the base and frame for psychological scaling. These rules provide the breakdown of the observed response into its functional components, as represented by the scale values and weights of the various pieces of information (Anderson & Cuneo, 1978). This is used to identify the kind of rules underlying the judgements students make when provided with different rectangular shapes and asked to judge their areas on a rating scale. The scale positions for the rectangles (which are specific combinations of height and width) are represented graphically and then subjected to an analysis of variance. Conclusions regarding the kind of rule underlying the judgements are determined from the shape of the graphical plot, and the significance or nonsignificance of the main and interaction effects (Anderson, 1981).

The graphical plot of the responses is against the length of one of the dimensions of the rectangles. Thus, if the plot is an arrangement of parallel lines or parallel curves, the students' judgements are considered to be additively based, that is, they are tending to perceive area of a rectangle in terms of the *sum* of its dimensions. If the plot is fan shaped (expanding lines or curves), the students' judgements are considered to be multiplicative, that is, the students are tending to see area of a rectangle in terms of the *product* of its dimensions. Figure 2 presents hypothetical curves for these rules. If the plot lines or curves intersect, then an inference with regard to additivity or multiplicativity may not be possible.

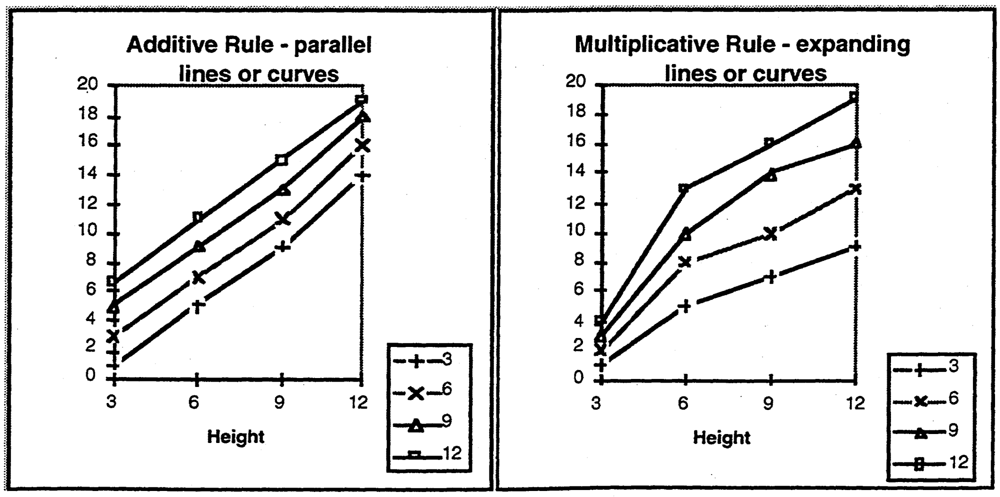

Figure 2. Hypothetical plots for additive and multiplicative based judgements

Factorial plots were drawn for each student for each experiment, as well as a group plot for each of the three grade levels. The plots were then compared to the hypothetical plots shown in Figure 2. On the basis of this comparison, additive or multiplicative integration rules were assigned to that student for that experiment.

The interviews were transcribed into protocols and the students' statements compared with their experiment results in an endeavour to provide a second option for explaining students' responses.

Student responses to the experiments

Understanding the instrument. All students appeared to understand the judgement they were required to make in terms of being happy or unhappy with pieces of chocolate in relation to the end anchors. They appeared able to express their judgement unambiguously using the 19 point scale. The understanding of the response scale was checked by having the student point to specific sections of the scale (for example, a section depicting a little bit of sadness), as well as making a verbal statement about the section being pointed to in relation to the size of the piece of chocolate (for example, "I would be a little bit sad").

The most important procedural detail concerns the establishment of the frame of reference. "The rating of any one stimulus is always relative to what other stimuli are being rated" (Anderson, 1980, p. 9). The standard device for setting up the frame of reference is the stimulus end anchors which are just noticeably more extreme, higher or lower, than the rectangular experimental stimuli. These end anchors define the stimulus range, and helps the student set up a frame of reference for using the rating scale. The end anchors also tie down the end responses so that the responses to the chocolate stimuli come from the interior of the scale thus avoiding end effects.

Each students' scale positions were analysed with the functional measurement methodology. Table 1 shows the averaged integration rules for the three grade levels (Grade 4, Grade 6 and Grade 8) and the three experiments (E1, E2 and E3). The symbol **X** is used to denote the multiplicative rule, and **+** the additive rule. **Total** gives the number of multiplicative and additive students in each grade.

Table 1 Grouped integration rules for area

	Grade 4			**Grade 6**			**Grade 8**		
	E1	**E2**	**E3**	**E1**	**E2**	**E3**	**E1**	**E2**	**E3**
Ave	X	+	X	X	X	X	+	X	X
Total	X = 4 + = 7 ? = 1	X = 6 + = 6	X = 7 + = 5	X = 6 + = 6	X = 8 + = 4	X = 5 + = 7	X = 6 + = 6	X = 7 + = 7	X = 5 + = 7

In the majority of cases the resulting plots were obviously additive with clear plots of parallel lines or curves, or multiplicative with clear plots of expanding curves or lines. In cases where the curves intersected (for example, the curve for a width of 6 cm crossed the curve for a width of 9 cm), the general shape of the plot was recorded, but a '**?**' was also recorded indicating a 'questionable' rule usage. It was not possible to determine a judgement rule for Ben, a Grade 4 student, doing Experiment 1. This particular plot had four intersecting locations and no obvious parallel curves or diverging lines.

The differences between the grades was not as obvious as could be expected. The perception of area of rectangle being related to the sum of the rectangles' dimensions is fairly consistent across the grades. The group of Grade 8 students tested do not seem to have advanced much beyond the Grade 4 or Grade 6 level. However, there were two interesting small changes. The first was the increase in multiplicativity in the Grade 4 results from Experiment 1 to Experiment 3. The second was the increase in multiplicativity from Experiment 1 to Experiment 2 across all Grades; and, except for the Grade 4 students, the decrease in multiplicativity from Experiments 1 and 2 to Experiment 3.

It is evident that judgement rules do change intra-individually. Ten students used an additive rule initially in experiment 1, but had altered this to a multiplicative rule by the conclusion of experiment 3. Surprisingly, 9 students did the reverse. They started using a multiplicative rule but changed to an additive rule in either the second or third experiment. Similar to the distribution of additivity and multiplicativity across the Grades, the changes in integration rule intra-individually within students) was also fairly constant across the grades with the number of students constant in their rule in each Grade remaining between 4 and 6 across the three Grades.

A preliminary analysis of the interview data reveals two interesting points. Firstly, of the ten children who had intra-individual rule changes from additive to multiplicative, seven showed a clear preference for a vertical alignment of the chocolate pieces rather than a horizontal alignment (See Figure 3a). Secondly, there was a strong tendency (88%) for children altering their judgement rule from multiplicative to additive to want to remove the part of the chocolate piece that had been altered (See Figure 3b).

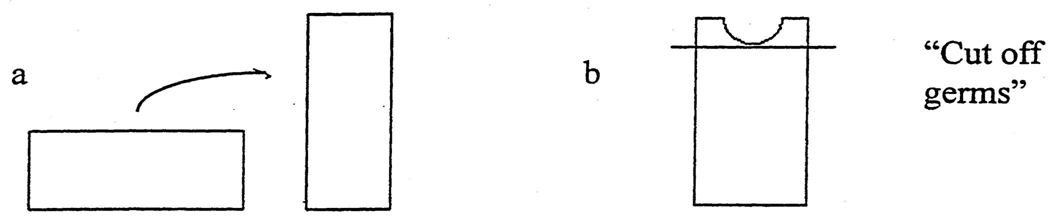

Figure 3. Alignment preferences and section removal

Discussion and conclusions

If the plot of the students' scale positions approximates parallel lines or curves, this reflects a perception of area where doubling the lengths of the sides of the rectangle can be seen as doubling the area. This reflects a perception of an additive relation between area of a rectangle and the dimensions of the rectangle. Thus, this perception can be considered as a confusion between area and perimeter for rectangles. This particular integration rule has been denoted as Area = Height + Width. By comparison, a plot which approximates a fan shape is reflecting a perception that doubling the sides more than doubles the area of the rectangle. This is seen as a correct perception and denoted as the Area = Height X Width integration rule. Plots with lines crossing indicate a very poor conception of area as this means that the student has judged a rectangle with smaller dimensions as having a larger area than a rectangle with larger dimensions.

This study has, therefore, supported the findings of Hirstein (1981) and Hirstein, Lamb and Osborne (1978) that there is confusion between area and perimeter. Around 50% of students from all Grades and in all experiments exhibited judgements that showed they were using the perimeter rule to determine area.

Experiments 2 and 3 presented modifications to the rectangles. The removal of a rectangular corner was found to reduce students use of the perimeter rule while the removal of a semicircular piece from an edge did not. The question is why? It could be argued that a rectangular piece out of a corner of a rectangle gives the effect of adding two more sides and thus the student tends to look at the amount of surface rather than the length and width. It could also be argued that the removal of the semicircular piece from a side has a lesser effect on how the rectangle is perceived than the removal of a corner, and that the addition of a semicircle to the factors that have to be taken into account in making area judgements adds weight to an additive focus on length. However, the reasons for students use of perimeter in the three experiments will have to wait until the interviews are fully analysed in relation to these experiments.

Over 50% of the students (22 out of 36) changed their integration rules across experiments. Once again the question is why? There appears to be no pattern in the changes: from additive to multiplicative, from multiplicative to additive, and sometimes in both directions. There appears to be no relation to Grade level.

The conversion to vertical alignment seems to be important to some children as it was adopted by those who made additive to multiplicative intra-individual rule changes. The children were allowed to handle the chocolate pieces, and they rotated pieces prior to making their judgements. One student explained that chocolate bars are stacked vertically on shop shelves, so you need them displayed here in the same way. Having rectangles represented in a familiar way may have some influence on a child's ability to correctly judge area.

The removal of a section from either the corner or side of a chocolate piece seems to confuse some children. A quarter of the children could correctly judge the area of rectangles, however after removing a section, they used an additive judgement rule. The interviews showed the majority of these children visualizing the chocolate pieces without the missing section, but shorter in one dimension. All such children claimed, either verbally or diagramatically, to cut off the altered section. One child explained it was to get rid of the germs where someone took a bite out of it. Why the imaginary act of cutting a piece down to size causes a rule change is not yet evident.

The author acknowledges the assistance of supervisor Dr Tom Cooper in the preparation of this paper.

References

Becker, & T. Trabasso (Eds.), *Information integration by children* (pp.1-45). Hillsdale, New Jersey: Lawrence Erlbaum.

Anderson, N.H. (1980). Information integration in developmental psychology. In F. Wilkening, J. Anderson , N.H. (1981). *Foundations of information integration theory.* New York: Academic Press.

Anderson, N.H., & Cuneo, D.O. (1978). The height + width rule in children's judgements of quantity. *Journal of Experimental Psychology: General, 107* (4), 335-378.

Baturo, A., & Nason, R.. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Education studies in mathematics.*

Bell, A, Costello, J, & Kuchmann, D. (1983). *Research on Learning and Teaching. Part A.* London; NFER Nelson.

Bell, D., Hughes, E. & Rogers, J. (1975). *Area weight and volume.* London: Thomas Nelson & Sons.

Blane, D., & Booth, L. (1989). *Moving through maths: Year 8.* Melbourne: Rigby.

Clark, T.B., Clark, L., Burza, C.G., & Conway, D.T. (1988). *Maths 2000 Book 1.* Melbourne: The Macmillan Co. of Australia.

Clements, M.A. & Ellerton, N. (1995). Assessing the effectiveness of pencil-and-paper tests for school mathematics. In B. Atweh & S. Flavel (Eds.), *Proceedings of the 18th Annual Conference of the Mathematics Education Research Group of Australiasia* (pp. 184-188). Darwin, NT: Mathematics Education Research Group of Australiasia

Duffy, E. & Murty, G. (1988). *Active Mathematics 8.* Melbourne: Longman Cheshire.

Hirstein, J.J. (1981). The second national assessment in mathematics: Area and volume. *Mathematics Teacher, 74* (9), 704-707.

Hirstein, J.J., Lamb, C.E., & Osborne, A. (1978). Student misconceptions about area measure. *Arithmetic Teacher, 25* (6), 10-16.

Kidman, G. & Cooper, T.J. (1996a). Assessing the major trends and directions of research into students' judgements of area. In P.C. Clarkson (Ed) *Proceedings of the 19th Annual Conference of the Mathematics Education Research Group of Australiasia* (pp. 337-344). Melbourne: Mathematics Education Research Group of Australiasia.

Kidman, G. & Cooper, T.J. (1996b). Children's perceptual judgement of area. In P.C. Clarkson (Ed) *Proceedings of the 19th Annual Conference of the Mathematics Education Research Group of Australiasia* (pp. 330-336). Melbourne: Mathematics Education Research Group of Australiasia.

Lautrey, J., Mullet, E., & Paques, P. (1989). Judgements of quantity and conservation of quatity: The area of a rectangle. *Journal of Experimental Child Psychology, 47,* 193-209.

Outhred, L. & Mitchelmore, M. (1996). Children's intuitive understanding of area measurement. In L. Puig & A. Gutierrez (Eds.) *Proceedings of the 20th Conference of the international group for the Psychology of Mathematics* Education, 4(pp. 91-98). Valencia, Spain: University of Valencia.

Priddle, A.G. & Davies, T.H. (1989). *Mathematics 8*. Brisbane:Brooks Waterloo.

Schlottman, A., & Anderson, N.H. (1994). Students' judgments of expected value. *Developmental Psychology, 30* (1), 56-66.

Silverman, I.W.., & Paskewitz, S.L. (1988). Developmental and individual differences in students' area judgment rules. *Journal of Experimental Child Psychology, 46,* 74-87.

Simon, M. & Blume, G. (1994). Building and understanding multiplicative relationships: A study of prospective elementary teachers. *Journal for research in mathematics Education, 25,* 472-494.

Wolf, Y. (1995). Estimation of Euclidean Quantity by 5- and 6-year-old students: Facilitating a multiplication rule. *Journal of Experimental Child Psychology, 59,* 49-75.

Woodward, E., & Byrd, F. (1983). Area: Included topic, neglected concept. *School Science and Mathematics, 83* (6), 8-13.